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Kalman filter performance in DGPS-based sea manoeuvring trial system

A relatively high accuracy and reliability of the Differential GPS system makes the receivers of its signal a very good sensor of a ship position and velocity. Among the input from the gyrocompass, a complete, self-contained and portable ship manoeuvring trial system is obtained. However, the ship instant velocity vector is not measured directly but estimated through the application of the Kalman filter (one of its functions). In case of the ship manoeuvring, this leads to a systematic deviation of such the estimate as compared to the real motion. The magnitude of this bias is being thoroughly investigated for different ship manoeuvres. It appears essential in ship manoeuvring mathematical model identification and/or validation based on full-scale trials.

INTRODUCTION

Last decade developments in the GPS (Global Positioning System) techniques of position and velocity estimation, particularly in case of moving (dynamic) objects, caused a great progress in performing ship manoeuvring sea trials. Highly reliable and accurate, both differential (DGPS) and real-time kinematic (RTK-GPS) systems have nearly completely superseded local terrestrial hyperbolic systems – see e.g. [1, 5, 7, 8]. The primary objectives of carrying out the ship manoeuvring trials (during a ship delivery) are:

- a crosscheck whether the ship is controllable within accepted standards (IMO manoeuvring standards),
- providing a rough data to the ship's crew on the ship manoeuvring capabilities at full ahead speed and open sea (being internationally required, albeit of little value during manoeuvring in restricted waterways),
- demonstrating a compliance with the owner's specific demands.

Such trials results, by virtue of lack of another data, are also used to identify and validate ship manoeuvring mathematical models (by means of e.g. stochastic methods or the least square optimisation of the mathematical model structure and parameters, the latter is widely used in the hydrodynamic science).

The required accuracy of position and velocity estimates in view of the three basic aims is relatively, just against a common notion, not high. A quite different matter is the accuracy of a positioning system in aspects of their application to the ship manoeuvring mathematical model (MM) design – i.e. the higher the better. The both GPS methods – DGPS and RTK – are potentially sufficient here, the static standard position error is of ca. a few metres and a few centimetres correspondingly. The other element of the state vector i.e. the velocity (in terms of direction and magnitude) is not measured but computed from position updates (those very often at 1[s] intervals). The role of 'differentiator' is played by the Kalman filter (KF) – a part of the GPS receiver data processing software. Beside random errors (well recognised and documented), there are essentially two major sources of systematic errors (biases):

- firstly, a sea current (mostly unknown) at the trial site (a fundamental survey of the current uncertainty and its effect upon the ship manoeuvring was made e.g. by [3]), we are interested more in the water-related motions than those in the original ground-related coordinates, it shall be mentioned here that even a half knot (0.25 [m/s]) current could be 'destructive' for manoeuvring data,
- secondly, the Kalman filter behaviour – its stiffness due to improperly assumed parameters in the KF matrices, the utilised particular KF architecture is protected by GPS receivers' manufacturers, and as a rule, is generally without input from the user (this is a common problem of commercial systems used for scientific purposes), the KF stiffness mostly relates to the velocity vector estimate.

The latter KF problem, in aspects of the required high accuracy of the motion state estimate, is mainly attributed to the DGPS system and it seems to have a low effect upon e.g. the RTK receiver dynamic performance. The DGPS system is still very popular and also for reasons of a great number of manoeuvring trials collected so far, there is a need to study the phenomenon of KF bias.

The primary aim of the paper is to have just a closer look upon the systematic errors due to a possible low elasticity of the KF as experienced in the DGPS receiver output while the ship is performing rather rapid manoeuvres both in course and speed. The magnitude of these errors should be accounted for during the ship manoeuvring MM optimisation process. Before dealing directly with the mentioned KF bias, a short explanation of the water current errors is initially given an interest for purpose of a complete characterisation of sea trial data quality – this problem touches however all kinds of satellite positioning sensors independent of their accuracy.

All hereafter investigations are carried out through numerical simulations as sufficient methods to depict all essential properties of the problems, anyhow the input data to the simulation follows a real-world pattern to some extent.

1. *The sea current effect*

The correction (conversion) of ground-related data (directly from GPS) to water-related ones can be performed by the following expressions:

$$\begin{bmatrix} \upsilon_x^{\text{water}} \\ \upsilon_y^{\text{water}} \end{bmatrix} = \begin{bmatrix} \upsilon_x^{\text{ground}} \\ \upsilon_y^{\text{ground}} \end{bmatrix} - \mathbf{D}^T(\psi) \begin{bmatrix} |\upsilon_c| \cos \gamma_c \\ |\upsilon_c| \sin \gamma_c \end{bmatrix}$$

where:

$$\mathbf{D}^T(\psi) = \begin{bmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{bmatrix},$$

$|\upsilon_c|, \gamma_c$ – ocean current parameters (velocity and direction) in the earth-fixed system, $\gamma_c = 0°$ stands for the north current (i.e. moving the ship north),

υ_x, υ_y – ship forward (surge) and lateral (sway) velocities in body system of axes.

The sea current mostly affects the ship sway velocity υ_y, as usually of a lower value while compared to the surge velocity magnitude, and this way also the drift angle β_A (being a very important factor in hydrodynamic analyses) according to the formula:

$$\operatorname{tg} \beta_A^{\text{water}} = - \upsilon_y^{\text{water}} / \upsilon_x^{\text{water}}$$

Figure 1 shows a simulated turning track of a chemical tanker of ca. 100 [m] in length (initial speed 7 [m/s]) rudder angle 35[°], and the associated ground-related drift angle in case of the constant south and west current of 0.25 [m/s]. It should be kept in mind here that the ship executes the same manoeuvre through the water in all these cases. This ship example is used further in the study as a basis of next KF computations.

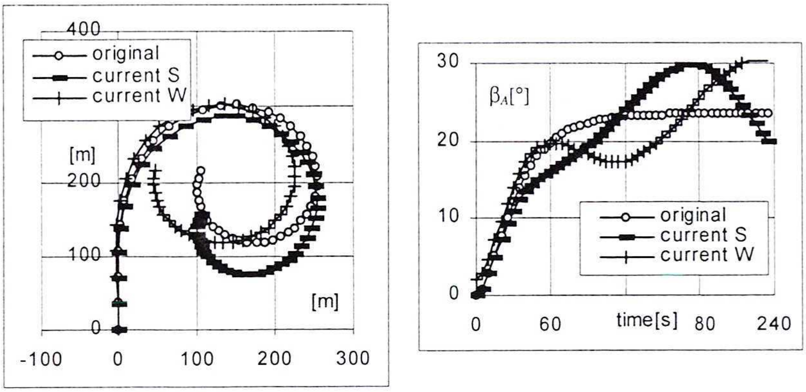

Fig. 1. Water current effect on turning test data

Another very good example of the sea current impact is the zigzag test where all motion states (displacements and velocities) are governed by some periodical non-harmonic functions. The current similarly affects here also the drift angle, specifically in a way the amplitude and phase lag is being altered. In hydrodynamic studies of the ship manoeuvring motion, of a major concern appears the correlation between the drift angle and the non-dimensional yaw velocity. In the zigzag test this relationship assumes the image of a closed 'hysteresis loop', rather narrow and prolonged one. The east or west current (the ship is assumed to proceed initially northwards) shifts this loop more towards a positive or negative region of drift angles, while the north (south) current governs a deflation/inflation of the loop. This phenomenon is more pronounced at the lower ship forward velocity. It shall be mentioned moreover that the sea current parameters at the trial site are comparatively harder to be assessed through the zigzag manoeuvre analysis than based on the turning test.

2. *Kalman filter operation based on DGPS measurements*

Let's assume a rather common 4-state KF architecture for stand-alone GPS sensors, as e.g. in [2], as follows:

$$\boldsymbol{x}^T = [\upsilon_{NS}, \upsilon_{EW}, x, y], \boldsymbol{z}^T = [x_{DGPS}, y_{DGPS}], \tag{1}$$

$$\begin{cases} \boldsymbol{x}_{k+1} = \boldsymbol{\Phi}\boldsymbol{x}_k + \boldsymbol{w} \\ \boldsymbol{z}_{k+1} = \mathbf{H}\boldsymbol{x}_{k+1} + \boldsymbol{\upsilon} \end{cases}, \tag{2}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \Delta t & 0 & 1 & 0 \\ 0 & \Delta t & 0 & 1 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

$$\mathbf{R} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \mathbf{Q} = \begin{bmatrix} \Delta t & 0 & \Delta t^2/2 & 0 \\ 0 & \Delta t & 0 & \Delta t^2/2 \\ \Delta t^2/2 & 0 & \Delta t^3/3 & 0 \\ 0 & \Delta t^2/2 & 0 & \Delta t^3/3 \end{bmatrix} S_p, \tag{4}$$

where:

$\boldsymbol{x}$ – state vector as earth-related velocity and position (the latter in [m] from a given origin),

$\boldsymbol{z}$ – measurement vector (position only),

Δt – time update step, assumed as 1 [s],

$\boldsymbol{\Phi}$ – state transition matrix,

$\mathbf{H}$ – measurement sensitivity matrix,

$\mathbf{Q}$, $\mathbf{R}$ – noise covariance matrices associated with the process and measurement random noises ($\boldsymbol{w}$ and $\boldsymbol{\upsilon}$ correspondingly),

S_p – spectral amplitude of PV random walk model.

The DGPS accuracy (see the **R** matrix) is represented here by the standard deviation (equal in north and east directions) of order 2 [m].

Due to the fact that matrices **Φ**, **H**, **R**, and **Q** are time-invariant, the well-known Kalman solution in the recursive form is:

– error covariance **P** and gain **K** matrices:

1. $\mathbf{P}_{k+1} = \mathbf{\Phi P}_k \mathbf{\Phi}^T + \mathbf{Q}$
2. $\mathbf{K}_{k+1} = \mathbf{P}_{k+1}\mathbf{H}^T[\mathbf{HP}_{k+1}\mathbf{H}^T + \mathbf{R}]^{-1}$
3. $\mathbf{P}_{k+1} = [\mathbf{I} - \mathbf{K}_{k+1}\mathbf{H}]\mathbf{P}_{k+1}$ and back to step 1.

– state estimate $\boldsymbol{x}$:

1. $\boldsymbol{x}_{k+1} = \mathbf{\Phi}\boldsymbol{x}_k$
2. $\boldsymbol{x}_{k+1} = \boldsymbol{x}_{k+1} + \mathbf{K}_{k+1}[\boldsymbol{z}_{k+1} - \mathbf{H}\boldsymbol{x}_{k+1}]$ and back to 1.

After a relatively rapid convergence of $\mathbf{P}_k$ and $\mathbf{K}_k$ matrices into $\mathbf{P}_\infty$ and $\mathbf{K}_\infty$ (the intial investigations show that even less than 30 cycles are just enough to be computed off-line and being practically independent from the initial $\mathbf{P}_0$ elements), the only state evolution needs to be estimated- see also e.g. [4], [6]:

$$\boldsymbol{x}_{k+1} = \mathbf{\Phi}\boldsymbol{x}_k + \mathbf{K}_\infty[\boldsymbol{z}_{k+1} - \mathbf{H}\cdot\mathbf{\Phi}\boldsymbol{x}_k] \quad (5)$$

All subsequent calculations are based upon the above equation, substituting each time a newly converged gain matrix as connected with the S_p parameter in **Q** matrix.

If **Q** is approaching zero, the ship manoeuvring (generally non-linear) could be considered a deterministic one i.e. able to be modelled by physical laws with the **Φ** matrix as the dynamic model. However, due to the linear kinematic model assumed in **Φ** matrix (as of low accuracy in the ship manoeuvring- the velocities are obviously evaluated as being constant all the time), we need to take account for it just by increasing **Q** elements. This action enables to 'follow' more accurately motion velocities as being variable during manoeuvring. Three characteristic **Q** cases, represented by S_p scalar, are possible here:

- S_p too low – systematic errors occur both in position and velocity, no noise is experienced in outputs because the over-smoothing takes place,
- S_p optimal – no bias in position appears but still exists in velocity, some noise (higher for velocity) is observed,
- S_p too high – a small bias in velocity is additionally achieved, anyhow a very high noise both in position and velocity (no filtering at all) is the fact.

Based on the common sense, no distortion to the originally measured *x-y* position (*x* means a ship translation northwards, while the *y* indicates her translation eastwards) is the highest priority while selecting S_p value. In view of the dynamics of our ship (the same chemical tanker is analysed further), an optimal S_p is around 0.01. Figure 2 and 3 present the simulation of the KF performance in the adopted above specific instances of its matrices. The predicted (by means of a verified hydrodynamic mathematical model) turning test data at half ahead speed and 65° rudder (of Schilling type) is brought into play here. Other architectures of the KF were also analysed, constituting e.g. of the 6-state vector comprising accelerations and the PVA model of the **Q** matrix. The problems have become more or less similar.

Reverting to Fig. 2 and 3, in all the cases an identical normal random noise sequence (by means of a random number generator) was imposed upon the original *x-y* data thus providing a simulation of the measurement vector *z*. Despite the *x-y* position (the trajectory in the left part of Fig. 2, the *x*-axis points up), the following variables are also displayed:

$$v_{xy} = \sqrt{v_{NS}^2 + v_{EW}^2} = \sqrt{v_x^2 + v_y^2} \tag{6}$$

$$\beta_A = \mathrm{arctg}(-v_y / v_x) = \psi - \mathrm{arctg}(v_{EW} / v_{NS}) \tag{7}$$

where:

v_{xy} – the magnitude of total velocity vector (the track velocity),

β_A – the drift angle (the actual course angle ψ is supplied to this formula) as representing in a certain manner the track direction.

Figure 2 illustrates that at relatively low S_P values a good smoothing is reached both in velocity and direction. Anyhow, those properties are occupied by the undesirable systematic errors in the trajectory – see Fig. 2 (the left part), where these errors assume even 10 [m] – the *x*-*y* estimates follow too much the assumed linear kinematic model of the ship dynamics (represented by the ϕ matrix).

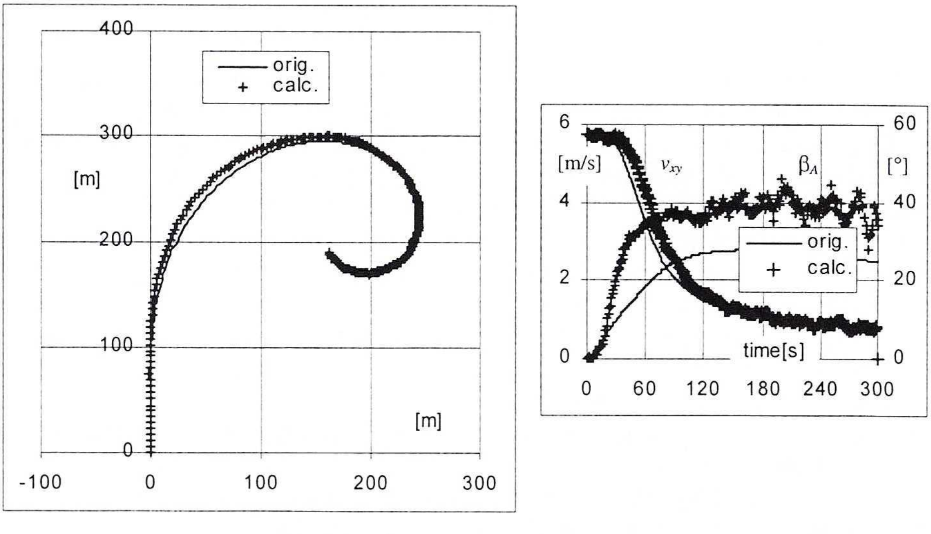

Fig. 2. KF performance at $S_P = 0.001$ – HAH 65°

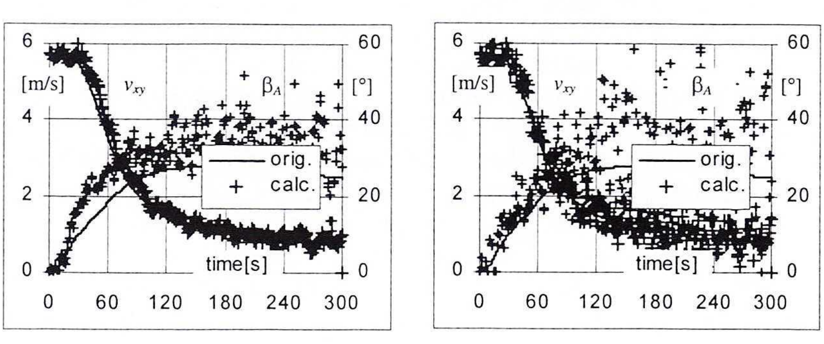

Fig. 3. KF performance at $S_P = 0.01$ (left) and $S_P = 0.1$ (right) – HAH 65°

The optimal tune-up of the KF in our DGPS-based exemplary architecture (i.e. at $S_P = 0.01$) gives maximum biases of order 0.5 [m/s] in velocity and 10 [°] in drift angle as observed at the initial stage of the manoeuvre – Fig. 3 (the left part). Both are significant in the ship manoeuvring MM identification and/or validation. The *x-y* trajectory is not included here because it is not systematically distorted, however there is a little noise filtration in it (the *x-y* estimates follow highly the measured values). In case the S_P value is much lower (e.g. 0.1), the bias in velocity and direction goes down, but both are of no practical advantage due to the noise „amplification" – see Fig. 3 (the right part).

3. *Other examples of manoeuvres*

Below there are a few principal manoeuvres studied, which are often included in the sea trial program. This is also the case for the chemical tanker in our concern, where the following runs were made:

- turning test at 35° starboard rudder,
- zigzag test 20°/20°,
- crash stop (full astern propeller pitch), the ship alters the course to starboard due to the propeller lateral thrust,
- coasting stop (zero pitch), the ship turns to port for the same reasons.

All of them are relevant to the initial full ahead speed (FAH) and are illustrated in Fig. 4 to 7. The same computation scheme is used throughout as before in this study. The originally measured *x-y* track data (after being transformed to the origin amidships) are assumed however to constitute a basis for the simulation of the measurement vector $\boldsymbol{z}$ (the appropriate noise of the standard deviation 2 [m] is to be imposed only). The optimal value of S_P parameter in the $\boldsymbol{Q}$ matrix, equal to 0.01, is taken as the reference to the KF performance calculations.

The major points to be emphasised here from the analysis of Fig. 4 to 7 are:

- the velocity υ_{xy} being output from the DGPS-KF could be higher than in reality when the manoeuvre experiences a speed loss, in other more general words – the KF loses the velocity magnitude dynamics, the absolute bias is sometimes of order one knot (0.5 [m/s]), however during e.g. the steady stages of turning tests (the overall linear velocity is constant) the bias is not conspicuous, but the directly estimated background state components υ_{NS} and υ_{EW} (changing according to sine and cosine functions) are prone to a horizontal shift as compared to the original curves,
- the computed drift angle is always higher (by even 10 [°], independent nearly from the type of manoeuvre) than the actually existing one, this bias seems to be more important than the error in velocity while identifying or validating the ship manoeuvring mathematical model.

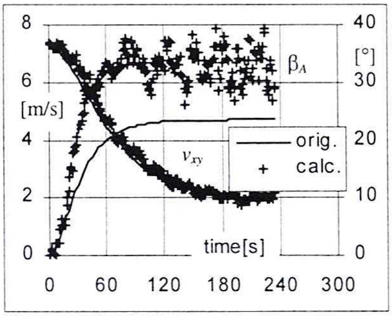

Fig. 4. Turning test FAH35°

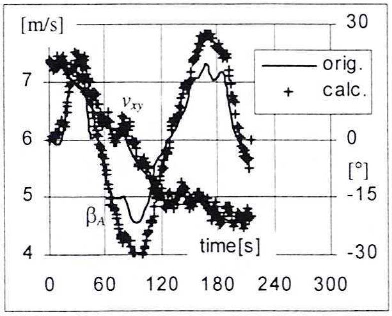

Fig. 5. Zigzag test FAH 20°/20°

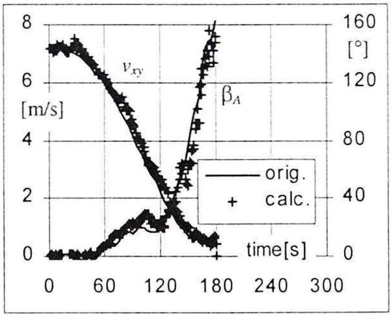

Fig. 6. Crash stop (FAH-FAS)

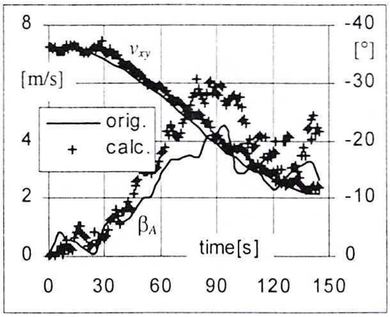

Fig. 7. Coasting stop (FAH-STOP)

CONCLUSIONS

In the present work, the problem of the velocity vector (in terms of its direction and magnitude) as estimated through the Kalman filter has been undertaken with regard to the acquired DGPS-based ship positions. Though the focus has been made on manoeuvring sea trials and the mathematical model identification, the raised issues invoke also the real-time ship state estimation and control tasks as well.

As the instant pivot point of a ship performing rudder/engine manoeuvres is moved somehow near to the bow, the antenna aft location experiences more rapid manoeuvres than the midship. If such a data is to be analysed, the bias in velocity vector would be more pronounced. This should be born in mind while interpreting the original velocity estimate – as the GPS receiver (and thus the KF contained therein), normally processes geographical co-ordinates of the antenna (as located aft) positions.

To avoid any bias in the velocity vector, it is recommended either:

- to bypass the KF in the DGPS receiver (if such an interference is possible) and post-process the unfiltered *x-y* position data stochastically in the off-line mode (by another KF or other methods), or
- to post-process only the KF position output in view of the velocity vector estimate.

In the latter case, a direct numerical differentiation of the *x-y* data seems to supply better results.

To verify the points of the paper, it is also suggested to perform parallel comparative trials of the DGPS and RTK-GPS receivers onboard a ship sustaining hard manoeuvres – the RTK system to be treated as the benchmark.

The assumed value 0.01 for S_P, though keeping the unbiased (unfouled) x-y track, does not remove the noise from the velocity estimate. As a lot of DGPS velocity data are rather smooth, the question of the bias magnitude also in position is of utmost importance.

REFERENCES

[1] Bieniek N., Cieszynski A., *The System for Testing Ship Manoeuvrability and Propulsion Properties Based on NAVSTAR-GPS Global Satellite System.* Polish Maritime Research, vol. 2, no. 1, pp. 14-17, 1995.

[2] Brown R.G., Hwang P.Y.C., *Introduction to Random Signals and Applied Kalman Filtering (with Matlab Exercises and Solution).* 3rd edition, John Wiley&Sons, New York, 1997.

[3] Crane C.L., *Maneuvering Trials of a 278,000 DWT Tanker in Shallow and Deep Waters.* SNAME Transactions, vol. 87, pp. 251-283, 1979.

[4] Grewal M.S., Andrews A.P., *Kalman Filtering: Theory and Practice.* Prentice Hall, Upper Saddle River (N.J.), 1993.

[5] Hirata N., Kose K., et al., *Application of a Kinematic GPS to Ship Maneuvering Trials.* WJSNA Transactions, no. 94 (Aug), pp. 111-120, 1997 (in Japanese).

[6] Minkler G., Minkler J., *Theory and Application of Kalman Filtering.* Magellan Book Company, Palm Bay, 1993.

[7] Pinkster J., Journee J.M.J., *Sea Trials with Regard to Design and Operational Limits of Fast Pilot Vessel MS Voyager.* International Conference „Surveillance, Pilot and Rescue Craft for the 21st Century' (SURV-IV), May 13-14, Gothenburg (Sweden), 1997.

[8] Yum D.J., Lee T.I., Lee H.Y., Lee H.W., *New Manoeuvring Sea Trial System Using DGPS.* Marine Simulation and Ship Manoeuvrability (Chislett M.S. (ed.), Proceedings of MARSIM '96 Conference, Sep 9-13, Copenhagen), A.A. Balkema, Rotterdam, pp. 333-340, 1996.

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Efektywność filtru Kalmana w systemie pomiarowym DGPS charakterystyk manewrowych statku

Streszczenie

Względnie wysoka niezawodność i dokładność (rzędu kilku metrów) różnicowego systemu pozycyjnego GPS (DGPS) sprawiła, że jest on szeroko wykorzystywany do wyznaczania charakterystyk manewrowych statku w czasie prób morskich. Typowy system pomiarowy składa się z przenośnego odbiornika DGPS, komputera i łącza do statkowego żyrokompasu. Wraz z informacją o pozycji statku, z samego odbiornika DGPS bardzo często jest również pobierany chwilowy wektor prędkości. Problem polega na tym, że ten wektor nie jest mierzony bezpośrednio, lecz estymowany (wyliczany) poprzez filtr Kalmana. To powoduje dość istotne błędy systematyczne w prędkości, szczególnie gdy zarejestrowane dane są wykorzystywane do identyfikacji czy też weryfikacji matematycznych modeli manewrowania statkiem. Niniejsza praca zawiera analizę błędów wektora prędkości, stosując symulację pracy filtru Kalmana, dla różnych manewrów wchodzących w zakres morskich prób manewrowych. Zasygnalizowano także ryzyko powstania systematycznych błędów nawet w pozycji statku, jeśli niewłaściwie dobrano parametry filtracji.

Ярослав Артышук

Эффективность фильтра Кальмана в системе измерений DGPS манёвренных характеристик судна

Резюме

Относительно высокая надёжность и точность (порядка нескольких метров) дифференциальной позиционной системы GPS (DGPS) привели к тому, что она широко используется для определения манёвренных характеристик суда во время морских испытаний. Типовая измерительная система состоит из портативного (переносного) приёмника DGPS, компьютера и канала к гирокомпасу судна. Вместе с информацией о позиции суда из самого приёмника DGPS очень часто берётся также кратковременный векто скорости. Проблема заключается в том, что этот вектор не измеряется непосредственно, а вычисляется с помощью фильтра Кальмана. Это вызывает довольно существенные систематические ошибки в скорости, особенно, когда зарегистрированные данные используются для идентификации или же верификации (проверки) математических моделей маневрирования судном. Настоящая работа содержит анализ ошибок вектора скорости, с применением симуляции работы фильтра Кальмана, для различных маневров, входящих в объём морских манёвренных испытаний. Сигнализируется также риск возникновения ошибок даже в позиции суда, если неправильно подобраны параметры фильтрации.