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New algorithms of GPS post-processing for multiple baseline models and analogies to classical geodetic networks

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Abstract: The paper presents new differencing algorithms for post-processing GPS data, using double or triple carrier phase differences and multiple baseline sessions. The characteristic feature of the new algorithms is, that they use full sets of Schreiber's type observation differences with theoretically proved diagonal weight matrices. The proposed estimation models are equivalent to the least squares estimation applied to the original system of un-differenced observation equations. The theoretical ground of the algorithms are the theorems on the properties of differencing equations of Schreiber's type. The theorems become practically useful mainly in case of functional models with triple-differences. In a classical approach, this task was simplified for the sake of necessity of inverting non diagonal covariance matrix, usually of a large dimension. Diagonal weight matrix is also obtained in case of multiple point observation session where correlation of the GPS vectors forces in practice the use of the simplified stochastic models. The proposed method eliminates also the problem of selection of a reference satellite. It is very important especially in case of long observation sessions. The algorithms are applied in professional software for GPS relative positioning.

Keywords: GPS, post-processing, carrier phase differences, triple-differences, multiple baseline session, elimination of nuisance parameters, BETA method, diagonal weight matrix

1. Introduction

In certain geodetic observation systems, such as static GPS observations, the pseudo-observations represented by differences of original observations are created to eliminate nuisance parameters or in the same sense – some systematic errors. The use of double-differences in processing GPS baselines eliminates satellite and receiver clock offsets, while the use of triple-differences eliminates in addition unknown linear combinations of ambiguities. A geodetic network with measured directions is the classical example of such problem; nuisance parameters are the orientation constants of directions for each observational station of the network. The orientation constants can naturally be eliminated after differencing the original observations.

Since in general case the pseudo-observations obtained after differencing original observations are not independent, the least squares unbiased estimation should be performed, generally, with a non diagonal weight matrix as the inverse or a pseudo-inverse of a non diagonal covariance matrix. For a large set of observations, it may lead to complicated numerical procedures and thus the strict algorithms are likewise replaced by the algorithms with various simplifications applied, e.g. by total neglecting the correlations. Analogical problems, for a case of multiple baseline GPS network with double phase differences as pseudo-observations were discussed (e.g. Ashkenazi and Yau, 1986; Beutler et al., 1986, 1987; Remondi, 1984; Eren, 1987). Numerical problems become significant in models of triple-differences, in particular for a multiple baseline network, where the inverse of the covariance matrix is a full matrix. Xu (2007) in his book writes "Taking the correlation between the baselines into account, an exact correlation description of the triple-differences of a GPS network turns out to be very complicated" and proposes some un-differenced alternatives, applying the elimination theory. Leick (2004) concludes: "Often the triple-difference solution serves as a pre-processor to get good initial position for the double-difference solution. The triple-differences have the advantage in that cycle slips are mapped as individual outliers in the computed residuals". On the other hand, it is known, that the correct double-difference solution depends on not always "sharp" numerically identified ambiguities, as the linear combination of integer values; the problems are discussed in literature (e.g. Rzepecka, 2004). In spite of the standard schemes, the triple-difference solution, cancelling the ambiguity functions as nuisance parameters (in the case of the ionosphere-free combination L1/L2 are not integer), still remains very attractive. The mentioned above numerical problems with mutually correlated differences are in the next part solved by using simple functional models, that are strictly equivalent to original task.

A special closed set of observation differences, leading strictly to the estimation procedure with diagonal weight matrix, which results are equivalent to the least squares estimators for the original observation system (un-differenced phase observations) is considered in this work. The set of observation differences is analogous to classical Schreiber's model of angle measurements (Schreiber, 1878). The application of such differences can be found in GPS phase data processing, especially for triple-differences. The main idea and properties of the set of Schreiber's type observation differences was presented in Kadaj (2006) and applied to the post-processing of GPS data (BETA method) (Kadaj, 2007, 2008) as well as in the automatic post-processing software for network of active multifunctional permanently operating GPS stations in Poland (Kadaj and Świętoń, 2008).

2. The set of Schreiber's type observation differences on the example of the classical geodetic network

Let $\{k_1, k_2, ..., k_m\}$ be the set of measured directions (Fig. 1a) at a station to some target points in a classical geodetic network. The matrix $\mathbf{Q}$ is the corresponding diagonal covariance matrix

$$\mathbf{Q} = \mathrm{diag}[\sigma_i^2 : i = 1, 2, ..., m] \tag{1}$$

with σ_i being an a priori mean error (standard deviation) for direction k_i, while the weight matrix $\mathbf{P} = \mathbf{Q}^{-1}$ is

$$\mathbf{P} = \mathrm{diag}[p_i : i = 1, 2, ..., m] = \mathbf{Q}^{-1} \tag{2}$$

with weights of directions $p_i = 1/\sigma_i^2$, and m is a number of directions observed at the station.

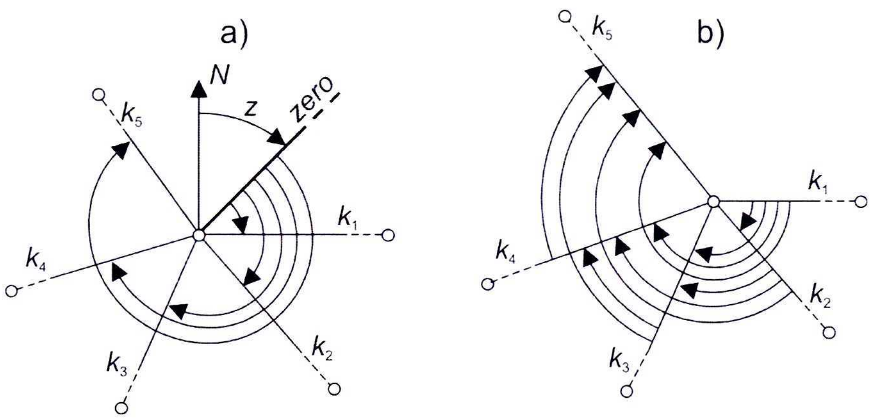

Fig. 1. The set of observed directions (a) and observation differences of Schreiber's type (b) (the pseudo-observations are computed from direction measurements)

The standard least squares problem for the subset of original observations is defined by equations

$v_i + k_i + z =$ (the function of estimated coordinates of a station and target points) (3)

where v_i are observation corrections as estimators of unknown observation errors ($i = 1, 2, \ldots, m$), z is the orientation constant as a nuisance parameter, and the sum

$$\sum_{i=1}^{m} p_i \cdot v_i^2$$

is the adequate part of the minimized least squares function for a network.

The performance of non-singular estimation process provides the unknown parameters, i.e. point coordinates and orientation constant z. Naturally, any differencing between measured directions (angles) eliminates the orientation constant; for example, creating the set of angles between adjacent directions

$$\{(k_j - k_i) : i = 1, 2, \ldots, m - 1; j = i + 1\} \tag{4}$$

Unfortunately, the covariance matrix corresponding to (4) is not diagonal and leads to complicated adjustment algorithms.

Classical geodetic literature describes the Schreiber's method of angle measurement (Fig. 1b) (Schreiber, 1878). The Schreiber's scheme can be used as a functional model, creating the system of differences

$$\alpha_{i,j} = k_j - k_i \text{ for all } (i, j) \in \boldsymbol{S} \tag{5}$$

with the Schreiber's scheme of differences

$$\begin{aligned}\boldsymbol{S} &= \{(i, j) : i = 1, 2, \ldots, m-1; j = i+1, i+2, \ldots, m\} \\ &= \{(i, j) : 1 \leq i < j \leq m\}\end{aligned} \tag{6}$$

The set of angles in the Schreiber's scheme is triangular (Fig. 2).

$$\begin{array}{ccccc} \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,4} & \cdots & \alpha_{1,m} \\ & \alpha_{2,3} & \alpha_{2,4} & \cdots & \alpha_{2,m} \\ & & \alpha_{3,4} & \cdots & \alpha_{3,m} \\ & & & & \cdots\cdots \\ & & & & \alpha_{m-1,m} \end{array}$$

Fig. 2. The Schreiber's set of angles $\alpha_{i,j}$: $(i, j) \in \boldsymbol{S}$

In the next part of this work it is proved, that the least squares adjustment process, using the Schreiber's set of differenced pseudo-observations (5) with the diagonal weight matrix

$$\begin{aligned}\mathbf{W} &= \text{diag}[w_{1,2}, w_{1,3}, \ldots, w_{1,m}, w_{2,3}, \ldots, w_{m-1,m}] \\ &= \text{diag}[w_{i,j} : (i, j) \in \boldsymbol{S}]\end{aligned} \tag{7}$$

where

$$w_{i,j} = p_i \cdot p_j \cdot c^{-1} \tag{8}$$

$$c = \sum_{i=1}^{m} p_i \tag{9}$$

gives results identical to least squares estimators for the original system of un-differenced observation equations (3).

The above property can be applied to algorithms applied for GPS relative positioning, in general for a multiple baseline session, by using double- or triple-differences. In each case of a set of Schreiber's type differences the diagonal weight matrix is obtained; it is mostly advantageous for triple-differences.

3. Un-differenced observation equations system and the Schreiber's model of differences

3.1. Initial assumption

Assume at first an uncorrelated observation system, divided on s groups

$$\mathbf{L}^{(r)} + \varepsilon^{(r)} = \mathbf{A}^{(r)} \cdot \mathbf{X} + \mathbf{J}^{(r)} \cdot y_r \quad \text{for } r = 1, 2, \ldots, s \tag{10}$$

where $m = m_1 + m_2 + \ldots + m_s$ is a number of observations as the sum from s groups,

$\mathbf{L}^{(r)} = [L_i^{(r)}]$ is $(m_r \times 1)$ observation vector,
$\mathbf{X} = [X_j]$ is $(n \times 1)$ unknown parameters vector,
y_r is an unknown parameter (nuisence parameter) for r-th group,
$\mathbf{A}^{(r)} = [A_{ij}^{(r)}]$ is $(m_r \times n)$ design matrix,
$\mathbf{J}^{(r)}$ is $(m_r \times 1)$ unit column matrix $[1, 1, \ldots, 1]^{\mathrm{T}}$,
$\boldsymbol{\varepsilon}^{(r)} = [\varepsilon_i^{(r)}]$ is $(m_r \times 1)$ vector of observation errors with the expected value $\mathrm{E}\{\boldsymbol{\varepsilon}^{(r)}\} = \mathbf{0}$,
$\mathbf{C}^{(r)} = \mathrm{E}\{\boldsymbol{\varepsilon}^{(r)} \cdot (\boldsymbol{\varepsilon}^{(r)})^{\mathrm{T}}\} = \mathrm{diag}[(\sigma_i^{(r)})^2]$ is the diagonal covariance matrix,
$\sigma_i^{(r)}$ is the standard deviation for i-th observation in r-th group, $i = 1, 2, \ldots, m_r$.

In general, the system (10) can be determined as a part of an integrated system.

The weighted, unbiased, least squares estimators $\mathbf{X}^{\wedge}$, $y_1^{\wedge}$, $y_2^{\wedge}$, ..., $y_s^{\wedge}$ of $\mathbf{X}$ and $y_1, y_2, \ldots, y_s$, respectively, minimize a global function (for an integrated system), with the part adequate to (10)

$$\sum_{r=1}^{s} (\mathbf{V}^{(r)})^{\mathrm{T}} \cdot \mathbf{P}^{(r)} \cdot \mathbf{V}^{(r)} \tag{11}$$

where

$$\mathbf{V}^{(r)} = [v_i^{(r)}] = \mathbf{A}^{(r)} \cdot \mathbf{X}^{\wedge} + \mathbf{J}^{(r)} \cdot y_r^{\wedge} - \mathbf{L}^{(r)} \tag{12}$$

is the estimator of $\boldsymbol{\varepsilon}^{(r)}$, and

$$\mathbf{P}^{(r)} = (\mathbf{C}^{(r)})^{-1} = \mathrm{diag}[p_i^{(r)}] \tag{13}$$

is $(m_r \times m_r)$ diagonal weight matrix, while

$$p_i = 1/\sigma_i^2 \tag{14}$$

is the weight of i-th observation.

For the defined estimator the following equations are fulfilled:

$$(\mathbf{A}^{(r)})^{\mathrm{T}} \cdot \mathbf{P}^{(r)} \cdot \mathbf{V}^{(r)} = \mathbf{0} \tag{15}$$

and

$$(\mathbf{J}^{(r)})^{\mathrm{T}} \cdot \mathbf{P}^{(r)} \cdot \mathbf{V}^{(r)} = \sum_{i}^{m_r} p_i^{(r)} \cdot v_i^{(r)} = 0 \tag{16}$$

Naturally, one can eliminate nuisance parameters y_r, creating the differences between simple scalar observation equations and (on the basis of the error propagation law) the corresponding stochastic model, comprising covariance and weight matrices. Unfortunately, in an optional case, the obtained system of pseudo-observations (observation differences) gives a result that is not identical with the original one, that concerns un-differenced task. Much more serious problem results from non-diagonal covariance matrix, particularly for a large number of observations. For example, in the GPS post-processing with successively defined (between adjacent epochs) triple-differences – the obtained pseudo-observation system has the non-diagonal covariance matrix and the weight matrix is a full matrix. We prove, that in each case of the solution based on phase differences (also in case of the multiple-point session), none generalization (simplification) of stochastic model is necessary, and the strict least squares solution with a diagonal weight matrix can be obtained, using Schreiber's type differences.

3.2. Schreiber's model of observation differences

The Schreiber's model of observation differences, concerning (10-16), is defined by the "triangular" set of pairs of observation indices (subscripts):

$$\boldsymbol{S}_r = \{(i,j) : i = 1,2, \ldots, m_r - 1; j = i+1, i+2, \ldots, m_r\},\ r = 1,2, \ldots, s \text{ (groups)} \tag{17}$$

determining the set of observation differences

$$L_i^{(r)} - L_j^{(r)} + v_i^{(r)} - v_j^{(r)} = \sum_{k=1}^{n} (A_{ik}^{(r)} - A_{jk}^{(r)}) \cdot x_k^{\wedge} \tag{18}$$

for each $(i, j) \in \boldsymbol{S}_r$ and for each group $r = 1, 2, \ldots, s$.
Comments:

- if $\boldsymbol{R}_r = \{1, 2, \ldots, m_r\}$ is the set of observation indices in r-th group then $\boldsymbol{S}_r \subset \boldsymbol{R}_r \times \boldsymbol{R}_r$ (Cartesian product),
- the subset $\boldsymbol{S}_r$ (for r-th group) has u_r elements

$$u_r = m_r \cdot (m_r - 1)/2 \tag{19}$$

but only $m_r - 1$ equations (18) are independent. The remaining $u_r - m_r + 1$ equations are linear combinations of first $m_r - 1$ equations,

- the product $\boldsymbol{R}_r \times \boldsymbol{R}_r$ has m_r^2 elements; containing u_r elements of Schreiber's model, u_r elements of symmetric triangular table (for $i > j$) (for $i = j$ the components of (18) reduce to zero) $m_r^2 = 2 \cdot u_r + m_r$.

  Denoting

$$\delta L_{ij}^{(r)} = L_i^{(r)} - L_j^{(r)}, \quad \delta v_{ij}^{(r)} = v_i^{(r)} - v_j^{(r)}, (i, j) \in \boldsymbol{S}_r \tag{20}$$

and

$$\boldsymbol{\delta}\mathbf{L}^{(r)} = [\delta L_{ij}^{(r)} : (i, j) \in_r], \quad \boldsymbol{\delta}\mathbf{v}^{(r)} = [\delta v_{ij}^{(r)} : (i, j) \in \boldsymbol{S}_r] \tag{21}$$

as $(u_r \times 1)$ vectors, and

$$\mathbf{a}^{(r)} = [(A_{ik}^{(r)} - A_{jk}^{(r)}) : (i, j) \in \boldsymbol{S}_r \text{ and } k = 1, 2, \ldots, n] \tag{22}$$

as $(u_r \times n)$ matrix of coefficients, the equations (18) can be written in the matrix form

$$\boldsymbol{\delta}\mathbf{L}^{(r)} + \boldsymbol{\delta}\mathbf{v}^{(r)} = \mathbf{a}^{(r)} \cdot \mathbf{X}^{\wedge}, r = 1, 2, \ldots, s \tag{23}$$

The Schreiber's sets are created independently for each group. On the ground of the general stochastic assumption for (10), if $r_1 \neq r_2$ ($r_1, r_2 = 1, 2, ..., s$) then the vectors $\boldsymbol{\delta}\mathbf{L}^{(r_1)}, \boldsymbol{\delta}\mathbf{L}^{(r_2)}$ of observation differences are uncorrelated, i.e.

$$E\{\boldsymbol{\delta\varepsilon}^{(r_1)} \cdot (\boldsymbol{\delta\varepsilon}^{(r_2)})^{\mathrm{T}}\} = \mathbf{0} \text{ (zero matrix of } (u_{r_1} \times u_{r_2}) \text{ dimension)} \tag{24}$$

where with $r = 1, 2, \ldots, s$

$$\boldsymbol{\delta}\mathbf{L}^{(r)} + \boldsymbol{\delta\varepsilon}^{(r)} = \mathbf{a}^{(r)} \cdot \mathbf{X} \tag{25}$$

$$\boldsymbol{\delta\varepsilon}^{(r)} = [(\varepsilon_i^{(r)} - \varepsilon_j^{(r)}) : (i, j) \in \boldsymbol{S}_r] \tag{26}$$

are observation equations for Schreiber's differences. Therefore, in the next formulae, for simplicity, the problem for only one Schreiber's group is considered, and the index of r-th group will be neglected.

3.3. Equivalence between original and differenced Schreiber's model

The two theorems show the properties of the Schreiber's model of observation differences and their equivalence with the original, un-differenced observation system.

Theorem 1 (two theses)

a) The equality

$$\boldsymbol{\delta}\mathbf{v}^{\mathrm{T}} \cdot \mathbf{W} \cdot \boldsymbol{\delta}\mathbf{v} = \mathbf{V}^{\mathrm{T}} \cdot \mathbf{P} \cdot \mathbf{V} \tag{27}$$

is fulfilled for the following $(u \times u)$ diagonal weight matrix:

$$\mathbf{W} = \text{diag}[w_{i,j} : (i, j) \in \boldsymbol{S}] \tag{28}$$

where

$$w_{i,j} = p_i \cdot p_j \cdot c^{-1};$$

$$c = \sum_{i=1}^{m} p_i \tag{29}$$

with p_i – weight of original observation, $i = 1, 2, \ldots, m$,
and
b) If $\mathbf{X}^\wedge$ is the least squares estimator for original system (10) then it is also the unbiased least squares estimator for the system (25).

Proofs:

The thesis a) can be written in the scalar form

$$c^{-1} \cdot \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} (v_i - v_j)^2 \cdot p_i \cdot p_j = \sum_{i=1}^{m} v_i^2 \cdot p_i \tag{30}$$

Considering $(v_i - v_i)^2 \cdot p_i^2 = 0$ and $(v_i - v_j)^2 \cdot p_i \cdot p_j = (v_j - v_i)^2 \cdot p_j \cdot p_i$ (symmetry) one obtains

$$\begin{aligned}
c^{-1} \cdot \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} (v_i - v_j)^2 \cdot p_i \cdot p_j = \\
&= \frac{1}{2} c^{-1} \cdot \sum_{i=1}^{m} \sum_{j=1}^{m} (v_i - v_j)^2 \cdot p_i \cdot p_j \\
&= \frac{1}{2} c^{-1} \cdot [\sum_{i=1}^{m} \sum_{j=1}^{m} v_i^2 \cdot p_i \cdot p_j + \sum_{i=1}^{m} \sum_{j=1}^{m} v_j^2 \cdot p_i \cdot p_j - 2 \sum_{i=1}^{m} \sum_{j=1}^{m} v_i \cdot v_j \cdot p_i \cdot p_j] \\
&= \frac{1}{2} c^{-1} \cdot [\sum_{i=1}^{m} v_i^2 \cdot p_i \cdot \sum_{j=1}^{m} p_j + \sum_{j=1}^{m} v_j^2 \cdot p_j \cdot \sum_{i=1}^{m} p_i - 2 \sum_{i=1}^{m} v_i \cdot p_i \cdot \sum_{j=1}^{m} v_j \cdot p_j] \\
&= \frac{1}{2} c^{-1} \cdot [2 \sum_{i=1}^{m} v_i^2 \cdot p_i \cdot c - 2 \sum_{i=1}^{m} v_i \cdot p_i \cdot \sum_{j=1}^{m} v_j \cdot p_j] \qquad (31) \\
&= \sum_{i=1}^{m} v_i^2 \cdot p_i \; - \; c^{-1} \cdot [\sum_{i=1}^{m} v_i \cdot p_i]^2 \\
&= \sum_{i=1}^{m} v_i^2 \cdot p_i \; - \; c^{-1} \cdot 0 \\
&= \sum_{i=1}^{m} v_i^2 \cdot p_i
\end{aligned}$$

In deriving (31) the equality $\sum_{i=1}^{m} v_i \cdot p_i = 0$ as in (16) is used.

The thesis b) of the theorem can be expressed as follows

$$\mathbf{a}^{\mathrm{T}} \cdot \mathbf{W} \cdot \boldsymbol{\delta}\mathbf{v} = \mathbf{a}^{\mathrm{T}} \cdot \mathbf{W} \cdot (\mathbf{a} \cdot \mathbf{X}^{\wedge} - \delta\mathbf{L}) = \mathbf{0} \tag{32}$$

i.e. the estimator $\mathbf{X}^{\wedge}$ as an optimum solution of the system (1) fulfils the normal equations corresponding to observation system (25), by the use of the diagonal matrix $\mathbf{W}$. For k-th equation of (32) one has

$$\begin{aligned}
\frac{1}{2}c^{-1} \cdot \sum_{i=1}^{m}\sum_{j=1}^{m} (A_{ik} - A_{jk}) \cdot p_i \cdot p_j \cdot (v_i - v_j) = \\
&= \frac{1}{2}c^{-1} \cdot \sum_{i=1}^{m}\sum_{j=1}^{m} A_{ik} \cdot p_i \cdot p_j \cdot (v_i - v_j) - \frac{1}{2}c^{-1} \cdot \sum_{i=1}^{m}\sum_{j=1}^{m} A_{jk} \cdot p_i \cdot p_j \cdot (v_i - v_j) \\
&= \frac{1}{2}c^{-1} \cdot \sum_{i=1}^{m}\sum_{j=1}^{m} A_{ik} \cdot p_i \cdot p_j \cdot (v_i - v_j) + \frac{1}{2}c^{-1} \cdot \sum_{i=1}^{m}\sum_{j=1}^{m} A_{jk} \cdot p_j \cdot p_i \cdot (v_j - v_i) \\
&= c^{-1} \cdot \sum_{i=1}^{m}\sum_{j=1}^{m} A_{ik} \cdot p_i \cdot p_j \cdot (v_i - v_j) \qquad (33) \\
&= c^{-1} \cdot \sum_{i=1}^{m}\sum_{j=1}^{m} A_{ik} \cdot p_i \cdot p_j \cdot v_i - c^{-1} \cdot \sum_{i=1}^{m}\sum_{j=1}^{m} A_{ik} \cdot p_j \cdot p_i \cdot v_j \\
&= c^{-1} \cdot \sum_{i=1}^{m} A_{ik} \cdot p_i \cdot v_i \cdot \sum_{j=1}^{m} p_j - c^{-1} \cdot \sum_{i=1}^{m} A_{ik} \cdot p_i \cdot \sum_{j=1}^{m} p_j \cdot v_j \\
&= c^{-1} \cdot c \cdot \sum_{i=1}^{m} A_{ik} \cdot p_i \cdot v_i - c^{-1} \cdot \sum_{i=1}^{m} A_{ik} \cdot p_i \cdot 0 \\
&= \sum_{i=1}^{m} A_{ik} \cdot p_i \cdot v_i = 0
\end{aligned}$$

because it results from the original least squares condition (15) $\mathbf{A}^{\mathrm{T}} \cdot \mathbf{P} \cdot \mathbf{V} = \mathbf{0}$, thus $\mathbf{a}^{\mathrm{T}} \cdot \mathbf{W} \cdot \boldsymbol{\delta}\mathbf{v} = \mathbf{0}$.

Theorem 2 (two theses)

a) The system of observation differences of Schreiber's type (25) is correlated and the covariance matrix of the pseudo-observation vector $\boldsymbol{\delta}\mathbf{L}$ has the structure

$$\mathrm{Cov}(\boldsymbol{\delta}\mathbf{L}) = \mathbf{C}_{\boldsymbol{\delta}\mathbf{L}} = [C_{(g,h)} : (g,h) \in \boldsymbol{S} \times \boldsymbol{S}\,(\text{Cartesian product})] \tag{34}$$

(the subscripts are the pairs: $g = (i,j) \in \boldsymbol{S}$; $h = (k,l) \in \boldsymbol{S}$), where the single element expressed using pseudo algorithmic language command is

$$C_{(g,h)} := \begin{array}{l} \text{if } (i = k) \wedge (j = l) \text{ then } p_i^{-1} + p_j^{-1} \\ \text{else if } (i = k) \text{ then } p_i^{-1} \\ \text{else if } (j = l) \text{ then } p_j^{-1} \\ \text{else if } (i = l) \text{ then } -p_i^{-1} \\ \text{else if } (j = k) \text{ then } -p_j^{-1} \text{ else } 0 \end{array} \tag{35}$$

b) The theoretical covariance matrix (corresponds to the cofactor matrix) $\mathbf{C_X}$ for the estimator $\mathbf{X}^\wedge$, as the least squares solution of differenced equations of Schreiber's type (25), is finally constructed by using of the diagonal weight matrix $\mathbf{W}$

$$\mathbf{C_X} = (\mathbf{a}^T \cdot \mathbf{W} \cdot \mathbf{a})^{-1} \cdot \mathbf{a}^T \cdot \mathbf{W} \cdot \mathbf{C_{\delta L}} \cdot \mathbf{W} \cdot \mathbf{a} \cdot (\mathbf{a}^T \cdot \mathbf{W} \cdot \mathbf{a})^{-1} = (\mathbf{a}^T \cdot \mathbf{W} \cdot \mathbf{a})^{-1} \tag{36}$$

Proofs:

The single element of the covariance matrix $\mathbf{C_{\delta L}} = \mathrm{E}\{\boldsymbol{\delta\varepsilon} \cdot (\boldsymbol{\delta\varepsilon})^T\}$ is expressed as:

$$C_{(g,h)} = \mathrm{E}\{(\varepsilon_i - \varepsilon_j) \cdot (\varepsilon_k - \varepsilon_l)\} = \mathrm{E}\{\varepsilon_i \cdot \varepsilon_k - \varepsilon_i \cdot \varepsilon_l - \varepsilon_j \cdot \varepsilon_k + \varepsilon_j \cdot \varepsilon_l\} \tag{37}$$

Considering the expected values $\mathrm{E}\{\varepsilon_i^2\} = \mu_i^2 = p_i^{-1}$, and if $i \neq j$ then $\mathrm{E}\{\varepsilon_i \cdot \varepsilon_j\} = 0$ one obtains the thesis a) of the theorem at once.

For the thesis b) it is enough to prove that

$$\mathbf{a}^T \cdot \mathbf{W} \cdot \mathbf{C_{\delta L}} \cdot \mathbf{W} \cdot \mathbf{a} = \mathbf{a}^T \cdot \mathbf{W} \cdot \mathbf{a} \tag{38}$$

Denoting

$$\mathbf{F} = \mathbf{a}^T \cdot \mathbf{W} \cdot \mathbf{C_{\delta L}} \cdot \mathbf{W} \cdot \mathbf{a} = [F_{r,s}]_{(n \times n)} \text{ and } \mathbf{B} = \mathbf{a}^T \cdot \mathbf{W} \cdot \mathbf{a} = [B_{r,s}]_{(n \times n)} \tag{39}$$

we prove that $\mathbf{F} = \mathbf{B}$ or $F_{r,s} = B_{r,s}$ (naturally: $\mathbf{C_X} = \mathbf{B}^{-1}$). From the definition of the covariance matrix $\mathbf{C_{\delta L}}$ one can express the matrix $\mathbf{F}$ as follows

$$\mathbf{F} = \mathbf{a}^T \cdot \mathbf{W} \cdot \mathrm{E}\{\mathbf{e} \cdot \mathbf{e}^T\} \cdot \mathbf{W} \cdot \mathbf{a} = \mathrm{E}\{\mathbf{a}^T \cdot \mathbf{W} \cdot \mathbf{e} \cdot \mathbf{e}^T \cdot \mathbf{W} \cdot \mathbf{a}\} = \mathrm{E}\{\mathbf{a}^T \cdot \mathbf{d} \cdot \mathbf{d}^T \cdot \mathbf{a}\} \tag{40}$$

where

$$\mathbf{d} = \mathbf{W} \cdot \mathbf{e} = c^{-1} \cdot [p_i \cdot p_j \cdot (\varepsilon_i - \varepsilon_j) : (i, j) \in \mathbf{S}]_{(u \times 1)} \tag{41}$$

with u being the number of Schreiber's differences.

We evaluate then the element of the matrix $\mathbf{F}$

$$\begin{aligned} F_{r,s} &= c^{-2} \cdot \mathrm{E}\{[\sum_{i,j} p_i \cdot p_j \cdot (A_{i,r} - A_{j,r})\cdot(\varepsilon_i - \varepsilon_j)][\sum_{k,l} p_k \cdot p_l \cdot (A_{k,s} - A_{l,s})\cdot(\varepsilon_k - \varepsilon_l)]\} \\ &= c^{-2} \cdot \sum_{i,j} \sum_{k,l} p_i \cdot p_j \cdot p_k \cdot p_l \cdot (A_{i,r} - A_{j,r})\cdot(A_{k,s} - A_{l,s}) \cdot \mathrm{E}\{(\varepsilon_i - \varepsilon_j) \cdot (\varepsilon_k - \varepsilon_l)\} \end{aligned} \tag{42}$$

(sums for (i, j), $(k, l) \in \mathbf{S}$). From the thesis a), the expected value $\mathrm{E}\{(\varepsilon_i - \varepsilon_j) \cdot (\varepsilon_k - \varepsilon_l)\}$ leads to corresponding values: $p_i^{-1} + p_j^{-1}, p_i^{-1}, p_j^{-1}, -p_i^{-1}, -p_j^{-1}$ or zero, for different relations of indices in the pairs (i, j), (k, l).

Considering that the rows of the matrix **a** are linearly dependent, after reduction of similar components, one obtains the element $F_{g,h}$ equal to $B_{g,h}$

$$\begin{aligned} F_{r,s} &= c^{-2} \cdot (\sum_{i=1}^{m} p_i) \cdot \sum_{i,j} p_i \cdot p_j \cdot (A_{i,r} - A_{j,r})\cdot(A_{i,s} - A_{j,s}) \\ &= c^{-2} \cdot c \cdot \sum_{i,j} p_i \cdot p_j \cdot (A_{i,r} - A_{j,r})\cdot(A_{i,s} - A_{j,s}) \\ &= c^{-1} \cdot \sum_{i,j} p_i \cdot p_j \cdot (A_{i,r} - A_{j,r})\cdot(A_{i,s} - A_{j,s}) = B_{r,s} \end{aligned} \tag{43}$$

Illustration for $m = 3$ (number of observations), $n = 2$ (number of unknowns)

$$\begin{aligned} F_{r,s} = [1/(p_l + p_2 + p_3)^2] \cdot [&p_1 \cdot p_2 \cdot p_1 \cdot p_2 \cdot (A_{1,r} - A_{2,r}) \cdot (A_{1,s} - A_{2,s}) \cdot (1/p_1 + 1/p_2) + \\ &p_1 \cdot p_3 \cdot p_1 \cdot p_3 \cdot (A_{1,r} - A_{3,r}) \cdot (A_{1,s} - A_{3,s}) \cdot (1/p_1 + 1/p_3) + \\ &p_2 \cdot p_3 \cdot p_2 \cdot p_3 \cdot (A_{2,r} - A_{3,r}) \cdot (A_{2,s} - A_{3,s}) \cdot (1/p_2 + 1/p_3) + \\ &p_1 \cdot p_2 \cdot p_1 \cdot p_3 \cdot (A_{1,r} - A_{2,r}) \cdot (A_{1,s} - A_{3,s}) \cdot (1/p_1) + \\ &p_1 \cdot p_2 \cdot p_2 \cdot p_3 \cdot (A_{1,r} - A_{2,r}) \cdot (A_{2,s} - A_{3,s}) \cdot (-1/p_2) + \\ &p_1 \cdot p_3 \cdot p_1 \cdot p_2 \cdot (A_{1,r} - A_{3,r}) \cdot (A_{1,s} - A_{2,s}) \cdot (1/p_1) + \\ &p_1 \cdot p_3 \cdot p_2 \cdot p_3 \cdot (A_{1,r} - A_{3,r}) \cdot (A_{2,s} - A_{3,s}) \cdot (1/p_3) + \\ &p_2 \cdot p_3 \cdot p_1 \cdot p_2 \cdot (A_{2,r} - A_{3,r}) \cdot (A_{1,s} - A_{2,s}) \cdot (-1/p_2) + \\ &p_2 \cdot p_3 \cdot p_1 \cdot p_3 \cdot (A_{2,r} - A_{3,r}) \cdot (A_{1,s} - A_{3,s}) \cdot (1/p_3)] = \\ = [1/(p_1 + p_2 + p_3)^2] \cdot [&p_1 \cdot p_2 \cdot (A_{1,r} - A_{2,r}) \cdot (A_{1,s} - A_{2,s}) \cdot (p_1 + p_2) + \\ &p_1 \cdot p_3 \cdot (A_{1,r} - A_{3,r}) \cdot (A_{1,s} - A_{3,s}) \cdot (p_1 + p_3) + \\ &p_2 \cdot p_3 \cdot (A_{2,r} - A_{3,r}) \cdot (A_{2,s} - A_{3,s}) \cdot (p_2 + p_3) + \\ &p_1 \cdot p_2 \cdot (A_{1,r} - A_{2,r}) \cdot (A_{1,s} - A_{3,s}) \cdot (p_3) + \\ &p_1 \cdot p_2 \cdot (A_{1,r} - A_{2,r}) \cdot (A_{2,s} - A_{3,s}) \cdot (-p_3) + \\ &p_1 \cdot p_3 \cdot (A_{1,r} - A_{3,r}) \cdot (A_{1,s} - A_{2,s}) \cdot (p_2) + \\ &p_1 \cdot p_3 \cdot (A_{1,r} - A_{3,r}) \cdot (A_{2,s} - A_{3,s}) \cdot (p_2) + \\ &p_2 \cdot p_3 \cdot (A_{2,r} - A_{3,r}) \cdot (A_{1,s} - A_{2,s}) \cdot (-p_1) + \\ &p_2 \cdot p_3 \cdot (A_{2,r} - A_{3,r}) \cdot (A_{1,s} - A_{3,s}) \cdot (p_1)] = \end{aligned} \tag{44}$$

$$= [1/(p_1 + p_2 + p_3)^2] \cdot (p_1 + p_2 + p_3) \cdot [p_1 \cdot p_2 \cdot (A_{1,r} - A_{2,r}) \cdot (A_{1,s} - A_{2,s}) + p_1 \cdot p_3 \cdot (A_{1,r} - A_{3,r}) \cdot (A_{1,s} - A_{3,s}) + p_2 \cdot p_3 \cdot (A_{2,r} - A_{3,r}) \cdot (A_{2,s} - A_{3,s})] = \\ = c^{-1} \cdot [p_1 \cdot p_2 \cdot (A_{1,r} - A_{2,r}) \cdot (A_{1,s} - A_{2,s}) + p_1 \cdot p_3 \cdot (A_{1,r} - A_{3,r}) \cdot (A_{1,s} - A_{3,s}) + p_2 \cdot p_3 \cdot (A_{2,r} - A_{3,r}) \cdot (A_{2,s} - A_{3,s})] = B_{r,s}$$

where $c = p_1 + p_2 + p_3$.

4. Application for double- and triple-differences in GPS post-processing

4.1. The double-differences and a multipoint session

The single- and double-differences for a pair of receivers (p, q) and a pair (i, j) of satellites at the epoch t_k (Fig. 3) are created as follows:

$$\Delta\Phi^i_{p,q}(t_k) = \Phi^i_p(t_k) - \Phi^i_q(t_k) \tag{45}$$

(in this version, the single phase differences eliminate satellite clock offsets)

$$\Delta\Delta\Phi^{i,j}_{p,q}(t_k) = \Delta\Phi^i_{p,q}(t_k) - \Delta\Phi^j_{p,q}(t_k) \tag{46}$$

(the double-differences eliminate receiver clock offsets).

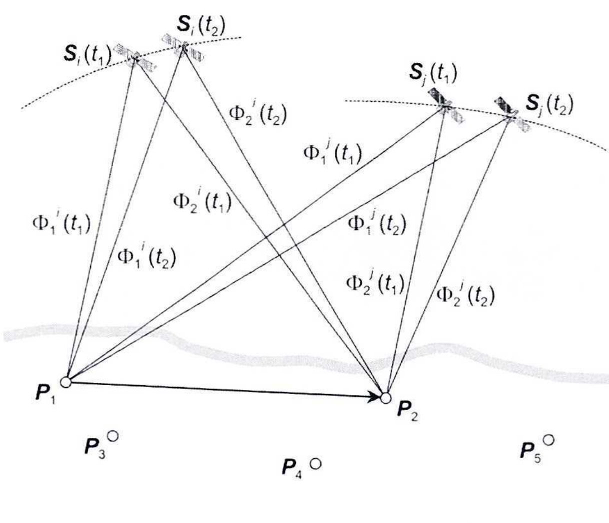

Fig. 3. Symbolic visualization of GPS – phase observations

Applying the ionosphere-free combination of L1, L2 carrier waves the single- and double-differences of corresponding phase observations Φ1, Φ2 are converted to

$$\Delta\Phi = \Delta\Phi 1 - (\lambda_1/\lambda_2)\cdot\Delta\Phi 2 \tag{47}$$

$$\Delta\Delta\Phi = \Delta\Delta\Phi 1 - (\lambda_1/\lambda_2)\cdot\Delta\Delta\Phi 2 \tag{48}$$

where λ_1, λ_2 are the wavelengths for L1, L2 carrier signals, respectively.

For the application of theorem 1, we define the Schreiber's sets for pairs of indices

$$\mathbf{P}_k = \{(p,q) : p < q \text{ and } p,q \in \boldsymbol{REC}(k) \subset \boldsymbol{REC}\} \text{ (for stations – receivers)} \tag{49}$$

$$\boldsymbol{S}_k = \{(i,j) : i < j \text{ and } i,j \in \boldsymbol{SAT}(k) \subset \boldsymbol{SAT}\} \text{ (for satellites)} \tag{50}$$

where the symbol ***REC*** denotes a set of integer indices (names) of r stations (receivers) used in GPS session, and ***REC*** (k) is the subset of indices of $r_k \le r$ active stations (receivers) in k-th epoch while the symbol ***SAT*** denotes a set of integer indices (names) of s satellites used in the GPS session, and ***SAT*** (k) is the subset of indices of $s_k \le s$ satellites used in k-th epoch.

All successive epochs in the observed time interval of the session create the set of epoch indices $\boldsymbol{K} = \{0, 1, 2, \dots\}$, but the effective observations, after elimination of defected phases (not corrected cycle slips, breaks of the phase registration, other outliers), correspond to some subset of epoch indices $\underline{\boldsymbol{K}} \subset \boldsymbol{K}$.

Let $\kappa(*)$ as the integer function, denotes the number of all elements of a set (*). For example (Fig. 4), for $s_k = 5$ satellites used in k-th epoch we have $\kappa(\boldsymbol{S}_k) = s_k \cdot (s_k - 1)/2 = 10$ double-differences for each $(p,q) \in \boldsymbol{P}_k$. The number of elements of the set $\boldsymbol{P}_k$ is $\kappa(\boldsymbol{P}_k) = r_k \cdot (r_k - 1)/2$. If for some k-th epoch (minimum) $r_k > 2$ then the observation system leads to multiple stations post-processing.

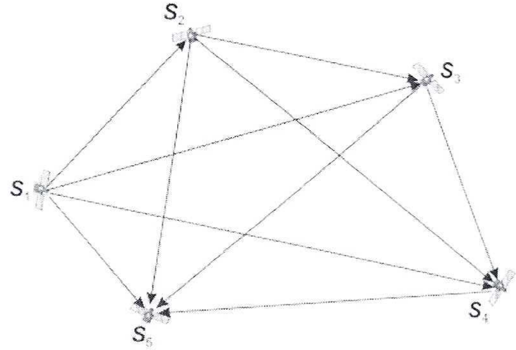

Fig. 4. The Schreiber's set of pairs for $s = 5$ satellites

In the above consideration the term "reference satellite" does not exist, because in the set of double-differences of the Schreiber's type, each satellite is treated equivalently. The number of all double-differences for n epochs is the sum of u_k for $k = 0, 1, 2, \ldots, n$. It arises from the theorem 1, stating that the weight matrix of the assumed set of double-difference observations is diagonal. Let the variance for simple phase, is a priori assumed as σ^2. Then the corresponding variance for simple differences, using the Schreiber's set, in k-th epoch, is $r_k \cdot \sigma^2$, because, on the basis of theorem 1, the weight of single difference equals to

$$\text{weight of } \Delta\Phi = \frac{(\text{weight of } \Phi)\cdot(\text{weight of } \Phi)}{(\text{sum of weights of } \Phi)} = \frac{\sigma^{-2}\cdot\sigma^{-2}}{r_k\cdot\sigma^{-2}} = (r_k\cdot\sigma^2)^{-1} \tag{51}$$

In some advanced stochastic models the variance of observed phase is multiplied by $1/\sin^2(\beta)$, where β is the elevation angle of the vector: receiver - satellite at an epoch. Constant variance may be used in order to simplify the illustration of the task discussed. In general case, however, theorem 1, allows for the use of arbitrary.

The set $u_k = s_k \cdot (s_k - 1)/2$ (where s_k is the number of satellites in k-th epoch) of double-differences of Schreiber's type in k-th epoch, has the weight sub-matrix $\mathbf{W}_k = w_k\ \mathbf{I}$, where $\mathbf{I}$ is $(u_k \times u_k)$ unit matrix, and

$$w_k = [1/(r_k\cdot\sigma^2)]\cdot[1/(r_k\cdot\sigma^2)]/[s_k/(r_k\cdot\sigma^2)] = (\sigma^2\cdot r_k\cdot s_k)^{-1} \tag{52}$$

also

$$\text{weight of } \Delta\Delta\Phi = \frac{(\text{weight of } \Delta\Phi)\cdot(\text{weight of } \Delta\Phi)}{(\text{sum of weights of } \Delta\Phi \text{ in } k\text{–th epoch})}$$

For each double-difference, in k-th epoch, the weight w_k is constant, and $m_k = s_k \cdot r_k$ is the number of all effective single differences (45) (without defected phases, non repaired cycle slips etc). If $r_k = 2$ for each effective epoch k, then the simple baseline vector is considered.

The observational equation system of double phase differences in Schreiber's "constellation" can be written as follows

$$\lambda_1\cdot\Delta\Delta\Phi^{i,j}_{p,q}(t_k) + \lambda_1\cdot\Delta\Delta N^{i,j}_{p,q} + \alpha\cdot\delta^{i,j}_{p,q}(t_k) + e^{i,j}_{p,q}(t_k) = \alpha\cdot\Delta\Delta\rho^{i,j}_{p,q}(t_k) \tag{53}$$

for $(i,j) \in \boldsymbol{S}_k$, $(p,q) \in \boldsymbol{P}_k$ and $k \in \underline{\boldsymbol{K}}$ (the set of indices of effective epochs) with quasi-diagonal weight matrix $\mathbf{W} = (1/\lambda_1)^2 \cdot \text{qdiag}[\mathbf{W}_k]$, where $\Delta\Delta N^{i,j}_{p,q}$ is the double-difference of integer ambiguities; for L1 carrier wave

$$\Delta\Delta N^{i,j}_{p,q} = \Delta\Delta N1^{i,j}_{p,q}$$

while for ionosphere-free combination of L1/L2

$$\Delta\Delta N^{i,j}_{p,q} = \Delta\Delta N1^{i,j}_{p,q} - (\lambda_1/\lambda_2)\cdot\Delta\Delta N2^{i,j}_{p,q} \tag{54}$$

with $\Delta\Delta N1^{i,j}_{p,q}$, $\Delta\Delta N2^{i,j}_{p,q}$ corresponding to L1, L2 double-differences of integer ambiguities (the final result (54) is not integer), and $\delta^{i,j}_{p,q}(t_k)$ is double-difference of systematic corrections as a sum of offset of antennas, reduction to the phase centre, troposphere and/or ionosphere corrections (in metres), $e^{i,j}_{p,q}(t_k)$ is a random error and $\Delta\Delta N1^{i,j}_{p,q}(t_k)$ is double-difference of distances from receivers (p, q) to satellites (i, j) at time t_k, defined as a function of vector coordinates connecting the terrestrial points in Cartesian geocentric frame. Satellite positions are interpolated at the moment $t_k - \tau$ of the signal detection, where τ is time interval for signal propagation. α is a constant factor; $\alpha = 1$ for L1 carrier wave and

$$\alpha = 1 - (\lambda_1/\lambda_2)^2 \tag{55}$$

for ionosphere-free combination of L1/L2.

For the detail theory of functional models see e.g. Hofmann-Wellenhof et al. (2001), Leick (2004), and Xu (2007).

The double-differences $\Delta\Delta^{i,j}_{p,q}(t_k)$ are defined as the non-linear functions of baseline vectors (considering the multiple point session). In the numerical realization of the least squares principle, the iterative Gauss-Newton procedure is used (in each iteration the non-linear functions of the functional model are linearised).

On the basis of theorem 1, the least squares adjustment of the Schreiber's set of double-differences with weights (52), is theoretically equivalent to the adjustment of the Schreiber's set of single-differences (45) with weights (51), by the additional estimation of receiver clock offsets. Simultaneously, the Schreiber's set of single-differences with weights (51) is equivalent to the original, un-differenced and uncorrelated, observation equations system. The Schreiber's set of double-differences is thus equivalent to the original system.

4.2. The triple-differences (BETA method)

The observation system in BETA method is constructed from triple phase differences

$$\Delta\Delta\Delta\Phi^{i,j}_{p,q}(t_{k_1}, t_{k_2}) = \Delta\Delta\Phi^{i,j}_{p,q}(t_{k_2}) - \Delta\Delta\Phi^{i,j}_{p,q}(t_{k_1}) \tag{56}$$

for $(p, q) \in \boldsymbol{P}_{k_1} \bigcap \boldsymbol{P}_{k_2}$ (the receivers p and q used in two epochs k_1, k_2),
$(i, j) \in \boldsymbol{S}_{k_1} \bigcap \boldsymbol{S}_{k_2}$ (the satellites i and j used in two epochs k_1, k_2),
$(k_1, k_2) \in \boldsymbol{T}$ (the Schreiber's set of pairs of epoch indices)

$$\boldsymbol{T} = \{(k_1, k_2) : k_1, k_2 \in \underline{\boldsymbol{K}} \text{ and } k_1 < k_2\} \tag{57}$$

All differential pseudo-observations for BETA algorithm are defined as

$$\boldsymbol{B} = \{[(p, q), (i, j), (k_1, k_2)] : (p, q) \in \boldsymbol{P}_{k_1} \bigcap \boldsymbol{P}_{k_2}, (i, j) \in \boldsymbol{S}_{k_1} \bigcap \boldsymbol{S}_{k_2}, (k_1, k_2) \in \boldsymbol{T}\} \tag{58}$$

The full observation system for triple differences of Schreiber's type has the form

$$\lambda_1 \cdot \Delta\Delta\Delta\Phi_{p,q}^{i,j}(t_{k_1}, t_{k_2}) + \alpha \cdot \delta_{p,q}^{i,j}(t_{k_1}, t_{k_2}) + e_{p,q}^{i,j}(t_{k_1}, t_{k_2}) = \alpha \cdot \Delta\Delta\Delta\rho_{p,q}^{i,j}(t_{k_1}, t_{k_2}) \quad (59)$$

for each $[(p, q), (i, j), (k1, k2)] \in \boldsymbol{B}$ (ambiguity differences are naturally eliminated).

The Schreiber's type set of triple-differences has the diagonal weight matrix

$$\mathbf{W} = (1/\lambda_1)^2 \cdot \operatorname{diag}[w_{k_1,k_2} : (k_1, k_2) \in \boldsymbol{T}] \quad (60)$$

with

$$w_{k_1,k_2} = w_{k_1} \cdot w_{k_2} \cdot c^{-1} \quad (61)$$

and

$$c = \sum_k w_k \text{ (the sum for } k \in \boldsymbol{K}) \quad (62)$$

Considering (52) the final form for the single weight component is obtained

$$w_{k_1,k_2} = (s_{k_1} \cdot r_{k_1} \cdot \sigma^2)^{-1} \cdot (s_{k_2} \cdot r_{k_2} \cdot \sigma^2)^{-1} \cdot [\sum (s_k \cdot r_k \cdot \sigma^2)^{-1}]^{-1}$$

$$= (1/\sigma^2) \cdot (s_{k_1} \cdot r_{k_1} \cdot s_{k_2} \cdot r_{k_2})^{-1} \cdot [\sum (s_k \cdot r_k)^{-1}]^{-1} \quad (63)$$

where s_{k_1}, s_{k_2} correspond to the number of satellites used in epochs k_1, k_2, respectively, and $k \in \boldsymbol{K}$.

Figure 5 illustrates the set of triple phase differences.

5. Conclusions

The classical algorithms for GPS post-processing, with differencing of carrier phase observations (for double- or triple phase differences), lead to the least squares procedure with some non diagonal covariance and weight matrices, for the pseudo-observation set used. Creating the special type (triangular, Schreiber's) set of differences we obtain the equivalent least squares estimation with the diagonal weight matrix. The advantages of the proposed method reflected mainly in the diagonality of weight matrices used are observed in GPS data processing models with the use of triple-differences and in case of multiple baseline sessions. The method is theoretically proved and practically implemented in GPS software.

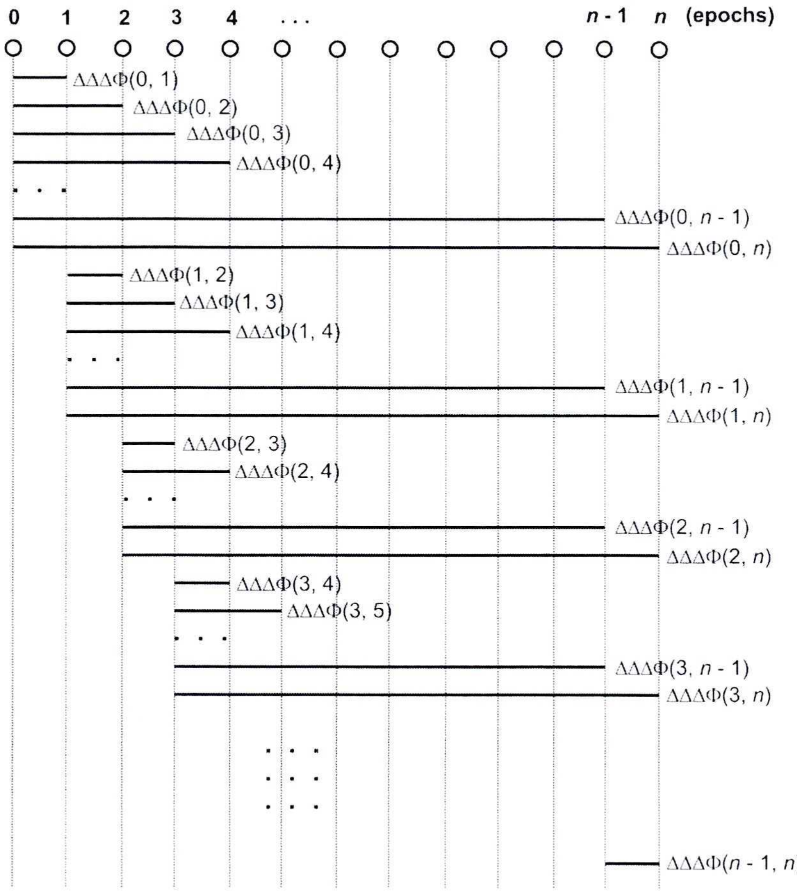

Fig. 5. The epoch distribution of triple phase differences in BETA method

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Nowe algorytmy postprocessingu GPS dla modeli wielo-bazowych oraz analogie do klasycznych sieci geodezyjnych

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Streszczenie

Praca prezentuje nowe algorytmy różnicowe dla postprocessingu GPS, z wykorzystaniem podwójnych lub potrójnych różnic faz oraz wielopunktowych sesji obserwacyjnych. Charakterystyczną cechą nowych algorytmów jest to, że wykorzystują one pełne zbiory (typu Schreibera) różnic obserwacyjnych z (teoretycznie) diagonalnymi macierzami wagowymi. Proponowane modele estymacji są równoważne do zadania najmniejszych kwadratów dla oryginalnego układu nie różnicowych równań obserwacyjnych. Teoretyczną podstawą algorytmów są twierdzenia o własnościach równań różnicowych typu Schreibera. Twierdzenia

mają praktyczne znaczenie zwłaszcza w odniesieniu do funkcjonalnego modelu potrójnych różnic faz. W ujęciu klasycznym takie zadanie było upraszczane ze względu na konieczność odwracania nie diagonalnej macierzy kowariancyjnej, zwykle o znacznych rozmiarach. Podobne korzyści otrzymuje się w przypadku wielopunktowej sesji obserwacyjnej, gdzie skorelowanie wektorów GPS zmusza w praktyce do użycia uproszczonych modeli stochastycznych. Proponowana metoda eliminuje także problem wyboru satelity bazowego (referencyjnego). Jest to bardzo ważne zwłaszcza dla długich sesji obserwacyjnych. Algorytmy nie wymagają selekcji i wyróżnienia jakiegokolwiek satelity – wszystkie „obserwowane" satelity pełnią względem siebie analogiczną funkcję. Algorytmy są zastosowane w profesjonalnych programach dla względnego pozycjonowania w systemie GPS.