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Inertial moments of generated TIN models for surface matching

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Abstract: The purpose of surface matching is to determine transformation parameters without known corresponding points for two data sets of spatial point coordinates obtained with use of different sensors. Instead of different features such as points of interest, lines, surface patches in the TIN (Triangle Irregular Network) or DEM model are used.

The paper presents an approach of using inertial moments of TIN models generated from two data sets of same terrain for surface matching. The inertial moments could easily be calculated for each triangle in the TIN using formulae given. Three moment invariants I_2, I_{max}, I_{min} that are used as the features of high level for surface matching are defined.

Keywords: Surface matching, inertial moments, moment invariants, TIN model

1. Introduction

Surface matching is an efficient tool for comparing two surfaces generated from two data sets of spatial point coordinates of same scene that are obtained with use of different sensors. Two data sets have in general different point distribution and they are referred to different systems. Determination of transformation parameters such as scale factor, rotation and translation between two surfaces generated from data sets is the essence of surface matching.

Surface matching is the effective technique for different purposes, especially for the investigation of damaged terrain due to an earthquake or subsidence (Ito et al., 2000; Zhou et al., 2000). To match surfaces the mutual correspondence has to be determined first and then transformation of two data sets can be performed. Those two operations could also be done simultaneously.

The first approach – the one of surface matching in two separate steps – is based on the features extracted from both surfaces. In that case the edge lines extracted in 3-D model or in 2-D image are often used. In the given line the dominant points with maxima curvatures are determined. The process of surface matching is then performed by comparison of dominant points in the corresponding lines (Han and Jang, 1990). The other method that uses the lines is implemented by transforming line equations into parametric representation in frequency domain. Surface matching will then be done using harmonic coefficients (Luong, 2003c). The method of surface matching with use of line moments has been recommended (Luong, 2004). Line moments are invariant under their transformation; they are therefore good means for surface matching.

The one step approach of surface matching is based on the points and surface patches being the elements of TIN (Triangle Irregular Network) models or DEM models. Target functions constructed on the conditions of minimizing differences in elevation between two surfaces or minimizing distances along surface normal have been investigated (Schenk et al., 2000). The use of normal vectors of TIN models and applying Hopfield's neural network for surface matching are recommended (Luong, 2003b). The other method of detecting surface deformations by least median squares matching was examined (Xu and Li, 2000).

This paper presents an approach of inertial moments of TIN models for surface matching. Inertial moments of TIN models are formed in each plane *OXY*, *OXZ*, *OYZ* to find transformation parameters with using "moment distances" or Hopfield's neural network.

2. Inertial moments of generated TIN models

A set of spatial point coordinates *X*, *Y*, *Z* obtained with different systems such as LIDAR, GPS, SAR etc. can simply be generated in TIN models (Fig. 1).

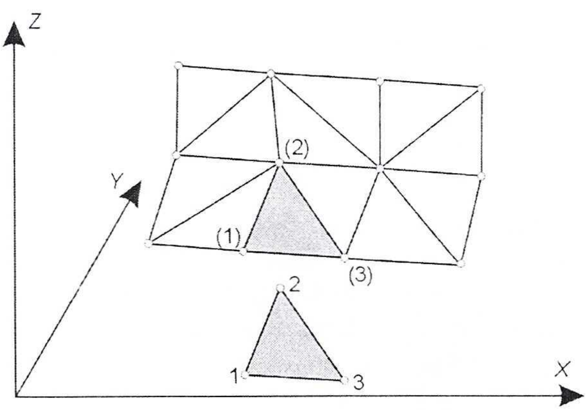


Fig. 1. Generated TIN models

For a single TIN model in 3-D space, e.g. the triangle (123) determined by three points (1), (2), (3), the equation of its plane is written as follows

$$\begin{vmatrix} X & Y & Z & 1 \\ X_1 & Y_1 & Z_1 & 1 \\ X_2 & Y_2 & Z_2 & 1 \\ X_3 & Y_3 & Z_3 & 1 \end{vmatrix} \equiv AX + BY + CZ + D = 0 \tag{1}$$

or

$$Z = aX + bY + c \tag{2a}$$

where

$$\begin{aligned} & a = -A/C,\ b = -B/C,\ c = -D/C \\ & A = (Y_2 - Y_1)(Z_3 - Z_1) - (Y_3 - Y_1)(Z_2 - Z_1) \\ & B = (X_2 - X_1)(Z_3 - Z_1) - (X_3 - X_1)(Z_2 - Z_1) \\ & C = (X_2 - X_1)(Y_3 - Y_1) - (X_3 - X_1)(Y_2 - Y_1) \\ & D = -AX_1 + BY_1 - CZ_1 \end{aligned} \tag{2b}$$

The area of a single triangle in the TIN model depicted with (2a) will be (Leitner, 1995)

$$S = \iint_S dS = \iint_\Delta \sqrt{Z_X^2 + Z_Y^2 + 1}\ dXdY = k \iint_\Delta dXdY \tag{3}$$

where $k = \sqrt{Z_X^2 + Z_Y^2 + 1} = \sqrt{a^2 + b^2 + 1};\ \frac{\partial Z}{\partial X} = Z_X = a;\ \frac{\partial Z}{\partial Y} = Z_Y = b$

S, Δ – the surface patches or triangles.

Figure 1 shows that the triangle (123) is limited by three straight lines (1–2), (2–3) and (3–1). The direction coefficients (m_{12}, m_{23}, m_{31}) of those lines are defined as follows

$$m_{12} = \frac{Y_2 - Y_1}{X_2 - X_1};\quad m_{23} = \frac{Y_3 - Y_2}{X_3 - X_2};\quad m_{31} = \frac{Y_1 - Y_3}{X_1 - X_3} \tag{4}$$

For any surface patch given by a function $F(X, Y, Z) = 0$ (1) with the surface density function $\rho(X, Y, Z)$, the components of the inertial moments of a triangle with respect to X–, Y–, Z– axes are defined as (Susłow, 1960)

$$\mu_{OZY}^{(2)} = \iint_S X^2 dS = \iint_\Delta X^2 \sqrt{Z_X^2 + Z_Y^2 + 1}\,\rho(X, Y, Z)\,dXdY \tag{5a}$$

$$\mu_{OZX}^{(2)} = \iint_S Y^2 dS = \iint_\Delta Y^2 \sqrt{Z_X^2 + Z_Y^2 + 1}\,\rho(X, Y, Z)\,dXdY \tag{5b}$$

$$\mu_{OXY}^{(2)} = \iint_S Z^2 dS = \iint_\Delta Z^2(X, Y) \sqrt{Z_X^2 + Z_Y^2 + 1}\,\rho(X, Y, Z)\,dXdY \tag{5c}$$

and mixed moments of inertia (deviation moments) are also defined

$$\mu_{XY}^{(1,1)} = \iint_S XYdS = \iint_\Delta XY \sqrt{Z_X^2 + Z_Y^2 + 1}\,\rho(X, Y, Z)\,dXdY \tag{6a}$$

$$\mu_{XZ}^{(1,1)} = \iint_S XZdS = \iint_\Delta XZ\sqrt{Z_X^2 + Z_Y^2 + 1}\,\rho(X, Y, Z)\,dXdY \tag{6b}$$

$$\mu_{YZ}^{(1,1)} = \iint_S YZdS = \iint_\Delta YZ\sqrt{Z_X^2 + Z_Y^2 + 1}\,\rho(X, Y, Z)\,dXdY \tag{6c}$$

Numerical values of the TIN model moments are found assuming $\rho(X, Y, Z) = 1$. The component $\mu_{OZY}^{(2)}$ (5a) of the moment of inertia can be computed for the triangle (123) (Fig. 1) as

$$\mu_{OZY}^{(2)} = \iint_S X^2\,dS = k\iint_\Delta X^2\,dXdY = k\left\{\int_{X_1}^{X_2} X^2\left(\int_{Y_1}^{Y_1+m_{12}(X-X_1)} dY\right)dX + \int_{X_2}^{X_3} X^2\left(\int_{Y_2}^{Y_2+m_{23}(X-X_2)} dY\right)dX + \right.$$

$$\left. + \int_{X_3}^{X_1} X^2\left(\int_{Y_3}^{Y_3+m_{31}(X-X_3)} dY\right)dX\right\} \tag{7a}$$

with

$$L_1 = \int_{X_1}^{X_2} X^2\left(\int_{Y_1}^{Y_1+m_{12}(X-X_1)} dY\right)dX,\quad L_2 = \int_{X_2}^{X_3} X^2\left(\int_{Y_2}^{Y_2+m_{23}(X-X_2)} dY\right)dX,\quad L_3 = \int_{X_3}^{X_1} X^2\left(\int_{Y_3}^{Y_3+m_{31}(X-X_3)} dY\right)dX \tag{7b}$$

(7a) becomes

$$\mu_{OZY}^{(2)} = k\{L_1 + L_2 + L_3\} = kL \tag{7c}$$

Integrating (7b) provides

$$\begin{cases} L_1 = m_{12}\left(\frac{1}{4}X_2^4 + \frac{1}{12}X_1^4 - \frac{1}{3}X_1X_2^3\right) \\ L_2 = m_{23}\left(\frac{1}{4}X_3^4 + \frac{1}{12}X_2^4 - \frac{1}{3}X_2X_3^3\right) \\ L_3 = m_{31}\left(\frac{1}{4}X_1^4 + \frac{1}{12}X_3^4 - \frac{1}{3}X_3X_1^3\right) \end{cases} \tag{7d}$$

Similarly, the component $\mu_{OZX}^{(2)}$ (5b) of the moment of inertia of a TIN model for the triangle (123), is

$$\mu_{OZX}^{(2)} = k\{M_1 + M_2 + M_3\} = kM \tag{8a}$$

where

$$\begin{cases} M_1 = \dfrac{1}{m_{12}}\left(\dfrac{1}{4}Y_2^4 + \dfrac{1}{12}Y_1^4 - \dfrac{1}{3}Y_1 Y_2^3\right) \\ M_2 = \dfrac{1}{m_{23}}\left(\dfrac{1}{4}Y_3^4 + \dfrac{1}{12}Y_2^4 - \dfrac{1}{3}Y_2 Y_3^3\right) \\ M_3 = \dfrac{1}{m_{31}}\left(\dfrac{1}{4}Y_1^4 + \dfrac{1}{12}Y_3^4 - \dfrac{1}{3}Y_3 Y_1^3\right) \end{cases} \tag{8b}$$

and the corresponding component $\mu_{OXY}^{(2)}$ (5c) of the moment of inertia of a TIN model is

$$\mu_{OXY}^{(2)} = \iint_S Z^2 dS = k\iint_\Delta Z^2(X, Y)\,dXdY = k\iint_\Delta (aX + bY + c)^2 dXdY = k\left\{a^2\iint_\Delta X^2 dXdY + \right.$$

$$\left. + b^2\iint_\Delta Y^2 dXdY + c^2\iint_\Delta dXdY + 2ab\iint_\Delta XYdXdY + 2ac\iint_\Delta XdXdY\right\}$$

Denoting

$$P = \iint_\Delta dXdY, \quad T = \iint_\Delta XYdXdY, \quad Q = \iint_\Delta XdXdY, \quad R = \iint_\Delta YdXdY$$

and computing integrals similarly to those in (7) the component $\mu_{OXY}^{(2)}$ of the moment of inertia of a TIN model becomes

$$\begin{aligned} \mu_{OXY}^{(2)} &= a^2\mu_{OZY}^{(2)} + b^2\mu_{OZX}^{(2)} + k\{c^2P + 2acQ + 2bcR + 2abT\} \\ &= k\{a^2L + b^2M + c^2P + 2acQ + 2bcR + 2abT\} \end{aligned} \tag{9a}$$

where

$$P = P_1 + P_2 + P_3 \quad \text{with} \quad \begin{cases} P_1 = m_{12}\left(\dfrac{1}{2}X_2^2 + \dfrac{1}{2}X_1^2 - X_1X_2\right) \\ P_2 = m_{23}\left(\dfrac{1}{2}X_3^2 + \dfrac{1}{2}X_2^2 - X_2X_3\right) \\ P_3 = m_{31}\left(\dfrac{1}{2}X_1^2 + \dfrac{1}{2}X_3^2 - X_3X_1\right) \end{cases} \tag{9b}$$

$$Q = Q_1 + Q_2 + Q_3 \quad \text{with} \quad \begin{cases} Q_1 = m_{12}\left(\frac{1}{3}X_2^3 + \frac{1}{6}X_1^3 - \frac{1}{2}X_1X_2^2\right) \\ Q_2 = m_{23}\left(\frac{1}{3}X_3^3 + \frac{1}{6}X_2^3 - \frac{1}{2}X_2X_3^2\right) \\ Q_3 = m_{31}\left(\frac{1}{3}X_1^3 + \frac{1}{6}X_3^3 - \frac{1}{2}X_3X_1^2\right) \end{cases} \tag{9c}$$

$$R = R_1 + R_2 + R_3 \quad \text{with} \quad \begin{cases} R_1 = \frac{1}{m_{12}}\left(\frac{1}{3}Y_2^3 + \frac{1}{6}Y_1^3 - \frac{1}{2}Y_1Y_2^2\right) \\ R_2 = \frac{1}{m_{23}}\left(\frac{1}{3}Y_3^3 + \frac{1}{6}Y_2^3 - \frac{1}{2}Y_2Y_3^2\right) \\ R_3 = \frac{1}{m_{31}}\left(\frac{1}{3}Y_1^3 + \frac{1}{6}Y_3^3 - \frac{1}{2}Y_3Y_1^2\right) \end{cases} \tag{9d}$$

and

$$T = T_1 + T_2 + T_3 \quad \text{with} \quad \begin{cases} T_1 = Y_1Q_1 + \frac{1}{2}m_{12}^2\left(\frac{1}{4}X_2^4 - \frac{1}{12}X_1^4 - \frac{2}{3}X_1X_2^3 + \frac{1}{2}X_1^2X_2^2\right) \\ T_2 = Y_2Q_2 + \frac{1}{2}m_{23}^2\left(\frac{1}{4}X_3^4 - \frac{1}{12}X_2^4 - \frac{2}{3}X_2X_3^3 + \frac{1}{2}X_2^2X_3^2\right) \\ T_3 = Y_3Q_3 + \frac{1}{2}m_{31}^2\left(\frac{1}{4}X_1^4 - \frac{1}{12}X_3^4 - \frac{2}{3}X_3X_1^3 + \frac{1}{2}X_3^2X_1^2\right) \end{cases} \tag{9e}$$

Next, the mixed moments (6a), (6b) and (6c) will be computed using (9a)

$$\mu_{XY}^{(1,1)} = k\iint_{\Delta} XYdXdY = k(T_1 + T_2 + T_3) = kT \tag{10a}$$

and

$$\mu_{XZ}^{(1,1)} = k\iint_{\Delta} XZdXdY = k\iint_{\Delta} X(aX + bY + c)\,dXdY = k\left\{a\iint_{\Delta} X^2dXdY + b\iint_{\Delta} XYdXdY + \right.$$
$$\left. + c\iint_{\Delta} XdXdY\right\}$$

$$\mu_{XZ}^{(1,1)} = a\mu_{OZY}^{(2)} + b\mu_{XY}^{(1,1)} + kcQ = k\{aL + bT + cQ\} \tag{10b}$$

and

$$\mu_{YZ}^{(1,1)} = k\iint_{\Delta} YZ(X, Y)dXdY = k\iint_{\Delta} Y(aX + bY + c)\,dXdY = k\left\{a\iint_{\Delta} XYdXdY + b\iint_{\Delta} Y^2 dXdY + \right.$$
$$\left. + c\iint_{\Delta} YdXdY\right\}$$

$$\mu_{YZ}^{(1,1)} = b\mu_{OZX}^{(2)} + a\mu_{XY}^{(1,1)} + kcR = k\{bM + aT + cR\} \tag{10c}$$

Moments of inertia $\mu_{XX}^{(2)}$, $\mu_{YY}^{(2)}$, $\mu_{ZZ}^{(2)}$ with respect to *X*–, *Y*–, *Z*– axes can now be determined (Susłow, 1960)

$$\mu_{XX}^{(2)} = \mu_{OXZ}^{(2)} + \mu_{OXY}^{(2)} \tag{11a}$$

$$\mu_{YY}^{(2)} = \mu_{OZY}^{(2)} + \mu_{OXY}^{(2)} \tag{11b}$$

$$\mu_{ZZ}^{(2)} = \mu_{OZX}^{(2)} + \mu_{OZY}^{(2)} \tag{11c}$$

For an easy implementation, the matching process related with the determination of transformation parameters will be performed in particular planes *OXY*, *OXZ*, *OYZ*. For each triangle in *OXY*, *OXZ*, *OYZ* planes the triplets of necessary moments are: $\mu_{XX}^{(2)}, \mu_{YY}^{(2)}, \mu_{XY}^{(1,1)}$; $\mu_{XX}^{(2)}, \mu_{ZZ}^{(2)}, \mu_{XZ}^{(1,1)}$; $\mu_{YY}^{(2)}, \mu_{ZZ}^{(2)}, \mu_{YZ}^{(1,1)}$, respectively.

Three moment invariants I_2, I_{max}, I_{min} can be determined in each plane. For example, basing on $\mu_{XX}^{(2)}, \mu_{YY}^{(2)}, \mu_{XY}^{(1,1)}$ of *OXY* plane they are (Luong, 2004)

$$I_1 = \mu_{XX}^{(2)} + \mu_{YY}^{(2)} \qquad I_{max} = \frac{I_1 + I_2}{2}$$
$$I_2 = \sqrt{(\mu_{XX}^{(2)} - \mu_{YY}^{(2)})^2 + 4(\mu_{XY}^{(1,1)})^2} \qquad I_{min} = \frac{I_1 - I_2}{2} \tag{12}$$

Two moment invariants I_{max}, I_{min} define two perpendicular directions with the origin in the centroid *C* of the triangle, where θ corresponds to the angle between the directions of I_{max} and *X*-axes (Fig. 2). The angle θ can be calculated as follows

$$tg2\theta = \frac{2\mu_{XY}^{(2)}}{\mu_{XX}^{(2)} - \mu_{YY}^{(2)}} \tag{13}$$

The moment invariants of TIN models in *OXZ* and *OYZ* planes can similarly be determined.

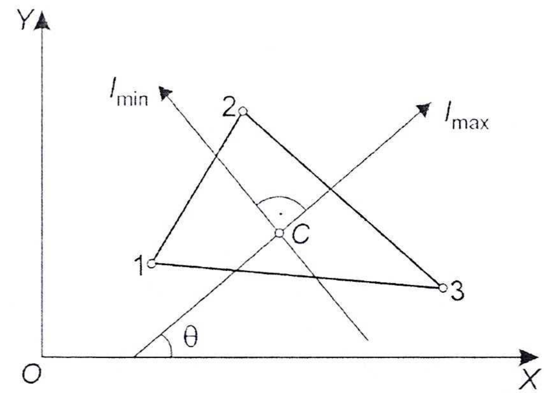


Fig. 2. The elements of single TIN model (C, S, θ) and two moment invariants I_{max}, I_{min}

Figure 2 shows that any TIN model can be represented by three elements: the centroid C of triangle, the area S and turning angle θ. The features characterizing each TIN model are three moment invariants I_{max}, I_{min}, I_2 that are suitable means for surface matching.

3. Surface matching problem

Surface matching problem deals with the derived using different technologies two data sets $S1$, $S2$ of point spatial coordinates that describe two models of same physical surface. Two sets of points referred to different reference systems and of different point distribution have in general no common points. The goal of surface matching is to find transformation parameters between two surfaces generated in TIN models from two data sets. The moments of inertia of TIN models treated as the features of high level are used.

Matching process using the moments of inertia of TIN models is the same as using line moments (Luong, 2004) that is briefly recalled. For each plane OXY, OXZ, OYZ, the transformation parameters: scale, rotation and translation have been determined. The "moment distances" $D_k(i, j)$ ($k = 1, 2, 3$) between i-th TIN model of the set $S1$ and j-th TIN model of the set $S2$ are defined using (12). For example in OXY plane

$$
\begin{aligned}
D_1(i, j) &= \frac{|I^{S1}_{\max(i)} - I^{S2}_{\max(j)}|}{|I^{S1}_{\max(i)} + I^{S2}_{\max(j)}|} \\
D_2(i, j) &= \frac{|I^{S1}_{\min(i)} - I^{S2}_{\min(j)}|}{|I^{S1}_{\min(i)} + I^{S2}_{\min(j)}|} \qquad (14) \\
D_3(i, j) &= \frac{|I^{S1}_{2(i)} - I^{S2}_{2(j)}|}{|I^{S1}_{2(i)} + I^{S2}_{2(j)}|}
\end{aligned}
$$

Best matching of two surfaces of same terrain is obtained when the following condition is fulfilled

$$\sum_{k=1}^{3}\sum_{i=1}^{m}\sum_{j=1}^{n} D_k(i,j) = \min \qquad \text{where} \quad \begin{cases} D_1(i,j) \le \delta_1 \\ D_2(i,j) \le \delta_2 \\ D_3(i,j) \le \delta_3 \end{cases} \tag{15}$$

where δ_1, δ_2, δ_3 – pre-defined threshold.

For every *i-th* TIN model of *S*1 and *j-th* TIN model of *S*2 the scale $t_{(i,j)}$, rotation differences $\Delta\theta_{(i,j)}$ and translation parameters $\Delta X_{C(i,j)}$, $\Delta Y_{C(i,j)}$ have been computed. For example, in *OXY* plane

$$\begin{cases} t_{XY(i,j)} = \sqrt{\dfrac{S^{S1}_{XY(i)}}{S^{S2}_{XY(j)}}} \\ \Delta\theta_{XY(i,j)} = \theta^{S1}_{XY(i)} - \theta^{S2}_{XY(j)} \\ \begin{cases} \Delta X_{C(i,j)} = X^{S1}_{C(i)} - X^{S2}_{C(j)} \\ \Delta Y_{C(i,j)} = Y^{S1}_{C(i)} - Y^{S2}_{C(j)} \end{cases} \end{cases} \tag{16}$$

where $S^{S1}_{XY(i)}$, $S^{S2}_{XY(j)}$ – the areas of *i-th* and *j-th* TIN models in *S*1 and *S*2 sets, respectively, that are calculated from coordinates of three points. Similar procedure is applied in *OXZ*, *OYZ* planes.

To determine the most likely transformation parameters a Hough transformation has been used (Adamos and Faig, 1992; Yun et al., 2000).

Three scale factors calculated in *OXY*, *OXZ*, *OYZ* planes determine the scale factors of affine transformation, and three rotations ($K \equiv \Delta\theta_{XY}$, Φ, Ω) in three planes determine mutual orientation of two surfaces generated from *S*1 and *S*2. After having computed orientation parameters, the affine transformation of the set *S*2 into the set *S*1 will be performed.

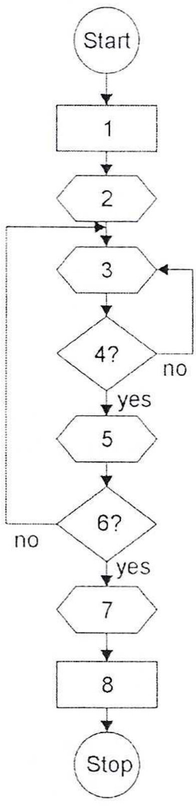
The analysis of reliability and accuracy of surface matching based on the moments of TIN models is carried out with use of a certain number of points exhibiting coordinate differences smaller than the pre-defined thresholds. The process of matching moment invariants is described in the block scheme of algorithm in Fig. 3.

Basing on the condition (15) the surface matching can be implemented with Hopfield's neural network. The energy function that consists of energy of target and constraint function (Luong, 2003a, 2003b) has to be defined. In the case considered the target function is given by (15). Its energy function is

$$E_T = \sum_{k=1}^{3}\sum_{i=1}^{m}\sum_{j=1}^{n} f(D_k(i,j) - \delta_k)y_{ij} = \sum_{k=1}^{3}\sum_{i=1}^{m}\sum_{j=1}^{n} f(x)_{kij}\,y_{ij} \tag{17}$$

$$\text{with} \quad f(x)_{kij} = \begin{cases} 1 & \text{for} \quad x > 0 \\ 0 & \text{for} \quad x \le 0 \end{cases}$$

where i, j – indices of neurons (i-th TIN model of $S1$ and j-th TIN model of $S2$),
k – the number of feature type (k = 1, 2, 3),
δ_k – the pre-defined thresholds,
y_{ij} – the neuron that is a function of total weighted input signal at i and j neurons (Luong, 2003b),
$f(.)_{kij}$ – the activation function (sigmoid unipolar function).



Explanation:

1. Data input.
2. Calculation of triangle areas, centroids of triangles, central moments $(\mu^{(2)}_{XX}, \mu^{(2)}_{YY}, \mu^{(2)}_{ZZ}, \mu^{(1,1)}_{XY}, \mu^{(1,1)}_{XZ}, \mu^{(1,1)}_{YZ})$ of triangles, moment invariants I_{max}, I_{min}, I_2 (7, 8, 9, 10, 11, 12, 13).
3. Computation of "moment distances" $D_1(i, j)$, $D_2(i, j)$, $D_3(i, j)$ (14).
4. Checking conditions $D_1(i,j) \le \delta_1$, $D_2(i,j) \le \delta_2$, $D_3(i,j) \le \delta_3$ (15).
5. Computation of transformation parameters of single matched triangles (16).
6. Control percentage of occurred gross errors of parameters [$\%(\Delta_{A0} \le 3m_{A0})$, $\%(\Delta t \le 3m_t)$]. If a response is *true*, perform next stage 7; if *not true*, return to 3 with the change of threshold values δ_1, δ_2, δ_3.
7. Computing final transformation parameters.
8. Transformation of $S2$ into $S1$. Results output.

Fig. 3. Algorithm of matching moment invariants

Two constraints read as follows:

- every TIN model of $S2$ belongs to one TIN model of $S1$ only (one-to-one constraint),
- after transforming the surface $S2$ into $S1$ all features have the same orientation (orientation consistency constraint).

Energy function built up on the basis of those two constraints has been presented (Luong, 2003b).

After determining total energy function the Hopfield Neural Network value will be calculated. The stable state of a system exhibits the best correspondence fitness between feature groups of i and j when the total energy of the system reaches minimum.

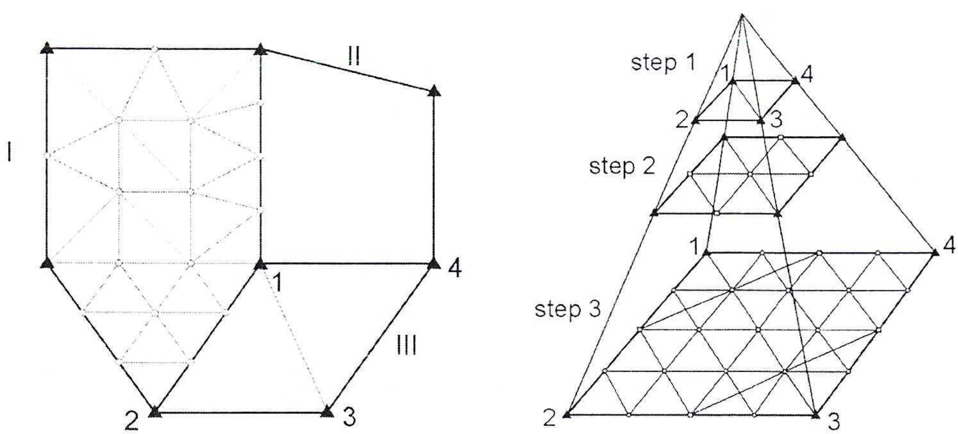
In practice, the number of points of an extensive area, recorded with sensor is very large. For the extensive and large area the strategy of step-by-step reduction of length of the TIN model sides is applied.

+ Extensive area is subdivided into small regions (regions I, II, III in Fig. 4a) of regular local slopes.

+ TIN models with long sides are generated in each region. Surface matching using moments of inertia will be carried out (step 1). For example, in the segment III four points with two generated triangles were taken in the first step (Fig. 4b). Transformation parameters determined in that step are treated as input to the next step.

+ In the second step the smaller TIN models are once again generated. Process of surface matching using moments of smaller TIN models is repeated. Parameters obtained will be more accurate than those in the first step.

Step-by-step reduction of length of the TIN model sides leads to determination of transformation parameters of required accuracy (step 3 and next). The concept of strategy of surface matching for extensive area is presented in Fig. 4b.



a) *b)*

Fig. 4. Strategy of step-by-step reduction of length of the TIN model sides

Parameter $k = \sqrt{Z_X^2 + Z_Y^2 + 1}$ in the integrals for TIN model is considered constant. If data set is generated in DEM (in the squares), the parameter k becomes a function of X, Y variables, i.e. $k = f(X, Y)$. Integrating process is complicated. Squares matching using least median for automated detection of surface deformations have been suggested (Xu and Li, 2000).

4. Conclusions

To determine transformation parameters between two data sets described by two models of the same scene, two tasks of correspondence and transformation have been carried out simultaneously. To realize this purpose the moments of inertia of TIN models generated from two data sets are calculated. They can be treated as the features of high level for surface matching. For each triangle the inertial moments are determined by readily proved formulas that are easy to use. In order to limit a complexity of calculation, the inertial moments of triangles are formed in each plane *OXY*, *OXZ*, *OYZ*. Scale, rotation and translation obtained in each plane determine transformation parameters that are necessary to transform second surface to first one.

The use of the moments of inertia for surface matching is very effective when two surfaces of same terrain, obtained with different technologies differ enough in orientation. The proposed approach does not require the knowledge of points corresponding in two surface systems. The moments of inertia of image objects could also be applied for updating maps and for performing image relative orientation.

The main disadvantage of use of the moments of inertia for surface matching is that the computation is time consuming. This problem will be the subject of further investigations.

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Wykorzystanie momentów bezwładności sieci generowanych trójkątów dla dopasowania powierzchni

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Streszczenie

Istotny problem dopasowania dwóch powierzchni generowanych w postaci TIN lub DEM na podstawie zbiorów punktów przestrzennych, otrzymanych przy pomocy różnych sensorów w różnych układach, polega na wyznaczeniu parametrów transformacji pomiędzy nimi bez wspólnych punktów w obu układach. Dla rozwiązania tego problemu wykorzystane są elementy powierzchniowe w formie trójkątów (model TIN) albo kwadratów (DEM).

Praca przedstawia propozycję analitycznego rozwiązania zadania „dopasowania" (matching) dwóch powierzchni na podstawie wykorzystania momentów bezwładności sieci generowanych trójkątów (model TIN). Proces dopasowania dwóch powierzchni zostaje zrealizowany na zasadzie warunku minimalizacji „długości momentów" liczonych pomiędzy *i*-tym modelem TIN pierwszej powierzchni i *j*-tym modelem TIN drugiej powierzchni na trzech płaszczyznach *OXY*, *OXZ*, *OYZ*. W ten sposób parametry transformacji zostaną wyznaczane bez punktów wspólnych i zadanie transformacji drugiej powierzchni na pierwszą zostanie zrealizowane.