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## Geometric models for satellite sensors

# Luong Chinh Ke, Wieslaw Wolniewicz

Warsaw University of Technology
Institute of Photogrammetry & Cartography
1 Plac Politechniki, 00-661 Warsaw, Poland
e-mail: lchinhke@gazeta.pl; w.wolniewicz@gik.pw.edu.pl

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**Abstract:** Since 2000 when first imageries of Space Imaging of one metre resolution satellite products appeared on the World market, many institutions started using them for cartographic production such as orthophotomaps on a large scale. A choice of the mathematic sensor models of imageries for their orthorectification in producing orthophotomaps is one of the main investigation directions. In order to restitute the functional relation between imageries and their ground space, the use of sensor models is required. They can be grouped into two classes, the generalized sensor models (geometric or replacement sensor models) and physical or parametric models.

The paper presents a brief overview of the geometric models such as RPC (Rational Polynomial Coefficients). Their properties, and in particular their advantages and disadvantages are discussed. Also the parametric models, developed by the authors are presented in this paper. They are based on time-dependent collinearity equation of the mathematic relation between ground space and its imageries through parameters describing the sensor position in satellite orbit and position of the orbit in the geocentric system.

**Keywords:** Very High Resolution Satellite, satellite sensors, geometric models, RPC model, orthorectification

## 1. Introduction

In conventional aerial and spaceborne photogrammetry the frame camera is frequently used for obtaining terrain information with films. In 1972 the MSS (Multi-Spectral Scanner) placed on the Landsat-1 has been employed for the acquisition of Earth surface information in a digital form. That kind of sensor belongs to dynamic photogrammetry. It means that sensor systems can only obtain a line or pixel image at an instant of time. For example, the pushbroom mode linear array of a CCD camera generates scanning images in the flight direction by the sensor flying along with the platform. The first digital camera for aerial photogrammetry – ADS40 – has been presented during the 19<sup>th</sup> ISPRS Congress in Amsterdam in 2000. In last years digital photogrammetry and remote sensing technology have been quickly developed for mapping and other applications. From 1999 up to now the new era with high resolution satellite imageries, such as Ikonos, QuickBird, EROS,

ORBIMAGE brings a new potential for producing orthophotomaps in large scale  $(1.5\,000-1.10\,000)$  and for updating the existing maps. It is often necessary to correct those imageries to the same geometric basis before it would be possible to use them.

Vast majority of the satellite imaging systems (including all very high resolution systems currently working and those planned ones) are based upon the concept of electro-optical scanner. In the plane of the optical system focal length (in the case of very high resolution systems it is the powerful reflecting telescope) there is a ruler of CCD detectors oriented crosswise with respect to the direction of flight and producing one line of image. The image in the direction of flight is produced in result of movement of the satellite and subsequent reading of signals from the CCD ruler. The image is thus created by two geometrical projections:

- central projection along the CCD ruler, i.e. crosswise to the direction of flight,
- parallel projection in the direction of flight (for the optical system vertically oriented, perpendicularly to the direction of flight, it will be an orthogonal projection).

The image is thus produced continuously along with the movement of satellite – one says that it is the dynamic image. Moreover, it is distorted by unstable flight – by changes of orientation angles or the orbit perturbations. It results in substantial distortions of dimensions that are many times larger than a pixel dimension. Thanks to the very high resolution, the image can be characterized with an excellent interpretation functionality, but at the same time it has very poor measurement quality. Geometrical correction is aimed at elimination of this disproportion. It is worth noting that this problem is very different for images obtained from satellite and aerial altitudes. The flight of satellite is rather stable, and the above-mentioned changes of orientation angles and trajectory perturbations are rather minor, but variable. It gives the chance for an effective elimination of distortions. In the case of an aircraft, the movement is far more dynamic what additionally complicates the problem of correction.

Several reasons of the satellite image distortions might be pointed out:

- camera (calibration errors, i.e. errors in determination of geometrical elements of external positioning, as well as the errors of electronic devices reading out and recording signals from the CCD ruler),
- satellite movement and orbit perturbations, variations of orbiting velocity,
- ongoing recording of the location in the orbit and of the platform angle of inclination (if such recording takes place at all),
- Earth rotation during imaging as well as the ground relief,
- targeted cartographic projection of adjusted image (relations between the geoid and ellipsoid, projection of ellipsoid onto the representation surface),
- atmosphere (refraction).

In the context of correction of very high resolution images, one should highlight the importance of the ground relief influence as the distortion factor. In earlier systems (like Landsat and SPOT), this problem did not occur so drastically. Angular field of view of those systems is narrow – just several degrees – and in the event of nadir representation, the influence of ground relief is rather minor, especially in relation to a pixel dimension.

Satellite camera metric systems exhibit a very narrow angular field of view (one up to a few degrees), but they also have the unique feature that they may be focused towards any

direction – with deflection from the vertical one up to even  $60^{\circ}$  – for imaging the area of interest. The impact of differences of heights of topography depends on the inclination angle, e.g. for inclination of system reaching  $45^{\circ}$ , the value of situational distortion will equal to the value of heights. This means that for correction of such images, high accuracy of the Digital Terrain Model is needed.

The very high resolution systems have also the option for continuous recording of elements of the camera external orientation, i.e. trajectory of the orbit defined from GPS measurements and from inclination angles defined on the basis of the star trackers. This enables for recording trajectory with accuracy of 2–3 m and of inclination angles with accuracy of 2″–3″. However, the system administrators are not interested in delivering decrypted hard data to the end users.

Geometrical models of VHRS imageries are very important for improving the orthorectification process.

In practice, for adjustment of the Very High Resolution Satellite imaging, one applies two basic approaches as follows.

- The first one is based upon the polynomial methods with the use of coefficients delivered with the image, or determined in the framework of the levelling process. In the first case, the coefficients are derivatively determined on the basis of images external orientation elements measured in a flight, while in the other case those coefficients are determined on the basis of a group of photopoints. The polynomial coefficients have no direct geometrical interpretation.
- The second approach is based upon the fundamental condition in photogrammetry, i.e. the co-linearity of the terrain point vector and reflecting imaging vector of the image on a line of CCD detectors. Those relations are described as functions of camera parameters and other elements of external orientation, that are variable in time. An example of such approach is the model functioning in the PCI commercial software that takes into consideration mathematical relations specified by Toutin (Toutin et al., 2004).

Commonly available software based on the adjustment methods operates on the basis of the "black box" without any basic photogrammetric description enabling the user to learn the relations in functioning of a model of a given type (Wolniewicz and Jaszczak, 2004, Wolniewicz et al., 2005).

The paper presents the fundamentals of the polynomial model based upon the use of RPC-type coefficients. Also, the description of the algorithm developed by the authors, based on co-linearity condition has been presented.

# 2. Methods of geometrical correction of VHRS imageries

In order to be able to thoroughly investigate the models' behaviour and their effects on the resulting quality of a great diversity of along-track and cross-track viewing angles, a general concept of geometrical model has been employed for creating two precisely determined data sets: the image co-ordinates (2D) and object co-ordinates (3D) mutually related by taking into consideration the viewing angles, the orbital position and the approximate physical characteristics and orientation of the respective satellite (Fig. 1).

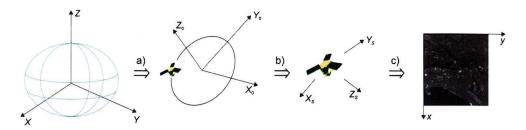


Fig. 1. The procedure flow consisting of the following steps:

- (a) the geocentric terrestrial object space coordinates are transformed into orbital system coordinates;(b) the latter are further transformed into satellite system coordinates;
  - (c) finally, the image coordinates are usually obtained by applying mapping function in addition to translation and rotation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \Rightarrow \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}, \quad \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \Rightarrow \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}, \quad \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix}$$
 (1)

The basis for geometrical correction is the definition of mathematical relation between ground coordinates X, Y, Z of the points and the coordinates x, y of their images. One applies here several substantially different approaches that result in different "geometrical models".

# 2.1. Rational polynomial functions based on sensor orientation - RPC

The purpose of a replacement model of camera is to provide a simple, generic set of equations to accurately represent the ground-to-image relationship of the physical camera. That relationship can be expressed as  $(x, y) = P(\varphi, \lambda, H)$ , where P(.) is the camera model function, (x, y) are image coordinates, and  $\varphi$ ,  $\lambda$ , H are ground coordinates. Ideally, one set of equations, with different coefficients, could model images from multiple camera designs.

A replacement model of camera must not only model the ground-to-image relationship accurately, but must also perform the tasks of a physical camera model. In the following sections the RPC camera models of high-resolution satellite and frame cameras will be described and the use of the RPC models for orthorectification will be presented.

The equations of Rational Polynomial Coefficients (RPC) constitute a replacement model of camera in that they are a generic set of equations that map object coordinates into image coordinates, for a variety of sensor systems. The RPC coefficients describe a particular image from a particular imaging system. The RPC coefficients are used in the RPC equations to calculate an image (sample, line) coordinates from an object (longitude, latitude, height) coordinates. For this model, image vendors describe the location of image positions as a function of the object coordinates (latitude, longitude) by the ratios of polynomials:

$$x_{ij} = \frac{P_i^1(\varphi, \lambda, H)_j}{P_i^2(\varphi, \lambda, H)_j}$$

$$y_{ij} = \frac{P_i^3(\varphi, \lambda, H)_j}{P_i^4(\varphi, \lambda, H)_j}$$
(2)

where  $x_{ij}$ ,  $y_{ij}$  are image coordinates;  $\varphi$ ,  $\lambda$ , H are latitude, longitude, and height; and the polynomial  $P_i^k$  (k = 1, 2, 3, 4) has the form (4).

The file given by vendors contains the coefficients for Rapid Positioning Capability, also called Rational Polynomial Coefficient (RPC). It represents mapping function from the object space to the image space. This mapping includes non-ideal imaging effects, such as lens distortion, light aberration, and atmospheric refraction.

RPC expresses the normalized column and row values in an image, as a ratio of polynomials of the normalized geodetic latitude  $\varphi$ , longitude  $\lambda$ , and height H. Normalized values are used instead of actual ones in order to minimize numerical errors in the calculation (Jacobsen et al., 2005).

$$\varphi = (\text{Latitude} - \text{LAT\_OFF})/\text{LAT\_SCALE}$$

$$\lambda = (\text{Longitude} - \text{LONG\_OFF})/\text{LONG\_SCALE}$$

$$H = (\text{Height} - \text{HEIGHT\_OFF})/\text{HEIGHT\_SCALE}$$

$$R = (\text{ROW} - \text{LINE\_OFF})/\text{LINE\_SCALE}$$

$$C = (\text{Column} - \text{SAMPLE\_OFF})/\text{SAMPLE\_SCALE}$$

Each polynomial is the third order one with respect to  $\varphi$ ,  $\lambda$ , H, and consists of as many as 20 terms. For example, for a generic set C of polynomial coefficients, the corresponding 20-term cubic polynomial has the form:

$$P(\varphi,\lambda,H) = C_1 + C_2\lambda + C_3\varphi + C_4H + C_5\lambda\varphi + C_6\lambda H + C_7\varphi H + C_8\lambda^2 + C_9\varphi^2 + C_{10}H^2 + C_{11}\varphi^2H + C_{12}\lambda^3 + C_{13}\lambda\varphi^2 + C_{14}\lambda H^2 + C_{15}\lambda^2\varphi + C_{16}\varphi^3 + C_{17}\varphi H^2 + C_{18}\lambda^2 H + C_{19}\varphi\lambda H + C_{20}H^3$$
(4)

This is a third-order rational function with 20-term polynomial that transforms point coordinates from the object space to the image space. Substituting *P* in (2) with the polynomials (4) and eliminating the first coefficient in the denominator, leads to a total of 39 Rational Function (RF) coefficients in each equation: 20 coefficients in the numerator and 19 in the denominator (Wolniewicz, 2004). Since each GCP produces two equations, at least 39 GCPs are required to solve for the 78 coefficients (Di et al., 2001, 2003a, 2003b). RPC are usually calculated by providers of satellite images without using GCPs. Instead, the object space is sliced in the vertical direction to generate virtual control points (Tao and Hu, 2001; Di et al., 2003a). For Ikonos images, ground coordinates derived from such RPC typically achieve an accuracy level similar to that of provided by Geoproducts (about 25 m). If quality GCPs are available, the accuracy of the determined points may reach the ground accuracy.

# 2.2. Parametrical model - reconstruction of the imaging geometry

Parametrical model describes in strictly geometrical terms the relations between the terrain and its image. The model has to take into consideration the above-mentioned multi-source distorting factors. In the event of classical photogrammetric image, such strict model is based on the assumption of co-linearity, that is fundamental for photogrammetry. The model includes the elements of external orientation as well as 6 elements of the image internal orientation, i.e. location in space and 3 inclination angles. Condition of co-linearity is also fundamental for the construction of the strict model of satellite images. However, in that case it might not be applied to the entire image, but just to a single line. So, the elements of satellite image orientation, in a sense as it is in the case of aerial photographs, cannot be discussed. Orientation elements are subjects to continuous change, so the function of those elements in relation to time should rather be discussed. Information on the construction of such models provided by different research centres is available in literature. However, the authors of published papers do not disclose the operational forms of the algorithms. The models quite often include a lot of unknown elements - parameters, which value for a given image is determined on the basis of the GCP's of known location on the ground and identified on the image.

After calibration of optical system, and in the case of precise determination of the camera external orientation elements that are variable in time, the elements of parametrical model are known. This provides for "straight" elaboration, e.g. "ortho-adjustment", without knowing the photopoints (but knowing the DTM), or for generation of DTM from stereoscopic images. Some distributors, who do not wish to disclose model parameters in the decrypted form, calculate – for a given image scene – the respective polynomial coefficients in quotient model, and they enclose these values to the images offered to end-users.

Because parametrical model describes the real geometrical relations, all model parameters have specific geometrical interpretation. Parametrical models should produce better results than the non-parametrical ones; they should be more resistant to distribution of GCP's, and possible errors in data, and they should also require less GCP's needed for determination of unknown parameters. The leading manufacturers of photogrammetric software supplement their products with the options enabling for elaboration of satellite images obtained from the basic systems, including recently the elaboration of the very high resolution satellite images. Usually, they offer optional selection between the strict model and quotient polynomial one. One should especially pay attention on the recent version of the Geomatica OrtoEngine package, offered by the Canadian company PCI. The package includes a "firmware" in the form of strict models of the most important satellite systems elaborated by Toutin from the Canada Centre for Remote Sensing - CCRS. The model enables correction of satellite images with a small number of GCP's (less than 10). The system administrator - Space Imaging - has not published, however, the strict model of Ikonos, but Toutin reconstructed this model on the basis of theoretical assumptions as well as on the basis of meta-data that constitute a standard attachment to distributed images. Today many research centers all over the world have established their own correction models based upon the co-linearity condition. The most modern are the models described

by Toutin et al., (2004), Zhang et al., (2004), Jacobsen et al., (2005). At the Institute of Photogrammetry and Cartography of the Warsaw University of Technology, a generic algorithm describing geometrical relations between an image and terrain based upon photogrammetric rules was elaborated.

In the following chapter the author's concept of the parametric model will be presented.

# 2.2.1. Colinearity of linear array imagery

Figure 2 presents a linear array sensor that is composed of a row of CCD elements perpendicular to the flying direction (Fig. 2a). There are two cases related with the sensor array tilts. The first is a sensor array tilted laterally on both sides by an angle  $\alpha$  (Fig. 2b), to obtain imagery from another strip, e.g. SPOT sensor. The collinearity equations are (Wang, 1990; Wolniewicz, 2005)

$$0 = f \frac{a_1(X - X_0) + a_2(Y - Y_0) + a_3(Z - Z_0)}{a_7(X - X_0) + a_8(Y - Y_0) + a_9(Z - Z_0)}$$

$$f \frac{y \cos \alpha + f \sin \alpha}{f \cos \alpha - y \sin \alpha} = -f \frac{a_4(X - X_0) + a_5(Y - Y_0) + a_6(Z - Z_0)}{a_7(X - X_0) + a_8(Y - Y_0) + a_9(Z - Z_0)}$$
(5)

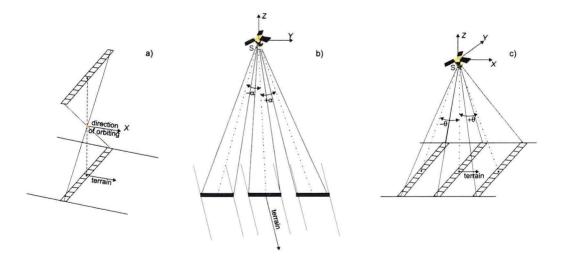


Fig. 2. Linear array elements: a) a row of CCD elements perpendicular to the flying direction, b) lateral tilts of sensor array, c) forward and backward tilts of sensor array

The second is a case when the sensor array is tilted forward or backward in the flying direction by an angle  $\theta$  (Fig. 2c), for example Ikonos, QuickBird, ORBVIEW etc. The collinearity equations have following form

$$f \operatorname{tg} \theta = -f \frac{a_1(X - X_0) + a_2(Y - Y_0) + a_3(Z - Z_0)}{a_7(X - X_0) + a_8(Y - Y_0) + a_9(Z - Z_0)}$$

$$\frac{y}{\cos \theta} = -f \frac{a_4(X - X_0) + a_5(Y - Y_0) + a_6(Z - Z_0)}{a_7(X - X_0) + a_8(Y - Y_0) + a_9(Z - Z_0)}$$
(6)

In (5) and (6) the coefficients  $a_i$  (i = 1, 2, 3, ..., 9) are the elements of rotational matrix **A** (see (7)); f is the calibrated focal length of camera; x, y are image coordinates;  $X_0, Y_0, Z_0$  are orbital coordinates of the exposure station corresponding to the ground point of X, Y, Z coordinates in the geocentric reference system;  $\alpha$ ,  $\theta$  are the lateral and forward (or backward) angles, respectively.

## 2.2.2. Construction of rotational matrix

For determining the elements  $a_i$  (i = 1, 2, 3, ..., 9) of rotational matrix **A** one has to determine the geometric relationship between imagery and Earth's surface in the geocentric reference system that is presented in Fig. 3.

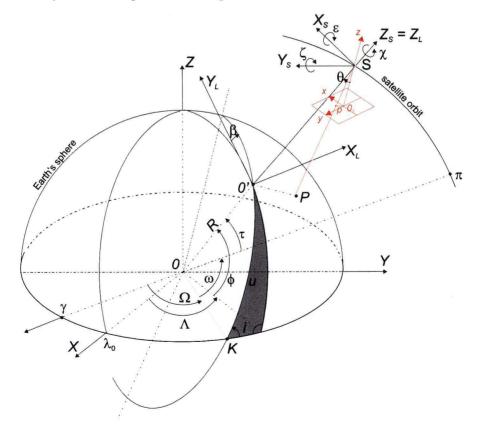


Fig. 3. Geometric relationship between imagery and Earth's surface in the geocentric reference system

where

 $\gamma$  – vernal equinox,

 $\lambda_0$  – longitude of Greenwich meridian,

K – ascending node,

 $\pi$  – perigee,

 $\Lambda$  – geocentric longitude,

OO' - Earth's radius,

i – inclination of orbital plane,

 $\Omega$  – right ascension,

 $\omega$  – argument of perigee,

 $\tau$  – true anomaly at time t,

 $\Phi$  – geocentric latitude,

O'S - orbital height,

R = OO' + O'S – geocentric radius of the satellite at time t.

There are four very important coordinate systems presented in Fig. 3:

 $O_1 xyz$ 

- imagery coordinate system,

 $SX_SY_SZ_S$ 

satellite coordinate system,

 $O'X_LY_LZ_L$  – local geodetic system,

- geocentric system.

In order to avoid the problem of map projection discontinuities, the geocentric coordinate reference system was adopted. The position of satellite  $X_S$ ,  $Y_S$ ,  $Z_S$ , also described in the geocentric coordinate system, can be computed for each array line. In the image coordinate system (x, y, z), x corresponds to the number of lines in the imagery along with flight direction (see Fig. 3); y to the number of samples in the cross-track direction; and z to the principal distance of the camera, perpendicular to the image. Since the imagery is linear, then x is a measurement of time variable and is assigned to zero (Fig. 3), while z takes the value (-f). Satellite orbit is determined on the basis of Kepler laws. Satellite position is determined by Keplerian orbit parameters:  $a, e, i, \Omega, u = (\omega + \tau)$ , where a is a semi-major axis, e - the eccentricity, and i,  $\Omega$ ,  $u = (\omega + \tau)$  are described in Fig. 3. Basing on i,  $\Omega$ , u angles, the rotational matrix C can be established for rotating satellite coordinate system  $SX_SY_SZ_S$  with respect of the geocentric reference system OXYZ. In the similar way, the rotational matrix B defining transformation of image coordinate system into satellite coordinate system can be determined. The elements of the rotational matrix **B** are functions of Eulerian parameters  $\varepsilon$ ,  $\zeta$ ,  $\chi$ . The image coordinate system  $O_1xyz$  can now be transformed into the geocentric system OXYZ with the rotational matrix A, determined as follows (Wolniewicz et al., 2005)

$$\mathbf{A} = \mathbf{B}^{\mathsf{T}} \mathbf{C}^{\mathsf{T}} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$$
 (7)

The elements  $a_i$  (i = 1, 2, 3, ..., 9) of the rotational matrix A are functions of  $i, \Omega, u$ ,  $\varepsilon$ ,  $\zeta$ ,  $\chi$ . They are later used in (5) and (6).  $X_0$ ,  $Y_0$ ,  $Z_0$  coordinates of the exposure station in (5) and (6) are computed as follows:

$$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} c_3 R \\ c_6 R \\ c_9 R \end{bmatrix}$$
 (8)

where  $c_3$ ,  $c_6$ ,  $c_9$  are the elements of a third column of rotational matrix C; and R = OO' + O'S (Fig. 3).

All scanning lines are recorded at different time t, therefore, image orientation parameters will be functions of time t. Image orientation parameters can efficiently be approximated using functions of time t:

$$R = R_0 + R_1 t + R_2 t^2, R = R_0 + R_1 n + R_2 n^2, 
\Omega = \Omega_0 + \Omega_1 t + \Omega_2 t^2, or \Omega = \Omega_0 + \Omega_1 n + \Omega_2 n^2, 
i = i_0 + i_1 t + i_2 t^2, i = i_0 + i_1 n + i_2 n^2, 
u = u_0 + u_1 t + u_2 t^2, u = u_0 + u_1 n + u_2 n^2,$$
(9a)

and

$$\varepsilon = \varepsilon_0 + \varepsilon_1 t + \varepsilon_2 t^2, \qquad \varepsilon = \varepsilon_0 + \varepsilon_1 n + \varepsilon_2 n^2, 
\zeta = \zeta_0 + \zeta_1 t + \zeta_2 t^2, \qquad \text{or} \qquad \zeta = \zeta_0 + \zeta_1 n + \zeta_2 n^2, 
\chi = \chi_0 + \chi_1 t + \chi_2 t^2, \qquad \chi = \chi_0 + \chi_1 n + \chi_2 n^2,$$
(9b)

where n is the number of scanning lines.

Basing on relations (6), (7), (8), and (9), a pair of equations can be written for each detector line, where  $F_x$ ,  $F_y$  are functions representing the relation between geocentric coordinates X, Y, Z of the ground point and its image coordinates x, y.

$$F_{x}(f, X, Y, Z, \theta, R_{0}, R_{1}, R_{2}, \Omega_{0}, \Omega_{1}, \Omega_{2}, i_{0}, i_{1}, i_{2}, u_{0}, u_{1}, u_{2}, \varepsilon_{0}, \varepsilon_{1}, \varepsilon_{2}, \zeta_{0}, \zeta_{1}, \zeta_{2}, \chi_{0}, \chi_{1}, \chi_{2}) = 0$$

$$(10)$$

$$F_{y}(f, y, X, Y, Z, \theta, R_{0}, R_{1}, R_{2}, \Omega_{0}, \Omega_{1}, \Omega_{2}, i_{0}, i_{1}, i_{2}, u_{0}, u_{1}, u_{2}, \varepsilon_{0}, \varepsilon_{1}, \varepsilon_{2}, \zeta_{0}, \zeta_{1}, \zeta_{2}, \chi_{0}, \chi_{1}, \chi_{2}) = 0$$

The observation equation in the matrix form is as follows

$$V = Dd - L$$
 with weight matrix P (11)

where  $\mathbf{V} = \begin{bmatrix} v_{x_1} & v_{y_1} & \dots & v_{x_N} & v_{y_N} \end{bmatrix}^T$  - vector of residuals,

 $\mathbf{L} = \begin{bmatrix} F_{x_1}^0 & F_{y_1}^0 & \dots & F_{x_N}^0 & F_{y_N}^0 \end{bmatrix}^T - \text{coefficient vector},$ 

D - matrix of partial derivatives,

d - unknown vector of parametric increments,

**P** – diagonal weight matrix,

N – number of points used.

The equation (10) is used to form the normal equation system. Its solution provides the elements of the vector  $\mathbf{d}$ ; then orientation parameters given by (9) are determined. Having determined parameters, any image point coordinates can be transformed into the geocentric reference system and then, into the geodetic reference system.

It is necessary to realize that all Ground Control Points (GCP's) have to be first transformed into the geocentric reference system, in which all operations will be done.

At the final step, the image points transformed into the geocentric reference system will further be transformed into geodetic system.

To determine the errors of sensor internal orientation, following formulae could be used

$$dx = dx_0 + \frac{x}{f}df + t_1xr^2 + t_2xr^4 + t_3xr^6 + p_1(y^2 + 3x^2) + p_22xy$$

$$dy = dy_0 + \frac{y}{f}df + t_1yr^2 + t_2yr^4 + t_3yr^6 + p_2(x^2 + 3y^2) + p_12xy$$
(12)

where dx, dy are the corrections to image coordinates;  $dx_0$ ,  $dy_0$ , df are the errors of internal orientation;  $t_1$ ,  $t_2$ ,  $t_3$  are the coefficients charactering error of symmetrical distortion; and  $p_1$ ,  $p_2$  are the coefficients charactering error of asymmetrical distortions. The number of unknown parameters in (8) will now be increased by 8  $(dx_0, dy_0, df, t_1, t_2, t_3, p_1, p_2)$ .

In order to reduce the influence of ground height differences h of GCP on the displacement of image point that decreases the accuracy of computed image orientation parameters, image point can be corrected as follows

$$dx_{h} = -\frac{x\cos\theta}{H''}h$$

$$dy_{h} = -\frac{y\cos\theta}{H''}h$$
(13)

where  $H'' = (Z - Z_0) \sin \theta + (Z - Z_0) \cos \theta$ ; h are height differences of ground control points; and  $Z_0$ , Z and  $\theta$  – are taken from (5) or (6).

The details of parametric model developed by the authors are described in the technical report (Wolniewicz et al., 2005).

#### 3. Conclusions

Available on the market correction models for the VHRS images based upon RPC concept, have numerous advantages. A minimum number of photopoints is required for orthoadjustment process. It is not necessary to know the parameters of sensor model. It is also not necessary to know the elements of external orientation of image and orbit parameters. Primary disadvantage of the RPC-based models is a missing physical interpretation of parameters included in meta-data of VHRS images. Parameters provided by the distributors describe only indirectly the relation between the image and terrain. In some cases the distributed RPC parameters seem not sufficiently precise. Then the use of parametric model would be recommended.

The main advantage of parametrical model consists in describing the relation between the image and terrain by mathematical equations. It is also possible to define all parameters involved in adjustment process. Parametrical model has one general disadvantage, namely a large number of photopoints are needed for adjustment (minimum 7–9 GCPs). It increases the costs of adjustment.

Parametric model developed by the authors might efficiently be applied for correcting very high resolution satellite imageries. The description of the mechanism of the model allows better understanding the complexity of the correction process what is beneficial for geometrical processing of VHRS data. Complete verification of the model developed is recently undertaken at the Warsaw University of Technology.

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#### Modele geometryczne systemów satelitarnych VHRS

#### Luong Chinh Ke, Wiesław Wolniewicz

Politechnika Warszawska
Instytut Fotogrametrii i Kartografii
Plac Politechniki 1, 00-661 Warszawa
e-mail: lchinhke@gazeta.pl; w.wolniewicz@gik.pw.edu.pl

#### Streszczenie

Rok 2000, w którym pierwszy obraz o metrowej rozdzielczości dostarczony przez satelitę Ikonos z Space Imaging zelektryzował światowe rynki komercyjne i ośrodki badawcze, można uważać za początek nowej ery technik obrazów satelitarnych. Jednym z głównych kierunków badań prowadzonych w zakresie praktycznego wykorzystania zobrazowań VHRS (Very High Resolution Satellite) jest optymalizacja modelu matematycznego obrazu dla jego korekcji geometrycznej i wytworzenia dokładnej ortofotomapy. Sprowadza się to do ustalenia poprawnych matematycznych zależności pomiędzy współrzędnymi zobrazowania i ich odpowiednikami w terenie. Rozróżnia się dwie zasadnicze grupy modeli. Pierwsza dotyczy zastępczej postaci modelu kamery – RPC (Rational Polynomial Coefficients), druga zaś uwzględnia fizyczne parametry kamery (skanera).

W artykule opisana jest koncepcja budowy modelu geometrycznego opartego na współczynnikach RPC. Są w nim przedstawione zasadnicze zalety i niedostatki wynikające ze stosowania tego modelu w praktyce ortorektyfikacji VHRS. W dalszej kolejności opisany jest model parametryczny, stworzony przez autorów. Na bazie fundamentalnego warunku kolinearności wektora terenowego punktu i odpowiadającego mu wektora obrazowego, przy pomocy kolejnej transformacji ustalono relacje między położeniem punktu w układzie terenowym, a jego obrazem tworzonym przez poruszający się (dynamiczny) skaner satelitarny, o ciągle zmiennym w czasie położeniu w przestrzeni.