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CONTROL AND INFORMATICS

Non-fragile event-triggered control of positive switched systems with random nonlinearities and controller perturbations

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Abstract. This paper investigates the non-fragile event-triggered control of positive switched systems with random nonlinearities and controller perturbations. The random nonlinearities and controller perturbations are assumed to obey Bernoulli and Binomial sequence, respectively. A class of linear event-triggering conditions is introduced. A switched linear co-positive Lyapunov function is constructed for the systems. For the same probability with respect to nonlinearities and controller perturbations in each subsystem, a non-fragile controller of positive switched systems is designed in terms of linear programming. Then, the different probability case is considered and the corresponding non-fragile event-triggered control is explored. Finally, the effectiveness of theoretical findings is verified via two examples.

Key words: positive switched systems; non-fragile event-triggered control; random nonlinearities; random controller perturbations.

1. INTRODUCTION

Positive switched systems, which comprise positive subsystems and a switching law that determines the active subsystem at the switching instant, have received wide attention in recent years [1–5]. This class of systems is widespread in many practical fields such as formation flying, tumor treatment, HIV mitigation therapy, and so on [6–9]. In the research on positive switched systems, stability and stabilization are the most important issues [10–12]. As we all know, quadratic Lyapunov functions (i.e., $V(t) = x^T(t)Px(t)$) and linear matrix inequalities are used for general switched systems (non-positive) for dealing with synthesis problems [13]. For positive switched systems, linear co-positive Lyapunov functions integrated with linear programming are more powerful than other approaches [14, 15]. The literature [16] discussed the stability of positive switched systems by introducing a switched co-positive Lyapunov function. A non-fragile controller was proposed in [17] for positive switched systems subject to actuator faults and saturation. Zhang et al. designed a novel controller by decomposing the controller gain into positive and negative parts [18]. Under sampling mechanism, the time-triggered control strategy was widely adopted in most literature. Such a design may refer to many useless samplings and thus results in waste of resources and heavy communication burden. In order to overcome these disadvantages, a so-called event-triggered mechanism was proposed in [19]. The key of event-triggered mechanism is that data is transmitted only when a specific event occurs. The event-triggered control has also been applied in many systems and verified to be effective. A co-design method of controller gains and triggering parameters was proposed in [20] for switched systems with time-varying delays. The event-triggered and self-triggered H_∞ controllers were derived in [21] for uncertain switched systems. Moreover, the problem of event-triggered networked fault detection for positive Markovian systems was studied in [22]. However, there are few studies on the event-triggered control of positive systems, let alone positive switched systems. The literature [23] presented an event-triggered state-feed back law of positive systems with input saturation. An event-triggering mechanism in the form of 1-norm was introduced for positive switched systems in [24]. In [25, 26], it has been shown that a non-fragile control strategy is an effective way to handle saturation and actuator faults. However, the non-fragile event-triggered control of positive switched systems has not been solved completely.

On the other hand, the nonlinearity is a non-negligible factor influencing the performances of systems. Indeed, the occurrence of nonlinearity holds a random property since nonlinearity may be induced by the random failure or repair of components and the sudden change of network environment [27]. This kind of nonlinearity is generally called random nonlinearity. Up to now, many related results have been reported in [28–30]. In [31], a novel adaptive event-triggered communication scheme was presented for networked systems with network-induced delays and random nonlinearities. The literature [32] investigated the stochastic synchronization of complex networks with nonlinearity obeys to Bernoulli distribution. When introducing nonlinearities into positive systems, how to ensure the positivity of systems is an important problem to be solved. Using a nonlinear Lyapunov-Krasovskii functional, absolute exponential L_1 stability of switched nonlinear positive systems with time-varying delay was studied in [33]. In [34], the saturation controller was designed for nonlinear positive Markovian jump systems subject to random actuator faults. In the literature men-

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tioned above, it is always assumed that random nonlinearities conform to Bernoulli sequence. In the Bernoulli process, a random variable can only take 0 or 1. Hence, the Bernoulli process looks like an on-off switch, where "on" means there is nonlinearity and "off" means there is no nonlinearity. However, it may contain different classes of nonlinearities in actual systems and the nonlinearities exist all the time. Such a class of simultaneous occurrence nonlinearities cannot be described by Bernoulli distribution. Thus, Binomial process is introduced for the multiple nonlinearities. Binomial distribution is a combination of multiple independent Bernoulli random variables. It is a generalization of the Bernoulli sequence. In addition, there are occasional random perturbations in many practical systems such as aircraft and electric circuits. Therefore, the perturbation will lead to the random uncertainties of controllers. To the authors' best knowledge, there are no results on random nonlinearities and controller perturbations of positive switched systems. Considering the advantages of the event-triggered strategy and the non-fragile control, it is significant to investigate the issues of random nonlinearities and controller perturbations of positive switched systems.

This paper is concerned with the problem of non-fragile event-triggered control of positive switched systems. The occurrence of nonlinearities conforms to Bernoulli sequence and controller perturbations are modeled as a set of Binomial sequences. The main contributions of this paper are as follows. A 1-norm based event-triggering mechanism is introduced. By construction of a switched linear co-positive Lyapunov function and utilization of a matrix decomposition technique, a non-fragile event-triggered controller is designed for the systems. The proposed approach can be extended to more general situations, where the probabilities of controller perturbations and nonlinearities in each subsystem are different. Under the designed controller, the presented conditions can be solved via linear programming. This paper is organized as follows. Section 2 gives the preliminaries. Section 3 presents the main results. Two examples are given in Section 4. Section 5 concludes this paper.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times r}$ are the sets of n-dimensional vectors and $n\times r$ matrices with real entries, respectively. Denote $\mathbb{N}$ (or $\mathbb{N}^+$) as the sets of nonnegative (or positive) integers. For $A=[a_{ij}]$ with $A\in\mathbb{R}^{n\times n}$, $A\succeq 0$ ($\succ 0$) means that $a_{ij}\geq 0$ ($a_{ij}>0$) $\forall i,j=1,\ldots,n$ and $A\preceq 0$ ($\prec 0$) means that $a_{ij}\leq 0$ ($a_{ij}<0$) $\forall i,j=1,\ldots,n$. Similarly, $A\succeq B$ ($A\preceq B$) means that $a_{ij}\geq b_{ij}$ ($a_{ij}\leq b_{ij}$) $\forall i,j=1,\ldots,n$. A^T is a transpose matrix of matrix A. x_i is the ith element of vector $x=(x_1,\ldots,x_n)^T$. Define I as an identity matrix with appropriate dimensions. $|\cdot|$ and $\|\cdot\|$ are the absolute value and Euclidean norm, respectively. The 1-norm $\|x\|_1$ and infinite-norm of a vector $x\in\mathbb{R}^n$ are defined as $\|x\|_1=\sum_{i=1}^n|x_i|$ and $\|x\|_\infty=\max(|x_1|,|x_2|,\ldots,|x_n|)$, respectively. Define $\mathbf{1}_r=(\underbrace{1,\ldots,1}_{r})^T$, $\mathbf{1}_r^{(\iota)}=(\underbrace{0,\ldots,0}_{\iota-1},1,\underbrace{0,\ldots,0}_{r-\iota})^T$, and let $\mathbf{1}_{n\times n}$ be the $n\times n$ matrix with all elements being 1. The symbol $\mathbb{E}\{\cdot\}$ refers to the mathematical expectation and $\mathrm{Prob}\{\cdot\}$ refers to probability. A matrix A is called Metzler matrix if its off-diagonal elements are all nonnegative real numbers.

2. PRELIMINARIES

Consider a class of discrete-time switched systems with random nonlinearities:

$$\begin{aligned} x(k+1)=A_{\sigma(k)}x(k)+B_{\sigma(k)}u^f_{\sigma(k)}(k)\\ +E_{\sigma(k)}\sum_{p=1}^{L}\aleph_{\sigma(k)p}(k)f_{\sigma(k)p}(x(k)), \end{aligned}\tag{1}$$

where $x(k)\in\mathbb{R}^n$ is system state and $u^f_{\sigma(k)}(k)\in\mathbb{R}^r$ is control input with random actuator faults. The switching signal $\sigma(k)$ takes values on the finite set $S=\{1,2,\ldots,N\}$, $N\in\mathbb{N}^+$. For simplicity, assume that the ith subsystem is activated when $\sigma(t)=i$. The function $\aleph_{ip}(k)$ indicates a random nonlinear process. The nonlinear function $f_{ip}(x(k))=(f_{ip1}(x_1(k)),\ldots,f_{ipn}(x_n(k)))^T$ is a vector-valued one. Throughout this paper, assume that $A_i\succeq 0$, $B_i\succeq 0$, and $E_i\succeq 0$.

Definition 1. ([6, 8]) A system is called positive system if its state and output are nonnegative for any nonnegative initial condition and input.

Lemma 1. ([6,8]) A system $x(k+1)=Ax(k)$ is positive if and only if $A\succeq 0$.

Definition 2. ([35,36]) Assume that the system (1) with $u^f_{\sigma(k)}=0$ is positive. The considered system is stochastically exponentially stable if the condition

$$\mathbb{E}\{\|x(k)\|_1\}\leq\tau\lambda^k\mathbb{E}\{\|x(0)\|_1\}$$

holds for any initial condition $x(0)\succeq 0$ and any switching signal $\sigma(0)\in S$, where $\tau>0$ and $0<\lambda<1$.

Assumption 1. The nonlinear function $f_{ip}(x(k))$ satisfies the following condition:

$$\alpha_1 x^2_{ipj}(k)\leq f_{ipj}(x_{ipj}(k))x_{ipj}(k)\leq\alpha_2 x^2_{ipj}(k),\tag{2}$$

where $0<\alpha_1<\alpha_2$.

This paper will design a non-fragile event-triggered controller:

$$u_i(k)=(F_i+\Delta F_i)\hat{x}(k),\ k\in[k_q,k_{q+1}),\tag{3}$$

where $q\in\mathbb{N}$, k_q represents the qth event-triggering instant ($k_0=0$), $\hat{x}(k)=x(k_q)$, $F_i\in\mathbb{R}^{r\times n}$ are normal controller gain matrices, ΔF_i are the gain perturbation matrices and $\Delta F_i=G_iH_i$ with $H_i\in\mathbb{R}^{r\times n}$ being unknown matrices and $G_i\in\mathbb{R}^{r\times r}$ satisfying $\theta_1 I\preceq G_i\preceq\theta_2 I$ for $0<\theta_1<\theta_2<1$.

Remark 1. It is necessary to state several points on the controller (3). First, there always exist modelling errors when describing a practical system. Due to the complexity of practical dynamic processes, it may be hard to establish an accurate model for a practical system. In such a case, an accurate controller for the error model is difficult to handle the practical system. Second, the structure of a system may change owing to unexpected internal and external factors. The controller of the original system is also hard to control the changed system. Third, the parameters of the controller may have fluctuations. Generally, actuators have limited implementation ability owing to limited capacity of elements. Therefore, it is not easy to activate the designed controller accurately. To solve these problems mentioned above, a non-fragile controller is introduced in (3), where a perturbation term ΔF_i is added for the normal controller $u_i(k) = F_i x(k)$. In addition, the controller (3) employs the event-triggering mechanism, that is, the control law only updates its state information when some prescribed event conditions (to be given later) are satisfied. Such a control strategy can reduce the update times of the control law and thus save the design cost of the controller. The event-triggered control is more practical than the traditional time-triggered control.

The considered controller with actuator fault is defined as:

$$u_i^f(k) = L_i u_i(k), \tag{4}$$

where $L_i = \mathrm{diag}(l_{i1}, l_{i2}, \ldots, l_{ir})$ are uncertainty fault matrices but bounded: $0 \preceq L_{di} \preceq L_i \preceq L_{ui} \preceq \rho L_{di}, \rho \geq 1$, L_{di} and L_{ui} are given diagonal matrices satisfying $L_{di} = \mathrm{diag}(l_{di1}, l_{di2}, \ldots, l_{dir})$ and $L_{ui} = \mathrm{diag}(l_{ui1}, l_{ui2}, \ldots, l_{uir})$, respectively.

The change of controller perturbations is random and dependent on Binomial sequence. Denote $\rho_i(k)$ as the stochastic variable. If $\rho_i(k) = m$, then the additive gain perturbation ΔF_i changes to $m\Delta F_i$, where $m = 0, 1, 2, \ldots, l$ and l is the number of changes. By (3) and (4), the non-fragile event-triggered controller with actuator fault is rewritten as:

$$u_i^f(k) = L_i(F_i + \rho_i(k)\Delta F_i)\hat{x}(k). \tag{5}$$

Remark 2. Robust control is a class of control methods that enhance the system's ability to resist interferences of the system. Non-fragile control refers to a control method that keeps the system stable when the controller parameters deviate from its design value. Generally speaking, non-fragile control is a kind of robust control. Compared with robust control, the non-fragile control is more specific since it aims to overcome the change of controller parameters caused by actuator faults [25, 26].

3. MAIN RESULTS

In this section, we will study the non-fragile event-triggered control of positive switched systems with random nonlinearities and controller perturbations. First, the nonlinearities of the systems with the same and different probabilities in each subsystem are considered, respectively. Then, the controller perturbations with the same occurrence probability and different occurrence probabilities in each subsystem are addressed, respectively.

The event-triggering condition is given as:

$$\|x_e(k)\|_1 > \eta \|x(k)\|_1 \,, \tag{6}$$

where $0 < \eta < 1$ and $x_e(k) = \hat{x}(k) - x(k)$ is the error. Given any initial state $x(k_0) \succeq 0$, it follows that

$$\| x_e(k_0) \|_1 \leq \eta(x_1(k_0) + \ldots + x_n(k_0)) = \eta \mathbf{1}_n^T x(k_0).$$

Thus,

$$-\eta \mathbf{1}_{n\times n} x(k_0) \preceq x_e(k_0) \preceq \eta \mathbf{1}_{n\times n} x(k_0). \tag{7}$$

Lemma 2. The system (1) is positive with $u_i^f = 0$.

Proof. By Assumption 1, we get

$$\alpha_1 x_{ipj}^2(k_0) \leq f_{ipj}(x_{ipj}(k_0)) x_{ipj}(k_0) \leq \alpha_2 x_{ipj}^2(k_0). \tag{8}$$

Given any initial state $x(k_0) \succeq 0$, we have

$$0 \leq \alpha_1 x_{ipj}(k_0) \leq f_{ipj}(x_{ipj}(k_0)) \leq \alpha_2 x_{ipj}(k_0),$$

which means that $f_{ip}(x_{ip}(k_0)) \succeq 0$. Since $\aleph_{ip}(k)$ takes values in the index set: $\{0, 1\}$, then $\aleph_{ip}(k_0) \geq 0$. By $A_i \succeq 0$ and $E_i \succeq 0$, it is clear that $x(k_0+1) = A_{\sigma(k_0)} x(k_0) + E_{\sigma(k_0)} \sum_{p=1}^{L} \aleph_{ip}(k_0) f_{ip}(x(k_0)) \succeq 0$. Using recursive induction, we can get $x(k) \succeq 0, \forall k \in \mathbb{N}$. So, the system (1) is positive by Definition 1. □

3.1. Random nonlinearities

First, we consider system (1) with $\Delta F_i = 0$. Then, the resulting closed-loop system is:

$$\begin{aligned} x(k+1) = {} & A_i x(k) + B_i L_i F_i x(k) + B_i L_i F_i x_e(k) \\ & + E_i \sum_{p=1}^{L} \aleph_{ip}(k) f_{ip}(x(k)), \end{aligned} \tag{9}$$

where $\aleph_{ip}(k)$ is a Bernoulli sequence and belongs to the index set: $\{0, 1\}$. Assume that the occurrence of nonlinearities in each subsystem is the same, that is, $\mathrm{Prob}\{\aleph_{ip}(k) = 1\} = \beta$, where $0 \leq \underline{\beta} \leq \beta \leq \overline{\beta} \leq 1$. Then $\sum_{p=1}^{L} \aleph_{ip}(k)$ satisfies

$$\mathbb{E}\left\{\sum_{p=1}^{L} \aleph_{ip}(k)\right\} = \sum_{p=1}^{L} \mathbb{E}\left\{\aleph_{ip}(k)\right\} = L\beta. \tag{10}$$

Theorem 1. If there exist constants $\mu > 0$, $\rho \geq 1$, $0 < \delta_1 < 1$, $\delta_2 \geq 1$ and $\mathbb{R}^n$ vectors $v_i \succ 0$, $\zeta_i^+ \succ 0$, $\zeta_{i\iota}^+ \succ 0$, $\underline{\zeta}_i^- \preceq \overline{\zeta}_i^- \prec 0$, $\zeta_{i\iota}^- \prec 0$ such that

$$\begin{aligned} & \rho \mathbf{1}_r^T L_{di}^T B_i^T v_i A_i + B_i L_{di} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} \\ & \quad + \rho B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} - \rho\eta B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} \mathbf{1}_{n\times n} \\ & \quad + \rho\eta B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} \mathbf{1}_{n\times n} \succeq 0, \end{aligned} \tag{11a}$$

$$\begin{aligned} & A_i^T v_j + \delta_2 \zeta_i^+ + \delta_1 \overline{\zeta}_i^- + \eta \delta_2 \mathbf{1}_{n\times n} \zeta_i^+ \\ & \quad - \eta \delta_2 \rho \mathbf{1}_{n\times n} \underline{\zeta}_i^- + \alpha_2 L \bar{\beta} E_i^T v_j - \mu v_i \prec 0, \end{aligned} \tag{11b}$$

$$\delta_1 v_i \preceq v_j \preceq \delta_2 v_i, \tag{11c}$$

$$\zeta_{i\iota}^{+} \preceq \zeta_{i}^{+}, \quad \underline{\zeta}_{i}^{-} \preceq \zeta_{i\iota}^{-} \preceq \overline{\zeta}_{i}^{-}, \quad \iota = 1,\ldots,r, \tag{11d}$$

hold $\forall (i,j) \in S \times S, i \neq j, s = 1,\ldots,r$, then under the control law (5) with

$$F_i = F_i^{+} + F_i^{-}, \tag{12}$$

and

$$F_i^{+} = \frac{\sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T}}{\mathbf{1}_r^T L_{ui}^T B_i^T v_i}, \quad F_i^{-} = \frac{\sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}, \tag{13}$$

the resulting closed-loop system (9) is positive and stochastically exponentially stable for arbitrary switching law.

Proof. Due to $\mathbf{1}_r \succeq 0$, $B_i \succ 0$, $0 \preceq L_{di} \preceq L_i \preceq L_{ui}$ and $v_i \succeq 0$, we get $0 < \mathbf{1}_r^T L_{di}^T B_i^T v_i < \mathbf{1}_r^T L_{ui}^T B_i^T v_i$. Since $F_i^{+} \succ 0$ and $F_i^{-} \prec 0$, it follows that

$$\begin{aligned} A_i + B_i L_{di} F_i^{+} + B_i L_{ui} F_i^{-} &\preceq A_i + B_i L_i F_i^{+} + B_i L_i F_i^{-} \\ &\preceq A_i + B_i L_{ui} F_i^{+} + B_i L_{di} F_i^{-}. \end{aligned} \tag{14}$$

By (7) and (12)–(14), we have

$$\begin{aligned} &A_i x(k) + B_i L_i F_i x(k) + B_i L_i F_i x_e(k) \\ &\succeq \big(A_i + B_i L_{di} F_i^{+} - \eta B_i L_{ui} F_i^{+} \mathbf{1}_{n\times n} \\ &\quad + B_i L_{ui} F_i^{-} + \eta B_i L_{ui} F_i^{-} \mathbf{1}_{n\times n}\big) x(k) \\ &= \left(A_i + \frac{B_i L_{di} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T}}{\mathbf{1}_r^T L_{ui}^T B_i^T v_i} - \eta \frac{B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} \mathbf{1}_{n\times n}}{\mathbf{1}_r^T L_{ui}^T B_i^T v_i} \right. \\ &\quad \left. + \frac{B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} + \eta \frac{B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} \mathbf{1}_{n\times n}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} \right) x(k) \\ &\succeq \left(A_i + \frac{1}{\rho} \frac{B_i L_{di} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} - \eta \frac{B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} \mathbf{1}_{n\times n}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} \right. \\ &\quad \left. + \frac{B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} + \eta \frac{B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} \mathbf{1}_{n\times n}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} \right) x(k). \end{aligned}$$

Together with (11a), it holds that

$$\begin{aligned} &A_i + \frac{1}{\rho} \frac{B_i L_{di} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} - \frac{\eta B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} \mathbf{1}_{n\times n}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} \\ &\quad + \frac{B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} + \frac{\eta B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} \mathbf{1}_{n\times n}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} \succeq 0. \end{aligned}$$

Thus, $A_i x(k) + B_i L_i F_i x(k) + B_i L_i F_i x_e(k) \succeq 0$ for each $i \in S$. Noting the fact $E_i \sum_{p=1}^{L} \aleph_{ip}(k) f_{ip}(x(k)) \succeq 0$ in Lemma 2, the positivity of the closed-loop system (9) is achieved.

Choose a switched linear co-positive Lyapunov function:

$$V(k) = x^T(k) v_{\sigma(k)}. \tag{15}$$

Then, $\mathbb{E}\{V(k+1)|V(k)\} = \mathbb{E}\{x^T(k+1) v_{\sigma(k+1)}\}$. Moreover,

$$\begin{aligned} \mathbb{E}\{\Delta V(k)\} &= \mathbb{E}\left\{x^T(k+1) v_j - x^T(k) v_i\right\} \\ &\leq \mathbb{E}\Big\{ x^T(k) (A_i^T v_j + F_i^{+T} L_{ui}^T B_i^T v_j \\ &\quad + F_i^{-T} L_{di}^T B_i^T v_j + \eta \mathbf{1}_{n\times n} F_i^{+T} L_{ui}^T B_i^T v_j \\ &\quad - \eta \mathbf{1}_{n\times n} F_i^{-T} L_{ui}^T B_i^T v_j - v_i) \\ &\quad + \alpha_2 \sum_{p=1}^{L} \aleph_{ip}(k) x^T(k) E_i^T v_j \Big\}, \end{aligned} \tag{16}$$

where $\sigma(k) = i$ and $\sigma(k+1) = j$ mean that the ith and jth subsystem is active at time instants k and $k+1$, respectively. By (11c), (11d) and (13),

$$\begin{aligned} F_i^{+T} L_{ui}^T B_i^T v_j &\preceq \frac{\sum_{\iota=1}^{r} \zeta_i^{+} \mathbf{1}_r^{(\iota)T} L_{ui}^T B_i^T v_j}{\mathbf{1}_r^T L_{ui}^T B_i^T v_i} \\ &\preceq \delta_2 \frac{\zeta_i^{+} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)T} L_{ui}^T B_i^T v_i}{\mathbf{1}_r^T L_{ui}^T B_i^T v_i} = \delta_2 \zeta_i^{+}, \end{aligned} \tag{17a}$$

$$\begin{aligned} F_i^{-T} L_{di}^T B_i^T v_j &\preceq \frac{\sum_{\iota=1}^{r} \overline{\zeta}_i^{-} \mathbf{1}_r^{(\iota)T} L_{di}^T B_i^T v_j}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} \\ &\preceq \delta_1 \frac{\overline{\zeta}_i^{-} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)T} L_{di}^T B_i^T v_i}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} = \delta_1 \overline{\zeta}_i^{-}, \end{aligned} \tag{17b}$$

$$\begin{aligned} F_i^{-T} L_{ui}^T B_i^T v_j &\succeq \frac{\sum_{\iota=1}^{r} \underline{\zeta}_i^{-} \mathbf{1}_r^{(\iota)T} L_{ui}^T B_i^T v_j}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} \\ &\succeq \delta_2 \rho \frac{\underline{\zeta}_i^{-} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)T} L_{di}^T B_i^T v_i}{\mathbf{1}_r^T L_{di}^T B_i^T v_i} = \delta_2 \rho \underline{\zeta}_i^{-}. \end{aligned} \tag{17c}$$

Substitute (17) into (16) yields that

$$\begin{aligned} \mathbb{E}\{\Delta V(k)\} &\leq x^T(k) \Big(A_i^T v_j + \delta_2 \zeta_i^{+} + \delta_1 \overline{\zeta}_i^{-} + \eta \delta_2 \mathbf{1}_{n\times n} \zeta_i^{+} \\ &\quad - \eta \delta_2 \rho \mathbf{1}_{n\times n} \underline{\zeta}_i^{-} + \alpha_2 L \beta E_i^T v_j - v_i \Big) \\ &\leq x^T(k) \Big(A_i^T v_j + \delta_2 \zeta_i^{+} + \delta_1 \overline{\zeta}_i^{-} + \eta \delta_2 \mathbf{1}_{n\times n} \zeta_i^{+} \\ &\quad - \eta \delta_2 \rho \mathbf{1}_{n\times n} \underline{\zeta}_i^{-} + \alpha_2 L \bar{\beta} E_i^T v_j - v_i \Big). \end{aligned}$$

By (11b), it is easy to obtain $\mathbb{E}\{V(k+1) - V(k)\} < -(1-\mu) V(k)$. Thus, $\mathbb{E}\{V(k)\} < \mu^k V(0)$. Moreover, $\mathbb{E}\{||x(k)||_1\} < \frac{\overline{\rho}}{\underline{\rho}} \mu^k \{||x(0)||_1\}$, where $\overline{\rho}$ and $\underline{\rho}$ are the minimal element and maximal element of $v_i, \forall i \in S$, respectively. By Definition 2, the system (9) is stochastically exponentially stable. $\square$

Remark 3. In [16], a switched co-positive Lyapunov function was constructed for positive switched systems. Multiple Lyapunov functions have less rigorous stability conditions but restricted dwell time conditions while common Lypaunov functions have rigorous stability conditions but less restricted dwell time conditions. Switched Lyapunov functions make a trade-off between stability and dwell time conditions. Finally, some less rigorous stability and dwell time conditions than common and multiple Lyapunov functions are obtained under switched Lyapunov functions, respectively. Considering these advantages mentioned, a switched linear co-positive Lyapunov function is employed in Theorem 1.

Remark 4. The literature [27–30, 32, 33] had considered the random issues concerning random saturation, random nonlinearities, and so on. It is always assumed in the literature that the random behavior obeys Bernoulli distribution. In this paper, the random behavior of nonlinearities in positive switched systems is assumed to confirm Binomial distribution, which is more general than Bernoulli distribution. A new even-triggered control framework for positive switched systems with Binomial distribution type of nonlinearities is established in Theorem 1 in terms of linear programming.

Remark 5. Nonlinearity is an interesting but challenging issue in the control field. How to determine the positivity of a nonlinear system is not an easy job [33, 34]. Particularly, few results are contributed to the event-triggered synthesis of positive systems [22–24]. There are still open issues in the event-triggered issues of positive systems. Under the event-triggered control framework, the positivity of nonlinear systems is more challenging. Compared with [33] and [34], a more general nonlinear description is introduced in Theorem 1 for positive switched systems and an event-triggered control strategy is proposed for the considered systems. Different from linear systems in [22–24], nonlinear positive switched systems are investigated in Theorem 1.

Theorem 1 considers the non-fragile event-triggered control design of the system (1) with $\Delta F_i = 0$. Based on Theorem 1, the different occurrence probabilities of nonlinearities in each subsystem is considered, that is, Prob$\{\aleph_{ip}(k) = 1\} = \beta_{ip}$, where $0 \le \underline{\beta}_{ip} \le \beta_{ip} \le \overline{\beta}_{ip} \le 1$. Then,

$$\mathbb{E}\left\{\sum_{p=1}^{L} \aleph_{ip}(k)\right\} = \sum_{p=1}^{L} \mathbb{E}\left\{\aleph_{ip}(k)\right\} = \sum_{p=1}^{L} \beta_{ip}. \tag{18}$$

Corollary 1. If there exist constants $\mu > 0$, $\rho \ge 1$, $\delta_2 \ge 1$, $0 < \delta_1 < 1$ and $\mathbb{R}^n$ vectors $v_i \succ 0$, $\zeta_i^+ \succ 0$, $\zeta_{i\iota}^+ \succ 0$, $\underline{\zeta}_i^- \preceq \overline{\zeta}_i^- \prec 0$, $\zeta_{i\iota}^- \prec 0$ such that

$$\begin{aligned}
&\rho \mathbf{1}_r^T L_{di}^T B_i^T v_i A_i + B_i L_{di} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} \\
&\quad + \rho B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} - \rho\eta B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} \mathbf{1}_{n\times n} \\
&\quad + \rho\eta B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} \mathbf{1}_{n\times n} \succeq 0,
\end{aligned} \tag{19a}$$

$$\begin{aligned}
&A_i^T v_j + \delta_2 \zeta_i^+ + \delta_1 \overline{\zeta}_i^- + \eta\delta_2 \mathbf{1}_{n\times n} \zeta_i^+ \\
&\quad - \eta\delta_2\rho \mathbf{1}_{n\times n} \underline{\zeta}_i^- + \alpha_2 \sum_{p=1}^{L} \bar{\beta}_{ip} E_i^T v_j - \mu v_i \prec 0,
\end{aligned} \tag{19b}$$

$$\delta_1 v_i \preceq v_j \preceq \delta_2 v_i, \tag{19c}$$

$$\zeta_{i\iota}^+ \preceq \zeta_i^+, \quad \underline{\zeta}_i^- \preceq \zeta_{i\iota}^- \preceq \overline{\zeta}_i^-, \quad \iota = 1, \ldots, r, \tag{19d}$$

hold $\forall (i, j) \in S \times S$, $i \ne j$, $s = 1, \ldots, r$, then under the control law (5), (12) and (13), the resulting closed-loop system (9) is positive and stochastically exponentially stable.

Sketch of Proof. By (19a), the positivity of the closed-loop system (9) can be proved using a similar method in Theorem 1. Choose the same switched linear co-positive Lyapunov function in (15), then

$$\begin{aligned}
\mathbb{E}\{\Delta V(k)\} \le \mathbb{E}\Big\{ & x^T(k)(A_i^T v_j + F_i^{+T} L_{ui}^T B_i^T v_j \\
& + F_i^{-T} L_{di}^T B_i^T v_j + \eta \mathbf{1}_{n\times n} F_i^{+T} L_{ui}^T B_i^T v_j \\
& - \eta \mathbf{1}_{n\times n} F_i^{-T} L_{ui}^T B_i^T v_j - v_i) \\
& + \alpha_2 \sum_{p=1}^{L} \aleph_{ip}(k) x^T(k) E_i^T v_j \Big\}.
\end{aligned}$$

Together with (17) and (18) gives

$$\begin{aligned}
\mathbb{E}\{\Delta V(k)\} \le x^T(k) \Big(& A_i^T v_j + \delta_2 \zeta_i^+ + \delta_1 \overline{\zeta}_i^- + \eta\delta_2 \mathbf{1}_{n\times n} \zeta_i^+ \\
& - \eta\delta_2\rho \mathbf{1}_{n\times n} \underline{\zeta}_i^- + \alpha_2 \sum_{p=1}^{L} \overline{\beta}_{ip} E_i^T v_j - v_i \Big).
\end{aligned}$$

By (19b), we have $\mathbb{E}\{\Delta V(k)\} < -(1-\mu)V(k)$. □

3.2. Random nonlinearities and controller perturbations

In Subsection 3.1, random nonlinearities are considered. Here, the random controller perturbations are further introduced for the systems. Then, the resulting closed-loop (9) is rewritten as:

$$\begin{aligned}
x(k+1) = {} & (A_i + B_i L_i F_i + \rho_i(k) B_i L_i \Delta F_i) x(k) \\
& + (B_i L_i F_i + \rho_i(k) B_i L_i \Delta F_i) x_e(k) \\
& + E_i \sum_{p=1}^{L} \aleph_{ip}(k) f_{ip}(x(k)).
\end{aligned} \tag{20}$$

In the process of system execution, there will be some disturbances due to the change of environment. In addition, when these disturbances exist, its subliminal degree in the controller varies according to the random change in the real environment. To solve the problem mentioned above, the controller parameter is assumed to be randomly changing in this subsection. This means ρ_i is a random process described by Binomial sequence. Assume that the occurrence probabilities of random nonlinearities of each subsystem is different and the occurrence probability of controller perturbations in each subsystem is the same.

Let $0 \leq \underline{\rho} \leq \rho \leq \overline{\rho} \leq 1$. Then, the stochastic variable $\rho_i(k)$ belongs to the index set of multiple elements: $\{0,1,2,\ldots,l\}$, and satisfies

$$\mathbb{E}\{\rho_i(k)\} = l\rho. \tag{21}$$

Theorem 2. If there exist constants $\mu > 0$, $\rho \geq 1$, $\delta_2 \geq 1$, $0 < \delta_1 < 1$ and $\mathbb{R}^n$ vectors $v_i \succ 0$, $\zeta_i^+ \succ 0$, $\zeta_{i\iota}^+ \succ 0$, $\xi_i^+ \succ 0$, $\xi_{i\iota}^+ \succ 0$, $\underline{\zeta}_i^- \prec \overline{\zeta}_i^- \prec 0$, $\zeta_{i\iota}^- \prec 0$, $\underline{\xi}_i^- \prec \overline{\xi}_i^- \prec 0$, $\xi_{i\iota}^- \prec 0$ such that

$$\begin{aligned}
&\rho \mathbf{1}_r^T L_{di}^T B_i^T v_i A_i - \rho\eta B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} \mathbf{1}_{n\times n} \\
&\quad + B_i L_{di} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} + \rho\eta B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} \mathbf{1}_{n\times n} \\
&\quad - \rho l\eta B_i L_{ui} G_i \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{+T} \mathbf{1}_{n\times n} \\
&\quad + \rho B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} + \rho l B_i L_{di} G_i \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{-T} \\
&\quad + \rho l\eta B_i L_{ui} G_i \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{-T} \mathbf{1}_{n\times n} \succeq 0,
\end{aligned} \tag{22a}$$

$$\begin{aligned}
&A_i^T v_j + \delta_2 \zeta_i^+ + \delta_1 \overline{\zeta}_i^- + \eta\delta_2 \mathbf{1}_{n\times n} \zeta_i^+ - \eta\delta_2\rho \mathbf{1}_{n\times n} \underline{\zeta}_i^- \\
&\quad + l\overline{\rho}\delta_2\theta_2 \xi_i^+ + l\underline{\rho}\delta_1\theta_1 \overline{\xi}_i^- + l\overline{\rho}\eta\delta_2\theta_2 \mathbf{1}_{n\times n} \xi_i^+ \\
&\quad - l\overline{\rho}\eta\delta_2\rho\theta_2 \mathbf{1}_{n\times n} \underline{\xi}_i^- + \alpha_2 \sum_{p=1}^{L} \bar{\beta}_{ip} E_i^T v_j - \mu v_i \prec 0,
\end{aligned} \tag{22b}$$

$$\delta_1 v_i \preceq v_j \preceq \delta_2 v_i, \tag{22c}$$

$$\begin{aligned}
&\zeta_{i\iota}^+ \preceq \zeta_i^+, \quad \underline{\zeta}_i^- \preceq \zeta_{i\iota}^- \preceq \overline{\zeta}_i^-, \quad \xi_{i\iota}^+ \preceq \xi_i^+, \\
&\underline{\xi}_i^- \preceq \xi_{i\iota}^- \preceq \overline{\xi}_i^-, \quad \iota = 1,\ldots,r,
\end{aligned} \tag{22d}$$

hold $\forall (i,j) \in S\times S, i \neq j, s = 1,\ldots,r$, then under the control law (5) with $F_i = F_i^+ + F_i^-, H_i = H_i^+ + H_i^-$, and

$$\begin{aligned}
F_i^+ &= \frac{\sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T}}{\mathbf{1}_r^T L_{ui}^T B_i^T v_i}, \quad F_i^- = \frac{\sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i}, \\
H_i^+ &= \frac{\sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{+T}}{\mathbf{1}_r^T L_{ui}^T B_i^T v_i}, \quad H_i^- = \frac{\sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{-T}}{\mathbf{1}_r^T L_{di}^T B_i^T v_i},
\end{aligned} \tag{23}$$

the system (20) is positive and stochastically exponentially stable under arbitrary switching law.

Proof. The proof of Theorem 2 can be seen in Appendix.

Remark 6. Due to limited capability of elements, the controller may be subject to parameter fluctuations when the running environment and status change. These fluctuations usually arise in the form of abrupt changes. Random process is suitable to be used for such fluctuations. In [25] and [26], the non-fragile control of positive Markovian jump systems has been explored. However, the parameter fluctuations are described in a determined way. In Theorem 2, it is assumed that the occurrence of parameter fluctuations obeys a stochastic process. This is more practical than the determined way.

Assume that the occurrence probability of controller perturbations for each subsystem is dependent on $\rho(k)$ satisfying

$$\mathbb{E}\{\rho_i(k)\} = \sum_{\hbar=1}^{l} \rho_{i\hbar}, \tag{24}$$

where $0 \leq \underline{\rho}_{i\hbar} \leq \rho_{i\hbar} \leq \overline{\rho}_{i\hbar} \leq 1$.

Corollary 2. If there exist constants $\mu > 0$, $\rho \geq 1$, $\delta_2 \geq 1$, $0 < \delta_1 < 1$ and $\mathbb{R}^n$ vectors $v_i \succ 0$, $\zeta_i^+ \succ 0$, $\zeta_{i\iota}^+ \succ 0$, $\xi_i^+ \succ 0$, $\xi_{i\iota}^+ \succ 0$, $\underline{\zeta}_i^- \prec \overline{\zeta}_i^- \prec 0$, $\zeta_{i\iota}^- \prec 0$, $\underline{\xi}_i^- \prec \overline{\xi}_i^- \prec 0$, $\xi_{i\iota}^- \prec 0$ such that

$$\begin{aligned}
&\rho \mathbf{1}_r^T L_{di}^T B_i^T v_i A_i + B_i L_{di} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} \\
&\quad - \rho\eta B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{+T} \mathbf{1}_{n\times n} + \rho B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} \\
&\quad + \rho\eta B_i L_{ui} \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_{i\iota}^{-T} \mathbf{1}_{n\times n} \\
&\quad + \rho l B_i L_{di} G_i \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{-T} \\
&\quad - \rho l\eta B_i L_{ui} G_i \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{+T} \mathbf{1}_{n\times n} \\
&\quad + \rho l\eta B_i L_{ui} G_i \sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \xi_{i\iota}^{-T} \mathbf{1}_{n\times n} \succeq 0,
\end{aligned} \tag{25a}$$

$$\begin{aligned}
&A_i^T v_j - \mu v_i + \delta_2 \zeta_i^+ + \delta_1 \overline{\zeta}_i^- + \eta\delta_2 \mathbf{1}_{n\times n} \zeta_i^+ \\
&\quad - \eta\delta_2\rho \mathbf{1}_{n\times n} \underline{\zeta}_i^- + \delta_2\theta_2 \sum_{\hbar=1}^{l} \overline{\rho}_{i\hbar} \xi_i^+ \\
&\quad + \delta_1\theta_1 \sum_{\hbar=1}^{l} \underline{\rho}_{i\hbar} \overline{\xi}_i^- + \eta\delta_2\theta_2 \sum_{\hbar=1}^{l} \overline{\rho}_{i\hbar} \mathbf{1}_{n\times n} \xi_i^+ \\
&\quad - \eta\delta_2\rho\theta_2 \sum_{\hbar=1}^{l} \overline{\rho}_{i\hbar} \mathbf{1}_{n\times n} \underline{\xi}_i^- + \alpha_2 \sum_{p=1}^{L} \overline{\beta}_{ip} E_i^T v_j \prec 0,
\end{aligned} \tag{25b}$$

$$\delta_1 v_i \preceq v_j \preceq \delta_2 v_i, \tag{25c}$$

$$\begin{aligned}
&\zeta_{i\iota}^+ \preceq \zeta_i^+, \quad \underline{\zeta}_i^- \preceq \zeta_{i\iota}^- \preceq \overline{\zeta}_i^-, \\
&\xi_{i\iota}^+ \preceq \xi_i^+, \quad \underline{\xi}_i^- \preceq \xi_{i\iota}^- \preceq \overline{\xi}_i^-, \quad \iota = 1,\ldots,r,
\end{aligned} \tag{25d}$$

hold $\forall (i,j) \in S\times S$, $i \neq j$, $s = 1,\ldots,r$, then under the control law (5) with $F_i = F_i^+ + F_i^-$, $H_i = H_i^+ + H_i^-$ and (23), the system (20) is positive and stochastically exponentially stable.

Sketch of Proof. By (25a), it is easy to get the positivity of the close-loop system (20). Using the switched linear co-positive Lyapunov function (15), we obtain

$$
\begin{aligned}
\mathbb{E}\{\Delta V(k)\} \leq \mathbb{E}\Big\{ & x^T(k)(A_i^T v_j + F_i^{+T} L_{ui}^T B_i^T v_j + F_i^{-T} L_{di}^T B_i^T v_j \\
& + \rho_i(k)(H_i^{+T} G_i^T L_{ui}^T B_i^T v_j + H_i^{-T} G_i^T L_{di}^T B_i^T v_j) \\
& + \eta \mathbf{1}_{n\times n} F_i^{+T} L_{ui}^T B_i^T v_j - \eta \mathbf{1}_{n\times n} F_i^{-T} L_{ui}^T B_i^T v_j \\
& + \rho_i(k)\eta \mathbf{1}_{n\times n} H_i^{+T} G_i^T L_{ui}^T B_i^T v_j \\
& - \rho_i(k)\eta \mathbf{1}_{n\times n} H_i^{-T} G_i^T L_{ui}^T B_i^T v_j \\
& + \alpha_2 \sum_{p=1}^{L} \aleph_{ip}(k) E_i^T v_j - v_i) \Big\}.
\end{aligned}
$$

Then, together with (23) and (24) gives

$$
\begin{aligned}
\mathbb{E}\{\Delta V(k)\} \leq\ & x^T(k)\Big(A_i^T v_j + \delta_2 \zeta_i^+ + \delta_1 \overline{\zeta}_i^- + \eta \delta_2 \mathbf{1}_{n\times n} \zeta_i^+ \\
& - \eta \delta_2 \rho \mathbf{1}_{n\times n} \underline{\zeta}_i^- + \delta_2 \theta_2 \sum_{\hbar=1}^{l} \rho_{i\hbar} \xi_i^+ + \delta_1 \theta_1 \sum_{\hbar=1}^{l} \rho_{i\hbar} \overline{\xi}_i^- \\
& + \eta \delta_2 \theta_2 \sum_{\hbar=1}^{l} \rho_{i\hbar} \mathbf{1}_{n\times n} \xi_i^+ - \eta \delta_2 \rho \theta_2 \sum_{\hbar=1}^{l} \rho_{i\hbar} \mathbf{1}_{n\times n} \underline{\xi}_i^- \\
& + \alpha_2 \sum_{p=1}^{L} \beta_{ip} E_i^T v_j - v_i \Big) \\
\leq\ & x^T(k)\Big(A_i^T v_j + \delta_2 \zeta_i^+ + \delta_1 \overline{\zeta}_i^- + \eta \delta_2 \mathbf{1}_{n\times n} \zeta_i^+ \\
& - \eta \delta_2 \rho \mathbf{1}_{n\times n} \underline{\zeta}_i^- + \delta_2 \theta_2 \sum_{\hbar=1}^{l} \overline{\rho}_{i\hbar} \xi_i^+ + \delta_1 \theta_1 \sum_{\hbar=1}^{l} \underline{\rho}_{i\hbar} \overline{\xi}_i^- \\
& + \eta \delta_2 \theta_2 \sum_{\hbar=1}^{l} \overline{\rho}_{i\hbar} \mathbf{1}_{n\times n} \xi_i^+ - \eta \delta_2 \rho \theta_2 \sum_{\hbar=1}^{l} \overline{\rho}_{i\hbar} \mathbf{1}_{n\times n} \underline{\xi}_i^- \\
& + \alpha_2 \sum_{p=1}^{L} \beta_{ip} E_i^T v_j - v_i \Big).
\end{aligned}
$$

By (25b), we have $\mathbb{E}\{V(k+1) - V(k)\} < -(1-\mu)V(k)$. Then, the stochastically exponential stability of system (20) with random nonlinearities and controller perturbations can be proved by using a similar method used in Theorem 1. □

Remark 7. Consider a switched system: $x(k+1) = Ax(k) + Bu(k), y(k) = Cx(k)$, where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^r$, $y(k) \in \mathbb{R}^s$. Theorems 1 and 2 present a matrix decomposition approach to design the controller of the system. Specifically, the controller gain matrix F is divided into the sum of $F^+ = \frac{\sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_\iota^{+T}}{\mathbf{1}_r^T B_i^T v}$ and $F^- = \frac{\sum_{\iota=1}^{r} \mathbf{1}_r^{(\iota)} \zeta_\iota^{-T}}{\mathbf{1}_r^T B^T v_i}$. It is necessary to point out that the design approach in Theorems 1 and 2 can be developed for the observer design of the considered system. Suppose that the gain matrix of Luenberger-type observer is L. One can design the gain matrix as $L = L^+ + L^- = \frac{\sum_{\iota=1}^{s} \mathbf{1}_s^{(\iota)} z_\iota^{+T}}{\mathbf{1}_s^T C^T v} + \frac{\sum_{\iota=1}^{s} \mathbf{1}_s^{(\iota)} z_\iota^{-T}}{\mathbf{1}_s^T C^T v_s}$. Then, we can obtain $C^T L^{+T} v \preceq z^{+T}$ and $C^T L^{-T} v \preceq z^{-T}$. Thus, $(A+LC)^T v \preceq A^T v + z^{+T} + z^{-T}$. Finally, the validity of the observer can be achieved if $A^T v + z^{+T} + z^{-T} \preceq 0$ holds.

Remark 8. This paper studies the non-fragile event-triggered controller of positive switched systems. It is assumed that the state is measurable. In practice, the state is often unmeasurable or unknown. This implies that it is necessary to design an observer of positive switched systems. Noting the statements in Remark 7, it is feasible to develop the matrix decomposition-based control approach for designing the observer of positive switched systems. The detail-deduced progress is complex but straightforward.

4. ILLUSTRATIVE EXAMPLES

The SEIR model is a mathematical model describing the generic behavior of epidemics. In this model, there are four classes of people, that is, the susceptible (S) who can contract the disease and become infectious; the exposed (E) and infectious (I) who can spread diseases; and the recovered (R) who have been immunized against the virus (including death). Moreover, the literature [37] proved that the transmission coefficient $\mathfrak{R}_0$ of virus is an important index to measure the infectious ability of a virus, and the disease can be almost eliminated if $\mathfrak{R}_0 < 1$, while the disease will spread if $\mathfrak{R}_0 > 1$. In real ecological systems, the population dynamics are often affected by the external environment. For example, the rate of disease transmission will be affected by the weather because the survival rate and infectivity of viruses and bacteria will be better in humid environment. In Fig. 1, the SEIR model switches in two cases ($\mathfrak{R}_0 < 1$ and $\mathfrak{R}_0 > 1$) , where ϖ_1 and ϖ_2 represent the probability of virus transmission from the susceptible to the exposed, latent rates ω_1 and ω_2 are the infection rates of latent individuals, γ_1 and γ_1 correspond to the recovery rates, and the parameters K_1 and K_2 represent the mortality rates. It should be noted that this is the simplest switched system which switches between two models. In fact, we can divide the basic reproduction number $\mathfrak{R}_0$ into n different intervals to express the infectivity of the disease. For example, $0 < \mathfrak{R}_0 < 0.25$, $0.25 \leq \mathfrak{R}_0 < 0.5$, $0.5 \leq \mathfrak{R}_0 < 1$, $1 \leq \mathfrak{R}_0 < 1.5$, $1.5 \leq \mathfrak{R}_0 < 2$, etc. Such a division method can help us to acquire more specific transmission status of virus. It is clear that this class of models can be represented by switched systems with n subsystems. What is more, since the state and outputs are all nonnegative, the SEIR model can be considered as a positive switched system. The fluctuations of natural birth and mortality can be regarded as a random non-

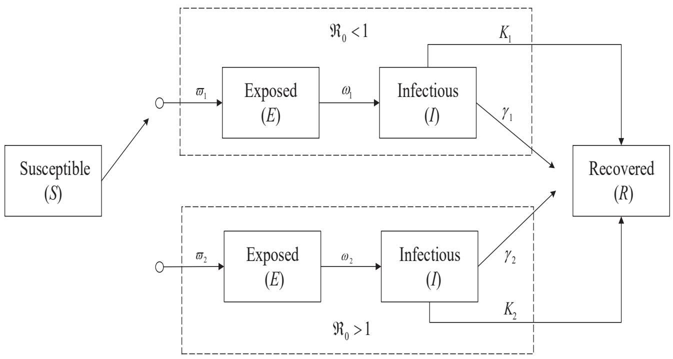

Fig. 1. The SEIR model framework

linear disturbance. Base on these points, an SEIR model containing n subsystems and multiple random nonlinearities is established, as shown in Fig. 2.

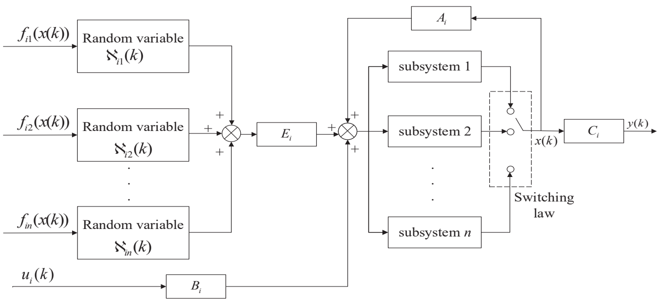

Fig. 2. Positive switched systems with random nonlinearities and controller perturbations

Example 1. Consider the system (9) with:

$$A_1 = \begin{pmatrix} 0.41 & 0.09 & 0.5 \\ 0.3 & 0.3 & 0.29 \\ 0.306 & 0.46 & 0.34 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.004 & 0.004 \\ 0.0026 & 0.0027 \\ 0.0029 & 0.0027 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0.3 & 0.4 & 0.25 \\ 0.38 & 0.2 & 0.47 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0.0029 & 0.003 \\ 0.0024 & 0.005 \\ 0.0026 & 0.0036 \end{pmatrix}.$$

Choose $E_1 = \operatorname{diag}(0.005\ \ 0.006\ \ 0.027)$ and $E_2 = \operatorname{diag}(0.007\ \ 0.009\ \ 0.008)$. The nonlinearities are selected as $f_{11}(x(k)) = 0.3\sin(x(k)) + 0.1x(k)$, $f_{12}(x(k)) = 0.1x(k) + 0.01x^2(k)$, $f_{13}(x(k)) = \frac{0.1x(k)}{0.01x^2(k)+1}$, $f_{21}(x(k)) = 0.3\sin(0.2x(k)) + 0.1x(k)$, $f_{22}(x(k)) = \frac{0.2x(k)}{0.02x^2(k)+2}$, $f_{23}(x(k)) = 0.2x(k) + 0.01x^2(k)$ and the corresponding probabilities are $\underline{\beta}_{11} = 0.15$, $\beta_{11} = 0.2$, $\overline{\beta}_{11} = 0.25$, $\underline{\beta}_{12} = 0.3$, $\beta_{12} = 0.3$, $\overline{\beta}_{12} = 0.35$, $\overline{\beta}_{13} = 0.43$, $\beta_{13} = 0.5$, $\overline{\beta}_{13} = 0.55$, $\underline{\beta}_{21} = 0.1$, $\beta_{21} = 0.2$, $\overline{\beta}_{21} = 0.2$, $\underline{\beta}_{22} = 0.3$, $\beta_{22} = 0.35$, $\overline{\beta}_{22} = 0.35$, $\underline{\beta}_{23} = 0.4$, $\beta_{23} = 0.45$, $\overline{\beta}_{23} = 0.5$. Set $\alpha_1 = 0.01$, $\alpha_2 = 0.4$, $\delta_1 = 0.98, \delta_2 = 1.01, \mu = 0.98$, $\eta = 0.19$ and $\rho = 1.2$. Let $L_{d1} = \operatorname{diag}(0.3\ \ 0.2)$, $L_{u1} = \operatorname{diag}(0.33\ \ 0.26)$, $L_{d2} = \operatorname{diag}(0.42\ 0.3)$, $L_{u2} = \operatorname{diag}(0.45\ 0.35)$. By Corollary 1, we get

$$v_1 = \begin{pmatrix} 3.5015 \\ 3.4736 \\ 2.4529 \end{pmatrix}, \quad \zeta_1^+ = \begin{pmatrix} 0.0020 \\ 0.3652 \\ 0.0020 \end{pmatrix}, \quad \underline{\zeta}_1^- = \begin{pmatrix} -0.3499 \\ -0.0030 \\ -1.7887 \end{pmatrix},$$

$$\overline{\zeta}_1^- = \begin{pmatrix} -0.3479 \\ -0.001 \\ -1.7867 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 3.4678 \\ 3.4402 \\ 2.4296 \end{pmatrix}, \quad \zeta_2^+ = \begin{pmatrix} 0.0020 \\ 0.0020 \\ 0.0020 \end{pmatrix},$$

$$\underline{\zeta}_2^- = \begin{pmatrix} -0.0276 \\ -0.0030 \\ -1.1795 \end{pmatrix}, \quad \overline{\zeta}_2 = \begin{pmatrix} -0.0010 \\ -0.0010 \\ -1.1775 \end{pmatrix}.$$

Then, the controller gains are

$$F_1 = \begin{pmatrix} -23.1288 & 20.3837 & -118.7568 \\ -23.1288 & 20.3837 & -118.7568 \end{pmatrix},$$

$$F_2 = \begin{pmatrix} -0.0420 & -0.0420 & -49.4599 \\ -0.0420 & -0.0420 & -49.4599 \end{pmatrix}.$$

Choose $L_1 = \operatorname{diag}(0.3\ 0.25)$ and $L_2 = \operatorname{diag}(0.43\ 0.32)$, then the resulting closed-loop system matrices are

$$A_1 + B_1 L_1 F_1 = \begin{pmatrix} 0.3591 & 0.1348 & 0.2387 \\ 0.2663 & 0.3297 & 0.1172 \\ 0.2703 & 0.4915 & 0.1565 \end{pmatrix},$$

$$A_2 + B_2 L_2 F_2 = \begin{pmatrix} 0.2999 & 0.3999 & 0.1408 \\ 0.3799 & 0.1999 & 0.3398 \\ 0.2999 & 0.3999 & 0.1877 \end{pmatrix}.$$

The simulation of system states is shown in Fig. 3 with the initial condition $x(t_0) = (4\ 3.5\ 3)^T$. Figure 4 shows the event-triggering signal and Fig. 5 is the state simulations with different initial conditions.

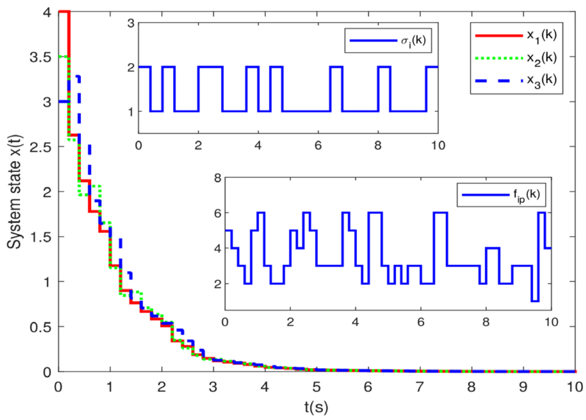

Fig. 3. The state simulations with $x(k_0) = (4\ 3.5\ 3)^T$

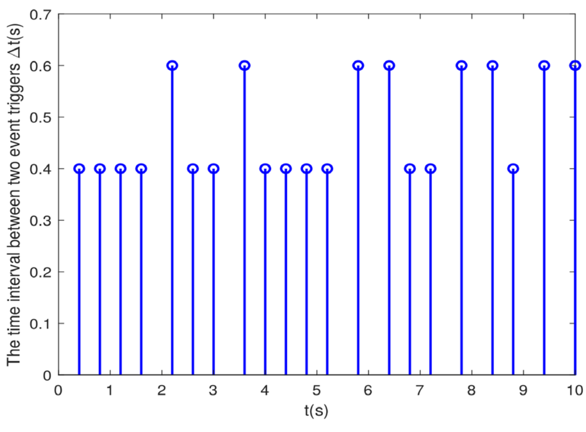

Fig. 4. The event-triggering signal

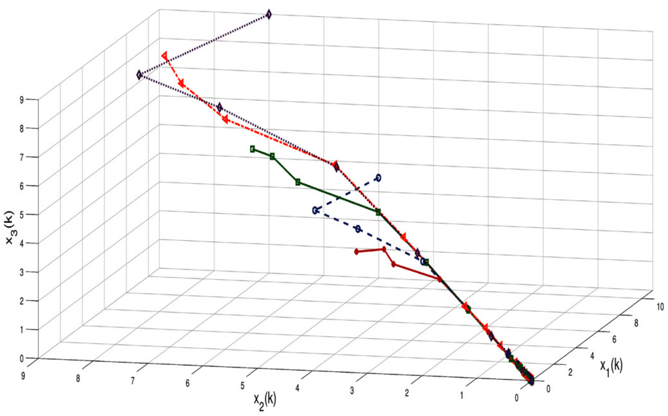

Fig. 5. The state simulations with different initial conditions

Example 2. Consider the system (20) with

$$A_1=\begin{pmatrix}0.48 & 0.1 & 0.49\\ 0.28 & 0.31 & 0.29\\ 0.31 & 0.45 & 0.33\end{pmatrix},\quad B_1=\begin{pmatrix}0.038 & 0.041\\ 0.025 & 0.027\\ 0.03 & 0.03\end{pmatrix},$$

$$A_2=\begin{pmatrix}0.3 & 0.4 & 0.25\\ 0.38 & 0.2 & 0.47\\ 0.3 & 0.4 & 0.32\end{pmatrix},\quad B_2=\begin{pmatrix}0.0027 & 0.0031\\ 0.0023 & 0.0045\\ 0.0028 & 0.0035\end{pmatrix}.$$

Let $E_1 = \mathrm{diag}(0.004\ \ 0.002\ \ 0.025)$ and $E_2 = \mathrm{diag}(0.0065\ \ 0.007\ \ 0.009)$. The corresponding parameters of nonlinearities and probabilities are the same as Example 1. In this Example, $\delta_1 = 0.95$, $\delta_2 = 1.01$, $\mu = 0.98$, $\eta = 0.13$, and $\rho = 1.2$. In subsystem 1, the probabilities of controller perturbations are $\underline{\rho}_{11} = 0.15$, $\rho_{11} = 0.2$, $\overline{\rho}_{11} = 0.3$, $\underline{\rho}_{12} = 0.3$, $\rho_{12} = 0.3$, $\overline{\rho}_{12} = 0.35$, $\underline{\rho}_{13} = 0.45$, $\rho_{13} = 0.45$, $\overline{\rho}_{13} = 0.53$. In subsystem 2, the probabilities of controller perturbations are taken as: $\underline{\rho}_{21} = 0.15$, $\rho_{21} = 0.15$, $\overline{\rho}_{21} = 0.25$, $\underline{\rho}_{22} = 0.25$, $\rho_{22} = 0.3$, $\overline{\rho}_{22} = 0.3$, $\underline{\rho}_{23} = 0.2$, $\rho_{23} = 0.52$, $\overline{\rho}_{23} = 0.55$. Choose $L_{d1} = \mathrm{diag}(0.29\ \ 0.2)$, $L_{u1} = \mathrm{diag}(0.33\ \ 0.29)$, $L_{d2} = \mathrm{diag}(0.41\ 0.28)$, $L_{u2} = \mathrm{diag}(0.44\ 0.34)$. By Corollary 2, we obtain

$$v_1=\begin{pmatrix}0.6307\\ 0.8339\\ 0.9382\end{pmatrix},\quad \zeta_1^+=\begin{pmatrix}0.0020\\ 0.0020\\ 0.0020\end{pmatrix},\quad \underline{\zeta}_1^-=\begin{pmatrix}-0.2751\\ -0.0030\\ -0.0030\end{pmatrix},$$

$$\overline{\zeta}_1^-=\begin{pmatrix}-0.2731\\ -0.0010\\ -0.0010\end{pmatrix},\quad \xi_1^+=\begin{pmatrix}0.0020\\ 0.0020\\ 0.0020\end{pmatrix},\quad \underline{\xi}_1^-=\begin{pmatrix}-0.0030\\ -0.0030\\ -0.0030\end{pmatrix},$$

$$\overline{\xi}_1^-=\begin{pmatrix}-0.0010\\ -0.0010\\ -0.0010\end{pmatrix},\quad v_2=\begin{pmatrix}0.6254\\ 0.8413\\ 0.9299\end{pmatrix},\quad \zeta_2^+=\begin{pmatrix}0.0020\\ 0.0020\\ 0.0020\end{pmatrix},$$

$$\underline{\zeta}_2^-=\begin{pmatrix}-0.2496\\ -0.0348\\ -0.0030\end{pmatrix},\quad \overline{\zeta}_2^-=\begin{pmatrix}-0.2476\\ -0.0328\\ -0.0010\end{pmatrix},\quad \xi_2^+=\begin{pmatrix}0.0020\\ 0.0020\\ 0.0020\end{pmatrix},$$

$$\underline{\xi}_2^-=\begin{pmatrix}-0.0401\\ -0.0030\\ -0.0030\end{pmatrix},\quad \overline{\xi}_2^-=\begin{pmatrix}-0.0010\\ -0.0010\\ -0.0010\end{pmatrix}.$$

Then, the controller gains and the gain perturbation matrices are

$$F_1=\begin{pmatrix}-7.4963 & -0.0332 & -0.0332\\ -7.4963 & -0.0332 & -0.0332\end{pmatrix},$$

$$F_2=\begin{pmatrix}-42.7349 & -5.6649 & -0.1726\\ -42.7349 & -5.6649 & -0.1726\end{pmatrix},$$

$$\triangle F_1=G_1H_1=\begin{pmatrix}-0.0010 & -0.0010 & -0.0010\\ -0.0007 & -0.0007 & 0.0007\end{pmatrix},$$

$$\triangle F_2=G_2H_2=\begin{pmatrix}-0.2765 & -0.0073 & -0.0073\\ -0.0005 & -0.0005 & -0.0005\end{pmatrix}.$$

Choose $L_1 = \mathrm{diag}(0.32\ 0.25)$ and $L_2 = \mathrm{diag}(0.43\ 0.3)$, then the resulting closed-loop system matrices are

$$A_1+B_1L_1F_1+\rho_1(k)B_1L_1G_1H_1 =\begin{pmatrix}0.3120 & 0.0992 & 0.4892\\ 0.1694 & 0.3095 & 0.2895\\ 0.1818 & 0.4494 & 0.3294\end{pmatrix},$$

$$A_2+B_2L_2K_2+\rho_2(k)B_2L_2G_2H_2 =\begin{pmatrix}0.2100 & 0.3881 & 0.2496\\ 0.2795 & 0.1867 & 0.4696\\ 0.2030 & 0.3872 & 0.3196\end{pmatrix}.$$

Take initial condition $x(t_0) = (4\ 3.5\ 3)^T$, $\rho_1(k) = 3$ and $\rho_2(k) = 2$, when $l = 3$. Figures 6 and 7 represent the system state and the event-triggering signal, respectively. Figure 8 shows state simulations with different initial conditions.

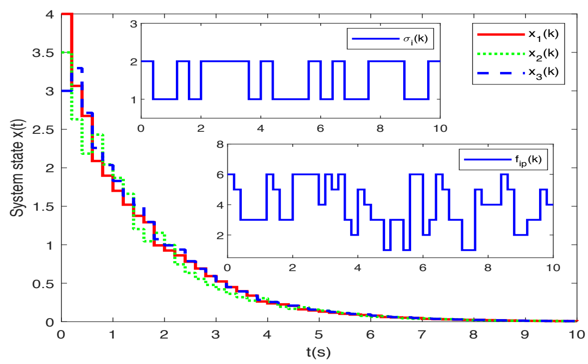

Fig. 6. The state simulations with $x(k_0) = (4\ 3.5\ 3)^T$

In this section, two examples are given to verify the effectiveness of designed controllers. Example 1 studies positive switched systems with random nonlinearities. Figure 3 shows that the system state will eventually be 0 under the non-fragile

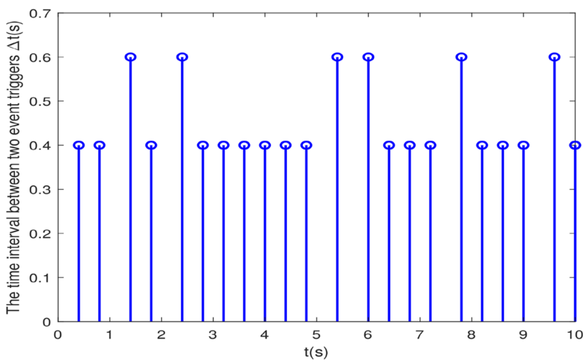

Fig. 7. The event-triggering signal

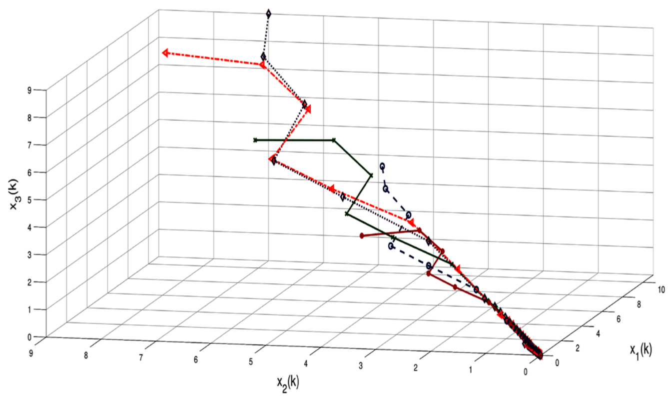

Fig. 8. The state simulations with different initial conditions

event-triggered controller. Figure 5 proves the stability of the system with different initial conditions. On the basis of Example 1, the random controller perturbations are additionally considered in Example 2. The simulation results prove that the system is stable under the designed controller.

5. CONCLUSIONS

This paper designs the non-fragile event-triggered controller for positive switched systems subject to randomly occurring nonlinearities and controller perturbations. Firstly, a non-fragile event-triggered controller for the positive switched system is formed by combining event-trigger mechanism and non-fragile control. The randomly occurring nonlinearities and perturbations are assumed to belong to the Bernoulli sequence and Binomial sequence, respectively. To drive main results, the suitable switched linear co-positive Lyapunov function is used to obtain the stability and stabilization conditions of positive switched systems. Then, the cases where the probabilities of nonlinearity and controller disturbance in each subsystem are different are considered.

ACKNOWLEDGEMENTS

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APPENDIX

The proof of Theorem 2:

Since A_i, B_i, G_i, F_i^+, and H_i^+ are nonnegative vectors, then $F_i^- \prec 0$ and $H_i^- \prec 0$. By $0 \preceq L_{di} \preceq L_{ui}$, we get

$$\begin{aligned}
&A_i + B_iL_{di}F_i^+ + B_iL_{ui}F_i^- + B_iL_{di}G_iH_i^+ + B_iL_{ui}G_iH_i^- \\
&\preceq A_i + B_iL_iF_i^+ + B_iL_iF_i^- + B_iL_iG_iH_i^+ + B_iL_iG_iH_i^- \\
&\preceq A_i + B_iL_{ui}F_i^+ + B_iL_{di}F_i^- + B_iL_{ui}G_iH_i^+ + B_iL_{di}G_iH_i^- . \quad \text{(A1)}
\end{aligned}$$

Together (7) with (A1) gives

$$\begin{aligned}
&A_ix(k) + B_iL_iF_ix(k) + B_iL_iF_ix_e(k) \\
&\quad + \rho_i(k)B_iL_i\Delta F_ix(k) + \rho_i(k)B_iL_i\Delta F_ix_e(k)) \\
&\succeq (A_i + B_iL_{di}F_i^+ + B_iL_{ui}F_i^- - \eta B_iL_{ui}F_i^+\mathbf{1}_{n\times n} \\
&\quad + \eta B_iL_{ui}F_i^-\mathbf{1}_{n\times n} + \rho_i(k)B_iL_{di}G_iH_i^+ + \rho_i(k)B_iL_{ui}G_iH_i^- \\
&\quad - \rho_i(k)\eta B_iL_{ui}G_iH_i^+\mathbf{1}_{n\times n} + \rho_i(k)\eta B_iL_{ui}G_iH_i^-\mathbf{1}_{n\times n})x(k).
\end{aligned}$$

Since $\rho_i(k)$ follows the Binomial sequence, $\rho_i(k)$ can take the value in the set $\{0, 1, 2, \ldots, l\}$. It is easy to get

$$\begin{aligned}
&\big(A_i + B_iL_{di}F_i^+ + B_iL_{ui}F_i^- - \eta B_iL_{ui}F_i^+\mathbf{1}_{n\times n} \\
&\quad + \eta B_iL_{ui}F_i^-\mathbf{1}_{n\times n} + \rho_i(k)B_iL_{di}G_iH_i^+ \\
&\quad + \rho_i(k)B_iL_{ui}G_iH_i^- - \rho_i(k)\eta B_iL_{ui}G_iH_i^+\mathbf{1}_{n\times n} \\
&\quad + \rho_i(k)\eta B_iL_{ui}G_iH_i^-\mathbf{1}_{n\times n}\big)x(k) \\
&\succeq \big(A_i + B_iL_{di}F_i^+ + B_iL_{ui}F_i^- - \eta B_iL_{ui}F_i^+\mathbf{1}_{n\times n} \\
&\quad + \eta B_iL_{ui}F_i^-\mathbf{1}_{n\times n} + lB_iL_{ui}G_iH_i^- \\
&\quad - l\eta B_iL_{ui}G_iH_i^+\mathbf{1}_{n\times n} + l\eta B_iL_{ui}G_iH_i^-\mathbf{1}_{n\times n}\big)x(k).
\end{aligned}$$

Together with $0 \preceq L_{di} \preceq L_i \preceq L_{ui} \preceq \rho L_{di}$ and (23) gives

$$\begin{aligned}
&\big(A_i + B_iL_{di}F_i^+ + B_iL_{ui}F_i^- - \eta B_iL_{ui}F_i^+\mathbf{1}_{n\times n} \\
&\quad + \eta B_iL_{ui}F_i^-\mathbf{1}_{n\times n} + lB_iL_{ui}G_iH_i^- \\
&\quad - l\eta B_iL_{ui}G_iH_i^+\mathbf{1}_{n\times n} + l\eta B_iL_{ui}G_iH_i^-\mathbf{1}_{n\times n}\big)x(k) \\
&\succeq \Bigg(A_i + \frac{1}{\rho}\frac{B_iL_{di}\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{+T}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} + \frac{B_iL_{ui}\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{-T}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} \\
&\quad - \eta\frac{B_iL_{ui}\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{+T}\mathbf{1}_{n\times n}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} + \eta\frac{B_iL_{ui}\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{-T}\mathbf{1}_{n\times n}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} \\
&\quad + l\frac{B_iL_{di}G_i\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{-T}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} - l\eta\frac{B_iL_{ui}\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{+T}\mathbf{1}_{n\times n}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} \\
&\quad + l\eta\frac{B_iL_{ui}G_i\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{-T}\mathbf{1}_{n\times n}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i}\Bigg)x(k).
\end{aligned}$$

By (22a), we have

$$\begin{aligned}
A_i &+ \frac{1}{\rho}\frac{B_iL_{di}\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{+T}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} + \frac{B_iL_{ui}\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{-T}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} \\
&- \frac{\eta B_iL_{ui}\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{+T}\mathbf{1}_{n\times n}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} + \frac{\eta B_iL_{ui}\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\zeta_{i\iota}^{-T}\mathbf{1}_{n\times n}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} \\
&+ \frac{lB_iL_{di}G_i\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{-T}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} - \frac{l\eta B_iL_{ui}G_i\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{+T}\mathbf{1}_{n\times n}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} \\
&+ \frac{l\eta B_iL_{ui}G_i\sum\limits_{\iota=1}^{r}\mathbf{1}_r^{(\iota)}\xi_{i\iota}^{-T}\mathbf{1}_{n\times n}}{\mathbf{1}_r^TL_{di}^TB_i^Tv_i} \succeq 0.
\end{aligned}$$

Thus, $(A_i+B_iL_iF_i+B_iL_i\Delta F_i)x(k)+(B_iL_iF_i+B_iL_i\Delta F_i)x_e(k)\succeq 0$ for $i\in S$. By Lemma 1, the positivity of the closed-loop system (20) is proved.

Consider the same switched linear co-positive Lyapunov function in (15), then

$$\begin{aligned}
\mathbb{E}\{\Delta V(k)\} \le \mathbb{E}\Big\{ & x^T(k)(A_i^Tv_j + F_i^{+T}L_{ui}^TB_i^Tv_j + F_i^{-T}L_{di}^TB_i^Tv_j \\
&+ \rho_i(k)(H_i^{+T}G_i^TL_{ui}^TB_i^Tv_j + H_i^{-T}G_i^TL_{di}^TB_i^Tv_j) \\
&+ \eta\mathbf{1}_{n\times n}F_i^{+T}L_{ui}^TB_i^Tv_j - \eta\mathbf{1}_{n\times n}F_i^{-T}L_{ui}^TB_i^Tv_j \\
&+ \eta\rho_i(k)\mathbf{1}_{n\times n}H_i^{+T}G_i^TL_{ui}^TB_i^Tv_j \\
&- \eta\rho_i(k)\mathbf{1}_{n\times n}H_i^{-T}G_i^TL_{ui}^TB_i^Tv_j - v_i \\
&+ \alpha_2\sum_{p=1}^{L}\aleph_{ip}(k)E_i^Tv_j)\Big\}.
\end{aligned}$$

Using (17), we have $F_i^{+T}L_{ui}^TB_i^Tv_j\preceq\delta_2\zeta_i^+$, $F_i^{-T}L_{di}^TB_i^Tv_j\preceq\delta_1\overline{\zeta}_i^-$ and $F_i^{-T}L_{ui}^TB_i^Tv_j\succeq\delta_2\rho\underline{\zeta}_i^-$. By (22c) and (22d), we obtain

$$\begin{aligned}
&H_i^{+T}G_i^TL_{ui}^TB_i^Tv_j \prec \delta_2\theta_2\xi_i^+, \\
&H_i^{-T}G_i^TL_{di}^TB_i^Tv_j \prec \delta_1\theta_1\overline{\xi}_i^-, \\
&H_i^{-T}G_i^TL_{ui}^TB_i^Tv_j \succeq \delta_2\rho\theta_2\underline{\xi}_i^-.
\end{aligned} \tag{A2}$$

By (18), (21) and (A3),

$$\begin{aligned}
\mathbb{E}\{\Delta V(k)\} \le x^T(k)\Big(& A_i^Tv_j + \delta_2\zeta_i^+ + \delta_1\overline{\zeta}_i^- + \eta\delta_2\mathbf{1}_{n\times n}\zeta_i^+ \\
&- \eta\delta_2\rho\mathbf{1}_{n\times n}\underline{\zeta}_i^- + l\overline{\rho}\delta_2\theta_2\xi_i^+ + l\underline{\rho}\delta_1\theta_1\overline{\xi}_i^- \\
&+ l\overline{\rho}\eta\delta_2\theta_2\mathbf{1}_{n\times n}\xi_i^+ - l\overline{\rho}\eta\delta_2\rho\theta_2\mathbf{1}_{n\times n}\underline{\xi}_i^- \\
&+ \alpha_2\sum_{p=1}^{L}\bar{\beta}_{ip}E_i^Tv_j - v_i\Big).
\end{aligned}$$

From (22b), we can get $\mathbb{E}\{V(k+1)-V(k)\} < -(1-\mu)V(k)$. Then, the exponentially stability of the system (1) with random nonlinearities and controller perturbations satisfying Binomial distribution can be proved by Definition 2. $\square$

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