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Identification of maximal complex boundaries on the basis of subregion description with directed graphs (digraphs)

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Abstract: The objective of the present study was to develop a method for automatic identification of boundaries of maximal complexes on the basis of boundary points of subcomplexes. According to the method proposed, regions are to be presented by means of geometric cyclic digraphs. The data on these digraphs are to be recorded in a neighbourhood matrix. The matrix notation, containing supplementary data, provides information sufficient to determine complexes according to the criteria adopted. The paper presents simple algorithms for data processing, enabling to detect data inconsistency.

Keywords: Region as a cyclic graph

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1. Introduction

The administrative division of Poland is multistage, comprising the following units: state, province, county, commune, district, registration parcel. There are also other systems of geographical space division into subregions, defined with boundary points. Lists of such boundary points are kept individually for particular units, which may result in certain divergences between datasets. In order to ensure the consistency and integrity of data and databases, complexes should be generated on the basis of boundaries of subregions forming these complexes.

The source lists of the coordinates of boundary points of various regions sometimes contain errors. It follows that adequate tools enabling their detection are needed. GIS provides certain tools to control the correctness of data entered into the system. However, the control is based on the results of a search within the neighbourhood of a given point only (Autodesk, 2000; Bentley, 2002; Esri, 2003). It would be better to detect errors before data are loaded into GIS. A tool applied for this purpose should allow for verifying lists of coordinates of administrative division units, and for introducing all necessary corrections.

This paper presents a method for determining boundary lines of maximal complexes on the basis of boundary points of subcomplexes. The method proposed is naturally connected with an internal control of spatial data consistency. This control is based on *a priori* structural properties of graphs. They result directly from the method of describing elementary components of complexes, adopted in the study, as well as from replacing selected directed edges with normal edges.

The introductory part of the paper provides general information on graphs, followed by a description of the method proposed. Theoretical considerations are illustrated by an example.

2. General information on graphs

The fundamentals of the graph theory can be found in mathematics (Ross and Wright, 2000) and computer science (Loudon, 1999; Gedgewick, 2003) textbooks. Graphs are generally associated with spatial analyses. The graph theory provides tools for solving specialized tasks, including travelling salesman problem, path analysis, network flow (Reingold et al., 1998; Grover, 2003; Wilson, 2000; Ford and Fulkerson, 1969). In geographical information systems (GIS), in the case of spatial data modelling (Molenaar, 1998), graphs are used for the visualization of topological data. They are also applied for representing neighbourhood relations between objects in topological network and area models (Gaździcki, 1990; 2001).

2.1. Ordinary graphs

Graphs are defined (Wilson, 2000; Kulikowski, 1986) as abstract structures representing a set of nodes and edges. Some nodes are interconnected by edges. If the coordinates of nodes are known, such a graph is referred to as geometric and can be represented graphically. From the mathematical perspective, graphs are objects described by triplets of ordered information:

$$G = [W, K, \varphi], \tag{1}$$

where W is a set of nodes, K is a set of edges, and φ is a mapping function of the Cartesian product in the form:

$$\varphi : W(G) \times W(G) \to K \tag{2}$$

indicating nodes belonging to edges, since edges have their origins and destinations in nodes. Mapping function φ, referred to as a neighbourhood relation, is a symmetric and reflexive relation. If there is a neighbourhood relation between nodes w_1w_2, there is also a neighbourhood relation between nodes w_2w_1, i.e.:

$$(w_1w_2) \in \varphi \Rightarrow (w_2w_1) \in \varphi \tag{3}$$

The neighbourhood relation φ may be expressed as a neighbourhood matrix, which is a symmetric $n \times n$ matrix, where n is the number of graph nodes. While generating this matrix for a graph, its nodes should be denoted by e.g. numbers from a set $\{1, 2, ..., n\}$. The element of the matrix with indices i, j corresponds to the number of edges connecting the node w_i to the node w_j.

2.2. Directed graphs – digraphs

Apart from graphs defined as sets of nodes and edges (Kulikowski, 1986), there exist directed graphs – digraphs. They consist of nodes and directed edges. Each directed edge has an origin and destination. In professional literature on the topic such edges are referred to as arcs. Digraphs are generally drawn with arrows on edges, to indicate a sense of direction. The notation of such a graph is presented as a mapping function φ:

$$G = [W, L, \varphi] \tag{4}$$

$$\varphi : W(G) \times W(G) \rightarrow L \tag{5}$$

where L is a set of arcs (directed edges) with a given origin and destination $\{(w_i, w_n) = l_{in}\} \subset L$. The mapping function may also be presented by means of a neighbourhood matrix, which in this case does not have to be a symmetric matrix.

If a given graph contains two arcs (directed edges) with opposite senses between the same nodes, they can be expressed as edges of an ordinary, non-directed graph. Such a situation can be described as follows:

$$\text{if } \varphi\,[w_1, w_2] = l_{ij} \text{ and } \varphi\,[w_2, w_1] = l_{ji} \Rightarrow \{l_{ij}, l_{ji}\} = k \subset K \tag{6}$$

A digraph may consist of nodes, ordinary edges and directed edges, assuming the form:

$$G = [W, K, L, \varphi] \tag{7}$$

where W is a set of nodes, K is a set of ordinary edges, L is a set of directed edges, and φ is a mapping function in the form:

$$\varphi : W \times W \rightarrow K \cup L \tag{8}$$

2.3. Cyclic graphs

Cyclic graphs are mappings α of a sequence of nodes $\{w_1, w_2, ..., w_n, w_1\}$. In this sequence the initial node w_1 comes also at the end. The mapping α allows for assigning an edge to each pair of neighbouring nodes in a sequence. In the case of a cyclic directed graph (cyclic digraph) $C = [W, L, \alpha]$, the mapping α allows for assigning directed edges $\{(w_1, w_2), (w_2, w_3), ..., (w_{n-1}, w_n), (w_n, w_1)\} \in L$ to a sequence of nodes $\{w_1, w_2, ..., w_n, w_1\}$: $w_i \in W$. Cyclic digraphs, just like ordinary graphs, can be represented geometrically provided that their nodes have specified coordinates.

A cyclic graph (digraph) may take the form of a simple cycle if none of its edges (directed edges) occurs more than once. Two cycles C' and C'' are referred to as mutually independent (Kulikowski, 1986) if they have no common edges (directed edges), and mutually strongly independent if they have no common nodes.

2.4. Graphs as carriers of supplementary information

Graphs can be presented in the following general form (Kulikowski, 1986):

$$K = [G, D, C] \tag{9}$$

where $G = [W, K, L, \varphi]$ is an ordinary graph if $L = \{\varnothing\}$, or a directed graph if $K = \{\varnothing\}$, D and C are supplementary functions describing graph elements by means of characteristics. These characteristics can be properties, or attributes of real objects, represented by nodes and edges of graphs.

D is a function:

$$D: W \to Y_i \tag{10}$$

which assigns values (function arguments, i.e. certain physical parameters of geographical space elements) to nodes:

$$\left.\begin{array}{c} y_1 = f_1(x) \\ y_2 = f_2(x) \\ \vdots \\ y_i = f_i(x) \end{array}\right\} \tag{11}$$

This allows to generate a vector of node attributes $Y_i = [y_1, y_2, ..., y_i]$.

Similarly, C is a function:

$$C: K \to Y_j \tag{12}$$

which assigns values y_j to edges. This allows to generate a vector of edge attributes $Y_j = [y_1, y_2, ..., y_j]$.

All of the considered functions of attributes, assigned to nodes and edges (directed edges), should be defined in a uniform database. In the case of edges, such a base is a set of graph vertices. The selection of concrete values of node and edge functions must be preceded by the selection of a subgraph $A \subset G$ fulfilling the set values of a function f_i.

3. Maximal complex

Regions, as all units of territorial-administrative division, can be described by defining a set of their boundary points. Let us assume that a sequence of points is consistent with a boundary line, and the direction of this boundary line is the clockwise direction in which the sequence of points defining the boundary is specified. Let us also assume that a graphic representation of the region discussed is a cyclic directed graph. The boundary points are nodes of this graph, and the boundary lines – directed edges define cyclic digraph $C = [W, L, \alpha]$.

Theoretically, a complex of regions G_{CR} is a sum of subregions, i.e.:

$$G_{CR} = \bigcup_{i=1}^{n} C_i = \bigcup_{i=1}^{n} [W_i, L_i, \alpha] = [W, L, \varphi] = [W, K, L, \varphi] \tag{13}$$

where i is an identifier of a region within the complex, $i = \{1, 2, ..., n\}$. A complex G_{CR} of n regions is also a region, so it can be presented as a digraph $G_{CM} \subset G_{CR}$ that might be represented by a cyclic digraph. A graph describing the boundaries of a complex of regions can be derived from matrix notations of digraphs representing particular subcomplexes.

A maximal complex can be a monoconnected region, or it can consist of many components represented by disconnected or node-connected subregions. In special cases subregions may include enclaves. If a maximal complex is defined as a region characterized by the properties determined by function (13), one obtains a graph $G_{CM} \subset G_{CR}$ whose components have the form of mutually independent simple cycles.

$$G_{CM} = \begin{cases} C_{CM} & \textit{(when } G_{CM} \textit{ is a monoconnected region)} \\ \sum_{p=1}^{n} C_{p_{CM}} & \textit{(when } G_{CM} \textit{ is a n-component region)} \end{cases} \tag{14}$$

3.1. Principles of defining maximal complexes

Let the registration parcels presented in Fig. 1 be an example of space division into regions. This is a simple example of dividing a connected complex into subcomplexes.

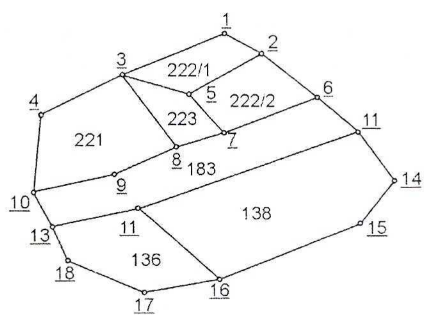
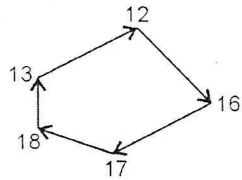


Fig. 1. Fragment of a cadastral map as an example of units of the territorial-administrative division of Poland

Generally speaking, a complex may constitute any planar composition of elemental subcomplexes (subregions). Let us assume that to each registration parcel may be assigned a set of coordinates, and a digraph $\bigcup_{i=1}^{n} C_i$, where n is the number of parcels within the complex. Figure 2 shows the graphic representation and matrix notation of such a cyclic digraph, $C = [W, L, \alpha]$, for a land parcel with a registration number 136.



$$
\begin{array}{c|ccccc}
 & 13 & 12 & 16 & 17 & 18 \\
13 & 0 & 1 & 0 & 0 & 0 \\
12 & 0 & 0 & 1 & 0 & 0 \\
16 & 0 & 0 & 0 & 1 & 0 \\
17 & 0 & 0 & 0 & 0 & 1 \\
18 & 1 & 0 & 0 & 0 & 0
\end{array}
$$

Fig. 2. Land parcel with a registration number 138 represented by a cyclic digraph and a neighbourhood matrix

A digraph of a complex of land parcels G_{CR} is a sum of cyclic digraphs defined for each of the parcels. This sum can be described by formula (13) and illustrated graphically (Fig. 3).

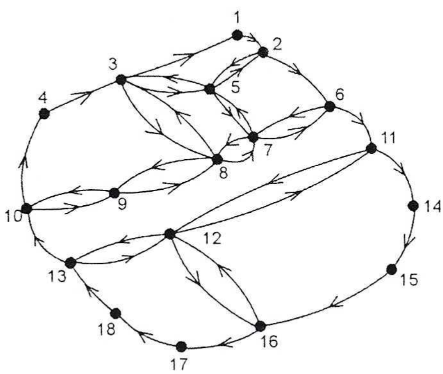


Fig. 3. Graph representing land parcels as a sum of directed subgraphs describing particular parcels

The digraph of a complex of land parcels, defined by mapping function $\varphi(W)$, may also be described by a neighbourhood matrix, which is a non-symmetric matrix as it defines directed edges not all of which have their counterparts with the opposite sense of direction. These directed edges are underlined in Fig. 4.

$$
\begin{array}{c|cccccccccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\
1 & 0 & \underline{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 & \underline{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & \underline{1} & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & \underline{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \underline{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 0 & 0 & 0 & \underline{1} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \underline{1} & 0 & 0 & 0 & 0 \\
12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \underline{1} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \underline{1} & 0 & 0 & 0 \\
15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \underline{1} & 0 & 0 \\
16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \underline{1} & 0 \\
17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \underline{1} \\
18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \underline{1} & 0 & 0 & 0 & 0 & 0
\end{array}
$$

Fig. 4. Neighbourhood matrix for the parcels shown in Fig. 1 and Fig. 3

If in this graph two directed edges with opposite senses, based on the same nodes, were replaced with edges of an ordinary graph, one would obtain the graph shown in Fig. 5 that can be described by the formula $G_{CR} = [W, K, L, \varphi]$.

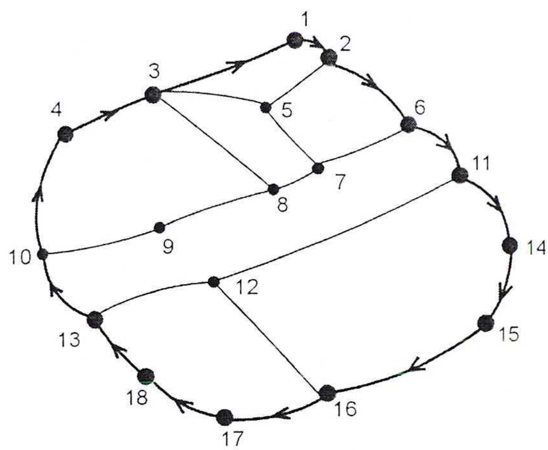


Fig. 5. Directed graph representing the contour line of a complex of land parcels obtained automatically on the basis of a graph being a sum of cyclic graphs describing particular parcels

Separating from this graph directed edges and nodes being the origins or destinations of directed edges one obtains a digraph $G_{CM} = [W_1, L_1, \varphi_1] \subset G_{CR}$. If data was ordered, the graph G_{CM} should be a cycle $C_{CM} = [W, L, \alpha]$ since it describes the monoconnected contour line of a complex of land. If one deletes the columns and rows containing data on edges from the neighbourhood matrix, and leave directed edges, one obtains a matrix describing the contour line of a complex of land parcels (Fig. 6a). To make legible matrix notation of a cycle, the sequences of columns and rows should be ordered. The process of graph matrix ordering may be presented as a simple cycle algorithm involving permutation of rows and columns aimed at arranging the vertices into a simple cycle sequence. In this example a simplified form of the algorithm can be used, since the maximal complex is described by means of a monoconnected region. This algorithm consists in determining the position for a value equal to 1 in consecutive rows of the matrix. Let us start searching in the first row ($i = 1$). The value 1 can be found in column j. Then column j is shifted to the position $i + 1$, i.e. 2. Analogically, the row identified by the value j is shifted to the position $i + 1$. After this modification, one can search for another value 1 in the next row ($i + 1$). When it is found, the process of matrix modification must be repeated. These operations are carried out until the value 1 is found in the second last row of the modified matrix. The result shown in Fig. 6 confirms that spatial data recorded in the form of graphs can be processed to obtain new data.

The above process replaces operations on sums of sets and allows for deriving the boundary line of a complex of land parcels in a simple way. This procedure can also be applied to documentation of land registration, in order to verify the correctness of description of particular parcels forming a complex, and the determination of the boundaries of this complex. This procedure enables to detect any errors in data records. If such errors occur, it is not possible to generate a cyclic graph. Matrix representation of the graph helps to reveal errors that can be visualized graphically.

a)

$$
\begin{array}{c|c}
 & \begin{array}{ccccccccccccc} 1 & 2 & 3 & 4 & 6 & 10 & 11 & 13 & 14 & 15 & 16 & 17 & 18 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 6 \\ 10 \\ 11 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \end{array} &
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

b)

$$
\begin{array}{c|c}
 & \begin{array}{ccccccccccccc} 1 & 2 & 6 & 11 & 14 & 15 & 16 & 17 & 18 & 13 & 10 & 4 & 3 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 6 \\ 11 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 13 \\ 10 \\ 4 \\ 3 \end{array} &
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

Fig. 6. Neighbourhood matrices describing the contour line of land parcels:
A – in the unordered form, *B* – in the ordered form

The information on the boundary line (new data) should be recorded permanently. It can be for example reproduced, i.e. recorded independently in the form of a new digraph with a full description. However, a better solution is to save the new data in the existing digraph, assigning supplementary characteristics to the elements of this digraph. In the solutions known to the author (Esri, 2003; Autodesk, 2000), the newly determined boundaries of a complex are recorded independently.

4. Region selection on the basis of data recorded in vectors of characteristics

As mentioned in Section 2.3, supplementary characteristics in the form of vectors of data can be assigned to nodes and edges of a digraph. This allows for assigning information on the type of boundaries to nodes and directed edges determining them. For instance, attributes informing about the boundary lines of the whole complex (e.g. a district, a commune, other geographical regions) can be assigned to nodes and edges representing the boundary lines of a land parcel. A digraph with characteristics providing information on the type of boundaries enables to choose any boundaries. Thus, it is also possible to choose any subcomplexes, if only data on their boundaries are included in the vectors of characteristics assigned to nodes and directed edges.

The selection of boundaries of parcel complexes results from adopting certain values of function f_k (12) which is connected with selecting subgraphs $A^{(k)} \subset G$ described with neighbourhood matrices $A^{(1)}, A^{(2)}, ..., A^{(k)}$. Defining the sums, intersections or differences of neighbourhood matrices $A^{(1)}, A^{(2)}, ..., A^{(k)}$ one can determine a matrix corresponding to many choices $f_1, f_2, ..., f_k$. The operations:

$$A^{(1)} \cup A^{(2)} = A^{(1+2)}, \quad A^{(1)} \cap A^{(2)} = A^{(1*2)}, \quad A^{(1)} / A^{(2)} = A^{(1/2)}$$

should be treated as determinations of the sum, intersection and difference of sets of matrix elements, respectively. For a sum of matrices:

$$A^{(1)} \cup A^{(2)} = A^{(1+2)} \tag{15}$$

$a_{ij}^{(i+j)} = 1 \Leftrightarrow (a_{ij}^{(1)} = 1 \text{ or } a_{ij}^{(2)} = 1)$; when both matrices assume the value 0, then $a_{ij}^{(1+2)} = 0$. Indices i, j correspond to row and column labels, which result from the adopted node enumeration, consistent with the enumeration of boundary points.

For an intersection of matrices

$$A^{(1)} \cap A^{(2)} = A^{(1*2)} \tag{16}$$

$a_{ij}^{(1*2)} = 1 \Leftrightarrow (a_{ij}^{(1)} = 1 \text{ or } a_{ij}^{(2)} = 1)$; in the other cases $a_{ij}^{(1*2)} = 0$.

For a difference of matrices:

$$A^{(1)} / A^{(2)} = A^{(1/2)} \tag{17}$$

$a_{ij}^{(1/2)} = 1 \Leftrightarrow (a_{ij}^{(1)} = 1 \text{ and } a_{ij}^{(2)} = 1)$; in the other cases $a_{ij}^{(1/2)} = 0$.

The operations: $A^{(1)} \cup A^{(2)} = A^{(1+2)}$ and $A^{(1)} \cap A^{(2)} = A^{(1*2)}$ correspond to a sum and intersection of sets, so they are permutable and associative:

$$A^{(1)} \cup A^{(2)} = A^{(2)} \cup A^{(1)} \tag{18}$$

$$(A^{(1)} \cup A^{(2)}) \cup A^{(3)} = A^{(1)} \cup (A^{(2)} \cup A^{(3)}) \tag{19}$$

$$A^{(1)} \cap A^{(2)} = A^{(2)} \cap A^{(1)} \tag{20}$$

$$(A^{(1)} \cap A^{(2)}) \cap A^{(3)} = A^{(1)} \cap (A^{(2)} \cap A^{(3)}) \tag{21}$$

$$(A^{(1)} \cup A^{(2)}) \cap A^{(3)} \cap A^{(4)}) = ((A^{(1)} \cap A^{(3)}) \cap A^{(4)}) \cup (A^{(2)} \cap (A^{(3)} \cap A^{(4)})) \tag{22}$$

Matrix elements resulting from k multiple operations, indicate the existence of connection between nodes i and j when

$$a_{ij}^{(1*2*....*k)} = 1 \Leftrightarrow (a_{ij}^{(1)} = 1 \text{ or } a_{ij}^{(2)} = 1, ..., a_{ij}^{(k)} = 1) \tag{23}$$

$$a_{ij}^{(1+2+,\ldots,+k)} = 1 \Leftrightarrow \left(a_{ij}^{(1)} = 1 \text{ or } a_{ij}^{(2)} = 1 \text{ or } ,\ldots, a_{ij}^{(k)} = 1\right) \tag{24}$$

$$a_{ij}^{(((1/2)/\ldots)/k)} = 1 \Leftrightarrow \left(a_{ij}^{(1)} = 1,\ a_{ij}^{(2)} = 0,\ a^{(3)} = 0,\ \ldots,\ a^{(k)} = 0\right) \tag{25}$$

The matrices $A^{(1)}, A^{(2)}, \ldots, A^{(k)}, A^{(1+2)}, A^{(i+j+k)}, A^{(i+\ldots+k)}$, as well as $A^{(1*2)}, A^{(i*l*k)}, A^{i*\ldots*k)}$, or even $A^{((1+2)*3)}, A^{((1*3)+(2*3)/4))}$, representing the results of operations on sets. They describe a graph $G_{CR}^{(i,\ldots,k)}$ with nodes and directed edges as well as with ordinary edges:

$$A^{(i,\ldots,k)} = [W, K, L, \varphi] \tag{26}$$

Deleting from the matrix $A^{(i,\ldots,k)}$ those rows and columns which contain zeros only and columns and rows describing ordinary edges (K), and leaving only the columns and rows describing directed edges, one can obtain a matrix notation of a maximal complex $A_{CM}^{(i,\ldots,k)} = G_{CM}^{(i,\ldots,k)}$, representing monoconnected regions (C_{CM}), or compositions $\left(\sum C_p\right)$ of disconnected or node-connected components, or even components with enclaves (Fig. 7):

$$A^{(i,\ldots,k)} \xrightarrow{\ \ ordered\ \ } A_{CM}^{(i,\ldots,k)} = G_{CM}^{(i,\ldots,k)} = \sum C_{p_{CM}} = \Sigma[W, L, \alpha] \tag{27}$$

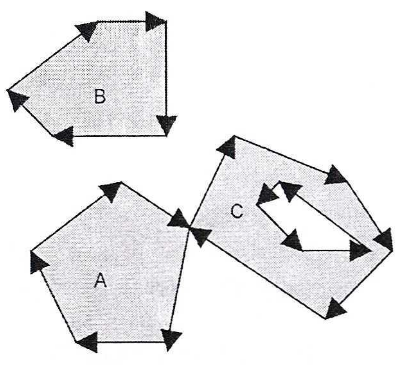


Fig. 7. Examples of components of a maximal complex: A and B – disconnected components, A and C – node-connected components, component C with an enclave (hole) not belonging to the complex

All components should be generated from the matrix $A_{CM}^{(1,\ldots,k)}$ in the form of a simple cycle. If a complex consists of one component only, one can apply the simple cycle algorithm presented above. In a general case, it is important to know whether a given maximal complex contains node-connected components. Thus, it is necessary to determine the degree of digraph nodes $A_{CM}^{(1,\ldots,k)}$, i.e. the number of edges that touch this node. If there are no nodes with $\deg(w) > 1$, the complex does not contain node-connected components, but it may contain disconnected components. In such a case, by ordering the matrix $A_{CM}^{(1,\ldots,k)}$ one can generate simple cycles describing each of the complex components separately. To achieve this, one has to modify the algorithm given above (Section 3.1) by adding some extra steps related to control over cycle closing, i.e. each time after shifting a column and row one has to check if it is possible to obtain the value 1, closing a simple cycle, in row i in the first column. If so, one can define another cycle ($i = i + 1$), starting from the next row and column, recording the initial position $a_1 = i$. In this way one can continue with shifting rows and columns, checking if the value 1, closing the next cycle, is present in the consecutive row $i = i + 1$ in column a. The procedure of matrix ordering is completed when $i + 1$ is equal to the number of nodes in the digraph $A_{CM}^{(1,\ldots,k)}$.

When a maximal complex contains node-connected components, it is possible to generate simple cycles on the basis of row and column permutation only if a given node connects two components only, provided that these components are mutually connected. In the first step one has to search for nodes with $\deg(w) = 2$. A row and column with labels of the first chosen node are shifted to the position 1. Then, according to the simple cycle algorithm, one has to find the position of the value equal to 1 in consecutive rows of the matrix, and shift certain columns and rows to the position $i = i + 1$ (this time the initial $i = 0$). Each time one has to check whether the value 1, closing the simple cycle, is present in the consecutive row $i = i + 1$ in the first column. If so, one can define another cycle, which is node-consistent with that defined before, because in the row closing the previous cycle there are two ones. The other one describes the first node of the next component of the complex. The procedure of row and column shifting must be repeated. If in the row closing the next component there is a single one only, it means that the process of defining connected components has been completed. In order to find another connected component, one has to check whether in the unordered part of the matrix there are any nodes of degree $n = 2$. The row and column describing such a node are to be shifted to the next position of the matrix. The previous sequence is repeated until all node-connected components have been described, adopting the initial $i = i$.

Another stage is to define simple cycles describing independent components. This is realized by repeating the process of row and column shifting, according to the procedure described above, checking each time if cycle closing has been achieved. The procedure of matrix ordering is completed when $i + 1$ is equal to the number of nodes in digraph $A_{CM}^{(1,\ldots,k)}$.

It may happen while defining a maximal complex that some component of this complex is described as a region with an enclave not belonging to the complex. In such a case the algorithm described above can be used to define the external and internal boundary of the complex. The external boundary is defined as a simple cycle specified in the clockwise direction, and the internal boundary – as a simple cycle specified in the counter-clockwise direction. This simple relationship enables to detect internal boundaries automatically, since the enclave area calculated on the basis of data for this cycle assumes a negative value. It is possible to assign this "hole" to a concrete component of the complex, using simple operations (complements), e.g. in the form of a bridge (Lewandowicz, 2004).

In the case of complicated compositions of maximal complex components, e.g. when nodes connect more than two components ($\deg(w) > 2$), or components are multi-connected, simple cycles of each component can be generated based on the method of column and row permutation, presented above. This is, however, associated with the need for selection submatrices for independent notation of each component separately.

4.1. Example of selecting maximal complexes with specified characteristics

Let us take the land parcels shown in Fig. 1 as an example of selecting regions with specified characteristics. Let us assume that attributes related to the form of land use and mean height above sea level (Table 1) were assigned to the parcels. The data included in Table 1 were assigned to directed edges of a digraph describing the parcels, by means of functions of attributes. In this case the function f_1 assumes four values only, and the function f_2 integer values.

Table 1. Characteristics of land parcels

No of a parcel	$[f_1]$ Form of land use	$[f_2]$ Mean height above sea level [m]
136	*R*	412
138	*R*	389
183	*dr*	379
221	*R*	422
222/1	*Ls*	401
222/2	*B*	389
223	*B*	398

Let us assume that our aim is to determine how much land located in this complex is to be afforested, on condition that trees are to be planted only on agricultural parcels situated 400 m above sea level or higher. Therefore, one has to define neighbourhood matrices fulfilling the following criterion:

$$A^{(((R)*(>400))+(Ls))} = (A^{(R)} \cap A^{(>400)}) \cup A^{(Ls)} \tag{28}$$

Making the choice one has to define neighbourhood matrices $A^{(R)}$, $A^{(>400)}$ and $A^{(Ls)}$ (Fig. 8, 9, 10), and then carry out operations of intersection and summation.

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccccccc} 1&2&3&4&5&6&7&8&9&10&11&12&13&14&15&16&17&18 \end{array} \\
\begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18 \end{array} &
\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0
\end{pmatrix}
\end{array}$$

Fig. 8. Neighbourhood matrix of agricultural land parcels $A^{(R)}$

$$
\begin{array}{c c}
 & \begin{array}{cccccccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \end{array} \\
\begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18 \end{array} &
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

Fig. 9. Neighbourhood matrix of land parcels situated 400 m above sea level or higher $A^{(>400)}$

$$
\begin{array}{c c}
 & \begin{array}{cccccccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \end{array} \\
\begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16\\17\\18 \end{array} &
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

Fig. 10. Neighbourhood matrix of agricultural land parcels $A^{(Ls)}$

The other matrices obtained according to the criterion adopted are given below. Making the choice on the basis of function $f_2 > 400$ m above sea level, one obtains matrix $A^{(>400)}$. The neighbourhood matrix $A^{(Ls)}$ fulfilling the condition is in this case a simple matrix describing only one land parcel, with a registration number 222/1. As a result of operations $(A^{(R)} \cap A^{(>400)}) \cup A^{(Ls)}$ one obtains a matrix $A^{(((R)*(>400))+(Ls))}$, presented in Fig. 11. Its ordered form, after transformation $A^{(((R)*(>400))+(Ls))} \longrightarrow A_{CM}^{(((R)*(>400))+(Ls))}$, is shown in Fig. 12.

The matrix $A_{CM}^{(((R)*(>400)+(Ls))}$ shows that the maximal complex obtained consists of three components, two of which are node-connected. The components are recorded in matrix $A_{CM}^{(((R)*(>400)+(Ls))}$ by means of three cycles $\{C_1, C_2, C_3\}$, two of which $\{C_1, C_2\}$ are node-connected. This can be presented in a simplified form:

$$
A_{CM}^{(((R)*(>400)+(Ls))} = \begin{pmatrix} [C_{1_{CM}} \cup C_{2_{CM}}] & 0 \\ 0 & C_{3_{CM}} \end{pmatrix} \tag{29}
$$

Each simple cycle may be generated independently as an ordered sequence of nodes $\{w_1, w_2, ..., w_n, w_1\}$ or edges $\{(w_1, w_2), (w_2, w_3), ..., (w_{n-1}, w_n), (w_n, w_1)\} \in L$ of digraph

$C_{P_{CM}} = [W, L, \alpha]$. This enables to calculate, in a simple way, both the area and length of boundaries of components of the complex.

$$
\begin{array}{r}
\\ 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 10\\ 11\\ 12\\ 13\\ 14\\ 15\\ 16\\ 17\\ 18
\end{array}
\begin{array}{c}
\begin{array}{cccccccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \end{array}\\
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

Fig. 11. Neighbourhood matrix $A^{(((R)*(>400))+(Ls))}$

$$
\begin{array}{r}
\\ 1\\ 2\\ 5\\ 3\\ 8\\ 9\\ 10\\ 4\\ 12\\ 16\\ 17\\ 18\\ 13
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccccc} 1 & 2 & 5 & 3 & 8 & 9 & 10 & 4 & 12 & 16 & 17 & 18 & 13 \end{array}\\
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

Fig. 12. Neighbourhood matrix $A_{CM}^{(((R)*(>400))+(Ls))}$ describing a maximal complex consisting of three components, two of which are node-connected

5. Conclusions

The existing space division can be introduced into a database using digraphs with characteristics describing edges and nodes. Then the boundary lines of maximal complexes can be generated automatically, based on defined functions. The method for identification of the boundaries of maximal complexes, presented in this paper, allows for making selections in a simple way. The boundaries of new complexes can be recorded in the existing database, adding e.g. a new function of attributes assigned to directed edges and nodes. These boundaries permit not only spatial visualization of selection of maximal complexes; they also allow for generating information on their numerical parameters (such as e.g. area, length of boundaries, percentage of a given region in the total area), which can help making decisions concerning land management.

Drawing general conclusions from the example presented in the paper it may be stated that describing units of territorial-administrative division by means of cyclic digraphs makes it possible to generate new data on boundaries of higher units in a simple way. In addition, such a representation enables to control automatically the internal consistency and integrity of spatial data concerning territorial-administrative division units.

The method for determining the boundaries of a maximal complex based on the graph theory can be easily implemented to GIS programs.

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Identyfikacja granic kompleksów maksymalnych na podstawie opisu podobszarów za pomoca digrafu

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Streszczenie

Celem niniejszej pracy jest nakreślenie pewnej metodyki automatycznego identyfikowania granic kompleksów maksymalnych na podstawie wykazów punktów granicznych pokompleksów. Proponowana metodyka wiąże się