BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 69(1), 2021, Article number: e136216
DOI: 10.24425/bpasts.2020.136216

POWER SYSTEMS AND POWER ELECTRONICS

An application of continuous wavelet transform and wavelet coherence for residential power consumer load profiles analysis

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Abstract. Load profiles of residential consumers are very diverse. This paper proposes the usage of a continuous wavelet transform and wavelet coherence to perform analysis of residential power consumer load profiles. The importance of load profiles in power engineering and common shapes of profiles along with the factors that cause them are described. The continuous wavelet transform and wavelet coherence has been presented. In contrast with other studies, this research has been conducted using detailed (not averaged) load profiles. Presented load profiles were measured separately on working day and weekend during winter in two urban households. Results of applying the continuous wavelet transform for load profiles analysis are presented as coloured scalograms. Moreover, the wavelet coherence was used to detect potential relationships between two consumers in power usage patterns. Results of coherence analysis are also presented in a colourful plots. The conducted studies show that the Morlet wavelet is slightly better suitable for load profiles analysis than the Meyer's wavelet. Research of this type may be valuable for a power system operator and companies selling electricity in order to match their offer to customers better or for people managing electricity consumption in buildings.

Key words: residential power consumers; load profiles; power demand; continuous wavelet transform; wavelet coherence.

1. Introduction

The intense interest in Smart Grids has led to increasing research of load profiles studies. In this new kind of a power network many objects are monitored and evaluated using measured time series. Nowadays designing new methods to analyse these data is a big challenge. With the widespread accessibility of the load profiles collection, it is very important to understand the necessity of further data analysis such as pattern recognition. This kind of knowledge can help discover hidden relationships and patterns concerning end users [1].

The growing potential of wavelet transform usage creates new opportunities for analysis of many data which may be extracted from measurements in distribution power networks. Residential consumer load profile can be an example of the data which, after applying a wavelet transform, will show its imperceptible properties. Such knowledge can be valuable in the future due to the widespread implementation of demand response programs or electric vehicles [2].

Load profiles play a significant role in understanding user's electric power consumption. They are usually presented as a variation of an active power in a function of 24-hour period. The final load profile is the result of the aggregation of individual load profiles of all appliances operating in a selected area. The demand for active power depends on a number of factors such as consumer attitude, type of appliances along with their working mode and time of using electricity. Therefore, each consumer has their own load profile which is closely associated with his all activities.

Most of the load profiles shapes are the results of the activities of the end users. Some residential power consumers focus on more conscious electricity usage, while the others are concerned only about satisfying needs. Thus, the final load profile of a selected household is a combination of several patterns. Moreover, different socio-economic factors will contribute to varied energy demand [3].

Despite these differences, all residential load profiles show three basic trends: morning and evening peaks with a little base demand during the rest of the time. In each not averaged residential load profile there are four basic types of shapes [4]. The first one is base load, which is caused mainly by small appliances in stand-by mode. The second type are regular oscillations, made mainly by fridges. The third type is temporary shape inducted by household appliances like a vacuum cleaner. The last type are ragged peaks made by ovens or electric kettles. Residential consumers may have a small influence on the first two types of shapes. However, they can strongly affect the temporary shapes and ragged peaks. It is worth highlighting the fact that usage of appliances making ragged peaks causes transient components of load profiles waveforms.

The shapes of active power waveform in load profile can be detailed or averaged. The consequence of averaging is the smoother waveform and less volatile load profile. Load profiles very often are presented as averaged or highly generalized [5–7]. This makes them more difficult for analysis.

The knowledge of detailed load profiles can be very useful for precise demand forecasting or grid development planning. Most electricity selling companies use the standard residential load profiles. Therefore, they can see only rough reflection

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Manuscript submitted 2020-09-09, revised 2020-10-23, initially accepted for publication 2020-11-23, published in February 2021

of individual patterns in electricity consumption. Understanding detailed load profiles analysis can lead to submitting new business strategies by clustering processed data into targeted groups. Detection of historical patterns can certainly play an important role in predicting future trends in electricity usage. This issue may become significant also when more renewable energy sources will be operating in the distribution grid. The lack of detailed load profiles analysis means that there is little awareness on how residential consumers use their appliances. Unfortunately, typical high resolution load profiles are nonstationary time series. The consequences of this may be difficulties in applying proper analysis tools. Thus, the obtained results may not be reliable for further studies and it will be hard to make the right decisions based on them.

In power engineering, wavelets are used for transient analysis [8], energy quality [9], fault detection [10], load forecasting [11] or appliances identification [12]. Wavelets are well-suited to analyse data with sharp discontinuities [13], for example caused by most household appliances.

The main advantage of applying wavelet transform is the ability to expose such signal properties as discontinuities or trends. As a result, the initial signal is decomposed into wavelets that are slim over high frequency and extensive over low frequency components [14]. This kind of signal processing is better than Fourier transform, which requires only stationary time series [15] and cannot provide good localization in both time and frequency.

Research on load profiles has been conducted for many years and still is a current issue. Related studies have also been conducted by [16], but they concerned the entire power system, not an individual residential consumer. Other papers like [1, 2, 5–7, 17] deal with averaged (not detailed) load profiles. Cruickshank et al., in their work [18] proposed wavelet analysis to capture and simulate time-frequency domain behaviour and presents load profiles with 15 minutes interval. In contrast to the above-mentioned papers, this paper shows detailed residential load profiles, all measured with 1 minute interval. Thanks to this approach, significant profile shapes can be shown and analysed with wavelets.

In this paper the application of continuous wavelet transform and wavelet coherence for residential consumer load profile analysis has been investigated. The proposed approach identifies the patterns of electricity usage and shows a potential coherence between two different users resulting from their load profiles. This paper is organized as follows: Section 2 describes residential load profiles and wavelets. Also detailed load profiles are shown. Section 3 presents the results of analysis. In Section 4 discussion and conclusions are presented respectively.

2. Residential load profiles and wavelets

2.1. Residential load profiles. Residential users in many countries account for about 20–30% of total electricity consumption. For example, in Poland it was about 23% in 2015 [19], while in Turkey it was about 26% in 2017 [20]. Average household share for European Union was around 30% [19]. Most published load profiles of residential consumers are with large time resolution, for example one hour. The final shape of each load profiles mostly depends on the interaction between patterns of electricity usage and time of occupancy. Residential load profiles are related to many factors such as personal status (age, number of occupants in the household, active working or retired person [17] and available appliances).

Active power demand was measured by an energy logger device running with each individual appliance. Recorded signals were stored into memory card and imported to a computer for further analysis. The dataset consists of load profiles from two different households obtained during winter and summer. The process of collecting each appliance data separately has allowed to save unique shapes of power demand. Theses shapes depend mostly on the selected operation mode and type of the appliance. It is worth mentioning that appliances which cause ragged peaks are used by the majority of users randomly.

In the following paper, load profiles were measured in two selected households located in a big city in Poland. Both households were located in the urban area, hence they do not require as much energy as these from rural areas. The first household was occupied by three persons, while the second only by two. Household 1 was unoccupied from 8 am to 4 pm during working days (from Monday to Friday). All inhabitants have full-time jobs. Their annual electricity consumption was around 2000 kWh. Household 1 was equipped with such receivers like: router, fridge, TV, PC computer, washing machine and hoover. Household 2 was unoccupied from 7 am to 6 pm during working days. All inhabitants have full-time jobs and other duties. Due to frequent stay away from home, their annual electricity consumption was below 500 kWh. Household 2 was equipped with such receivers like: fridge, electric kettle, laptop, washing machine and hoover. In both households domestic hot water was delivered from the public network thus, there was no need to use an electric boiler.

The manufactures, models and using manners of all appliances were different in both cases. In consequence, each load profile reflects the unique shapes caused by every household resident during everyday electricity utilization.

Figure 1 presents load profile form household 1 on a typical working day, while Fig. 2 on weekend (Saturday). Figure 3 presents load profile of household 2 on a typical working day, while Fig. 4 presents weekend (Saturday also).

The typical days have been selected for presentation because they were closest to the usual daily consumption patterns. All measured load profiles are typical for residential households in an urban area and reflect employment activities during working days and increased activity in appliances usage at the weekends. It is worth pointing out that there is an annual, seasonal or even daily variability of profiles. The base load form is almost constant. Then load increases during morning hours, decreases during evenings. The peak load appears in the late evening hours between 4–8 pm (household 1) and 6–9 pm (household 2). Load profiles were not much affected by the temperature. Only the profile of the refrigerator changed slightly depending on the year season. For example, during summer, power demand was

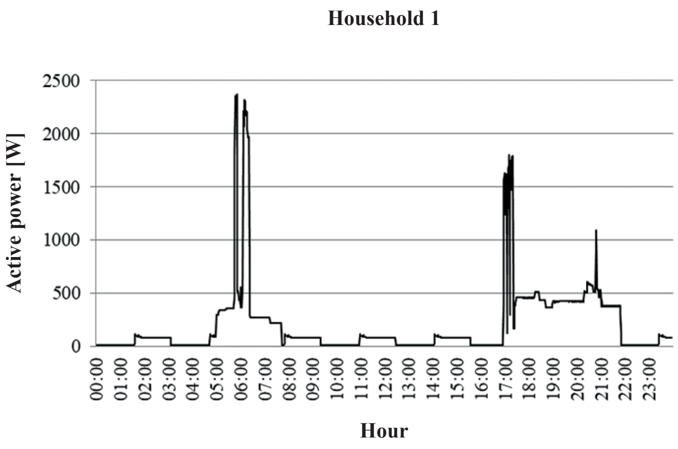

Fig. 1. Household 1 load profile, working day, winter. Based on [21]

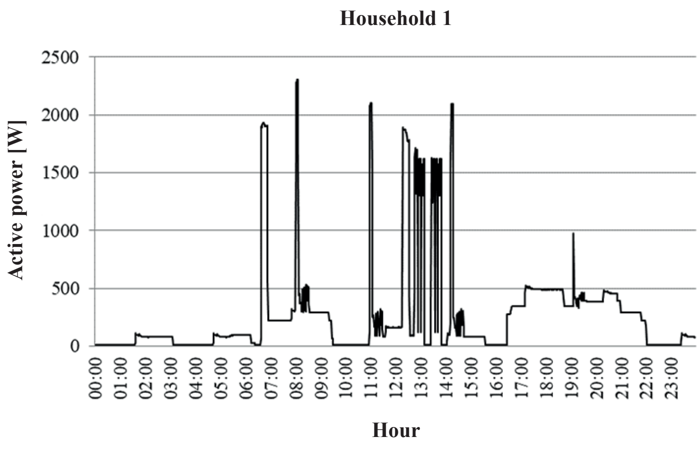

Fig. 2. Household 1 load profile, weekend, winter. Based on [21]

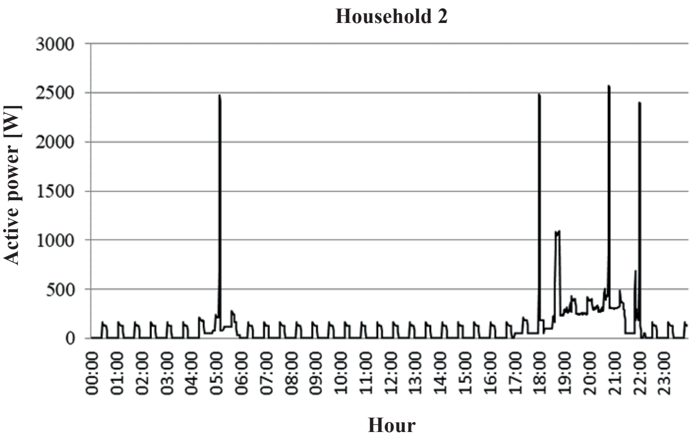

Fig. 3. Household 2 load profile, working day, winter. Based on [21]

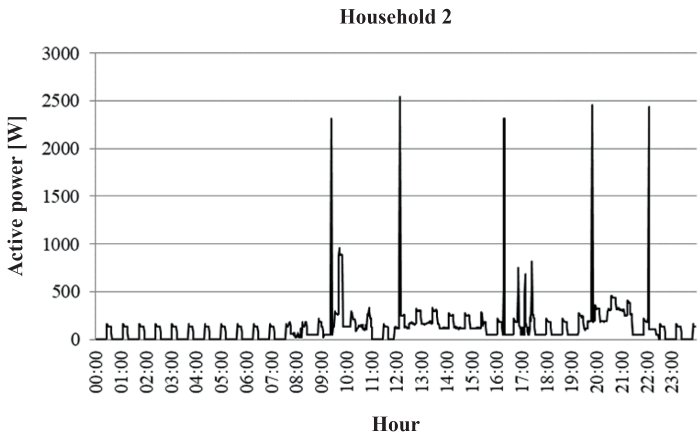

Fig. 4. Household 2 load profile, weekend, winter. Based on [21].

marginally higher than in the rest of the year. In each of the households, the usage of all receivers was almost independent of the outside temperature.

2.2. Wavelet analysis and continuous wavelet transform. Load profiles provide the information about power consumption over the time domain. Additional information about the properties of electricity usage can be obtained by representing the load profile using selected transformation, such as wavelet transform.

Wavelet analysis can be a relevant addition for power engineers who want to expand their knowledge about signal processing. The purpose of applying wavelet transform is changing the initial time series into scaled and shifted versions of a mother wavelet function [22]. The wavelet transform has the ability to identify scale or frequency components with their time location simultaneously. Time-frequency analysis can detect frequency components at certain time. The advantage of applying a wavelet transform is the possibility to analyse nonstationary signals.

Wavelets are relatively new as a signal processing technique. The wavelet transform can be seen as a microscope which can focus on analysed signal. Wavelets are finite functions which are similar to waves in nature [16]. Every wavelet is located in time and frequency simultaneously. The admissibility condition implies that wavelet has zero mean. In contrast to the sine waves, wavelets are irregular, asymmetric and decay to zero at both ends.

The most important aspect of using wavelet transform is to choose a mother wavelet. The mother wavelet function is a prototype for all further wavelets. There are several types of mother wavelets, where the most popular are: Haar, Morlet, Meyer and Mexican Hat. Meyer wavelet is orthogonal type while Morlet is nonorthogonal. Performing continuous wavelet transform (CWT) both types can be used, which is a big advantage when many load profiles have different shapes. In order to get a better performance, the chosen wavelet should have a similar shape to ragged peaks in measured load profile. The mother wavelet is dilated or contracted by changes of scale parameter s. The translation parameter l describes the position of the wavelet in time. By changing of l wavelet can be moved over the signal. Every wavelet coefficient is related to both scale and a point in time.

Many mother wavelets can be used in the continuous wavelet transform. For this kind of research Morlet and Meyer wavelets were chosen because their shapes are similar to the load profiles form. Although this is the continuous wavelet transform, it is determined on discrete data which is convenient in computer calculations. Features distinguishing a continuous wavelet transform from discrete transform are the possibilities of calculation for each scale value and continuity in a relation to the displacement.

The continuous wavelet transform of a time series is defined as the convolution of each value of the signal with scaled and translated mother wavelet and it is described by Eq. (1) [16]:

$$W_m(s) = \frac{\sigma_t}{\sqrt{s}} \sum_{n=0}^{N-1} x_n \varphi^* \left[\frac{(n-m)\sigma_t}{s} \right] \tag{1}$$

where: σ_t – the same interval between samples, s – scale factor, N – number of time series samples, x_n – n-th element of the time series (where $n = 1, 2, 3 \ldots, N$), φ – wavelet function, m – shift coefficient, * – conjugation of a complex number.

Hence, CWT compares the time series to shifted (delayed or hastened) and scaled (stretched or compressed) version of a mother wavelet function. The final transform coefficients depend on the choice of a mother wavelet and scales. It is worth emphasizing that a scale factor is inversely related to the frequency.

The continuous wavelet transform is a time-scale representation of the initial signal. It is able to find position and duration of an event such as the growth of demanded power. This transform can be described by the following 6 steps. Step 1 – place a wavelet at the beginning of the analysing signal. Step 2 – compare wavelet with the signal. Step 3 – calculate transform coefficients for a given part of signal. Step 4 – shift the wavelet through the whole signal (repeat previous steps, until the end of the signal). Step 5 – scale the wavelet (repeat previous steps – 1, 2, 3 and 4). Step 6 – repeat all previous steps for all scales. The result of applying CWT is a two-dimensional representation of the signal. The change in the coefficients over scales provides information for pattern matching. Wavelet coefficients are stored in 2D matrix.

2.3. Wavelet coherence. To study the interaction between two time series, wavelet coherence was adopted [22]. This kind of transformation can detect regions in the time-frequency space where both analysed time series show co-movement. Wavelet coherence is described by Eq. (2) [22]:

$$R_n^2(s) = \frac{\left| S\left(s^{-1} W_n^{XY}(s)\right)\right|^2}{S\left(s^{-1}\left|W_n^X(s)^2\right|\right) \cdot S\left(s^{-1}\left|W_n^Y(s)\right|^2\right)} \tag{2}$$

where: $R_n^2(s)$ – coherence coefficient, S – smooth operator, s – scale factor, W_n^X, W_n^Y – continuous wavelet transforms of X, Y signals expressed as a convolution of the n-th signal sample x_n with a scaled and normalized wavelet, $W_n^{XY} = W_n^X W_n^{Y^*}$ – cross wavelet transform of x_n and y_n samples.

The correlation between a wavelet and a part of the measured signal is represented by the wavelet coefficients. A large value of scales corresponds to a low frequency while small values of scales correspond to high frequency. Overall information about signal is given by large scales. Detailed information is given by small scales. The graphic representation of wavelet coefficients is called a scalogram. The scalogram is a graphical representation of the absolute values of the CWT coefficients. Scalogram vertical axis describes periods, while the horizontal axis describes the time. The more intensive phenomena, the more intensive colour on a scalogram. When wavelet coherence is close to zero both time series have a weak correlation. Strong correlation between time series exists when coherence is close to one.

Wavelet coherence analysis can also show phase differences which correspond to delays in the oscillations between both time series. Phase difference is described by Eq. (3) [23]:

$$\phi_{x,y} = \tan^{-1}\left(\frac{\mathrm{Im}\left(W_n^{XY}\right)}{\mathrm{Re}\left(W_n^{XY}\right)}\right) \tag{3}$$

where: $\phi_{x,y}$ – coherence phase included in the range $[-\pi, \pi]$, Re, Im – the real part and the imaginary part of W_n^{XY} respectively.

Wavelet coherence can detect transient relationship between considered time series. Differences are usually pointed as arrows on the final coherence plot. When arrows are directed to the right – both time series are in phase. The direction to the left means that time series are in anti-phase. Arrows directed up mean that first signal leads the second one by 90°, while arrows directed down mean opposite situation. In many measured signals arrows have intermediate positions.

3. Results of analysis

In order to obtain the presented below scalograms, Matlab and Wavelet Toolbox were used. The upper parts of Figs. 5–8 show the load profile of the measured household, while the lower part contains the values of wavelet coefficients in the form of a coloured scalogram. Colours illustrate abrupt changes in the load demand. The horizontal axis shows minutes, while the vertical axes show active power demand and wavelet scales respectively. Scales values constitute the degree of wavelet dilatation or compression. It can be seen that with low values of scales the wavelet is compressed (thin stripes at the bottom part of each scalogram), while with high values of scales the wavelet is stretched (wider stripes at the upper part of each scalogram).

By applying continuous wavelet transform it was possible to convert measured load profiles from the time domain to the time-scale domain. To analyse load profile of household 1 during weekend, Meyer (Fig. 5) and Morlet wavelets (Fig. 6) were used. Power consumption by a fridge was visible in the form of slightly brightened blue stripes.

The moments of increased power consumption are clearly marked by red colour. It was the effect of using a washing machine and a vacuum cleaner. The use of these appliances in a short time significantly increases the consumption of active power in relation to the power of other appliances such as devices on a stand-by mode. Figures 5 and 6 show that the maximum value of power demand occurs about scales from 21

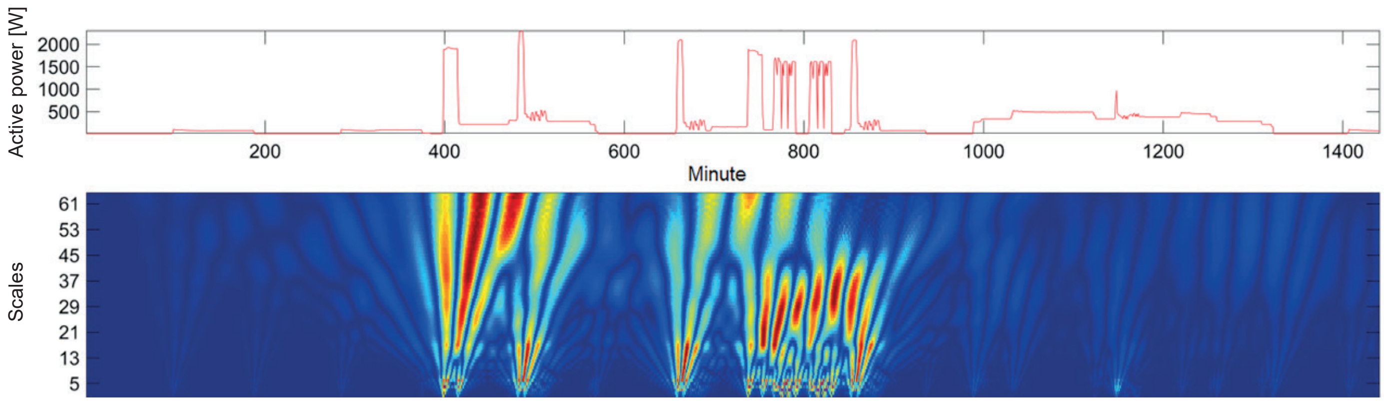

Fig. 5. Wavelet transform of household 1 load profile during weekend, winter. Mother wavelet: Meyer. Based on [21]

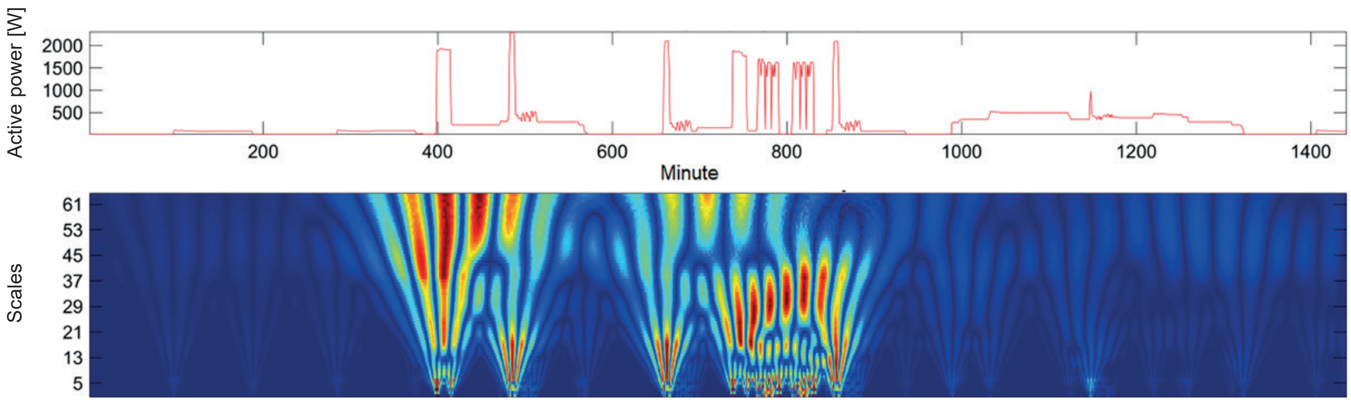

Fig. 6. Wavelet transform of household 1 load profile, weekend, winter. Mother wavelet: Morlet. Based on [21]

to 61 during the morning peak, while during afternoon peak for scales from 13 to 37. This relationship appears independently from the selected mother wavelet function.

The second case concerns a selected load profile of household 2 in winter during a working day. Both Morlet and Meyer wavelets show that there was a regular, oscillating power consumption – the effect of using a refrigerator. It was a completely different manufacturer and a model than in the household 1. Compared to household 1, these oscillations were more frequent, as shown in Figs. 7 and 8. In contrast to household 1, for household 2 in the afternoon peak, when the demand for power increased, the scale factors values were higher and ranged from 13 to 61.

It is worth paying attention to very small values of coefficients between 400 and 900 minutes in the long-term in the household 2 load profile (Figs. 7 and 8). Analysing this profile, one can guess that at this time the flat is probably empty. Wavelet analysis clearly marks this fact through smaller values of coefficients, and it can be observed regardless of the choice of the mother wavelet. During the absence of residents the scale

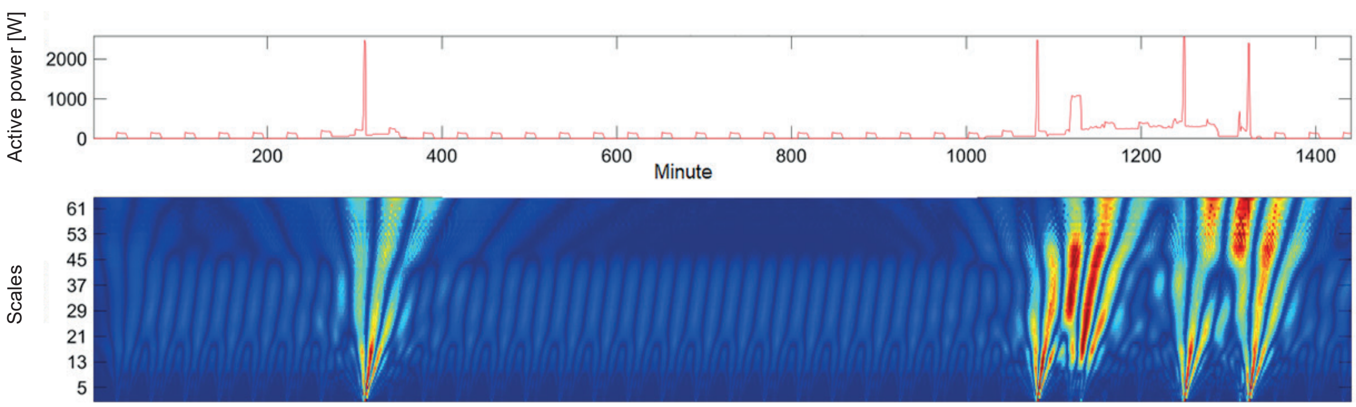

Fig. 7. Wavelet transform of household 2 load profile, working day, winter. Mother wavelet: Meyer. Based on [21]

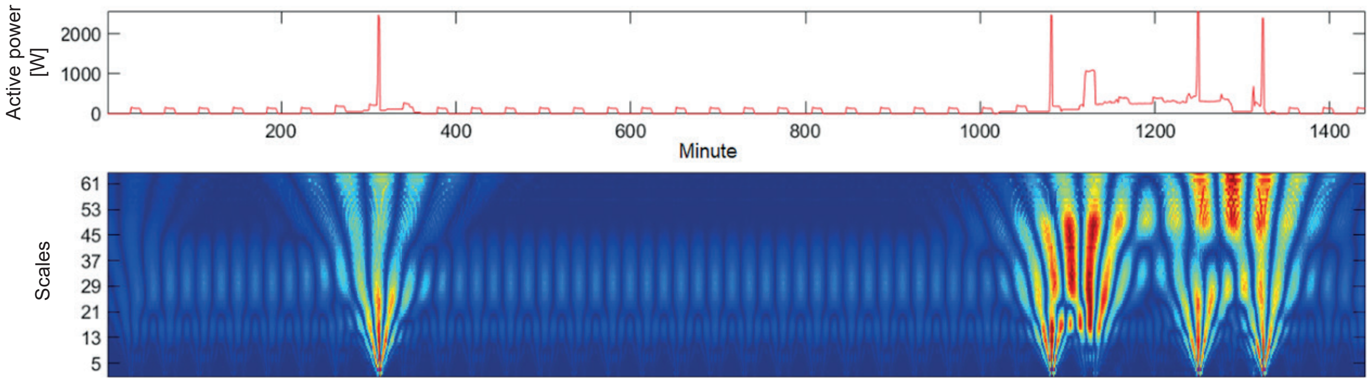

Fig. 8. Wavelet transform of household 2 load profile, working day, winter. Mother wavelet: Morlet. Based on [21]

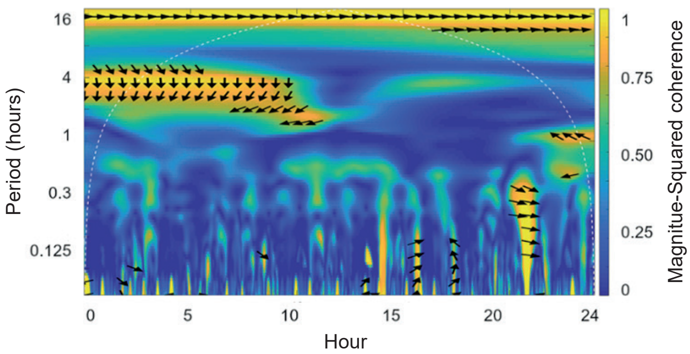

Fig. 9. Wavelet coherence of household 1 and household 2 load profiles during working day, winter

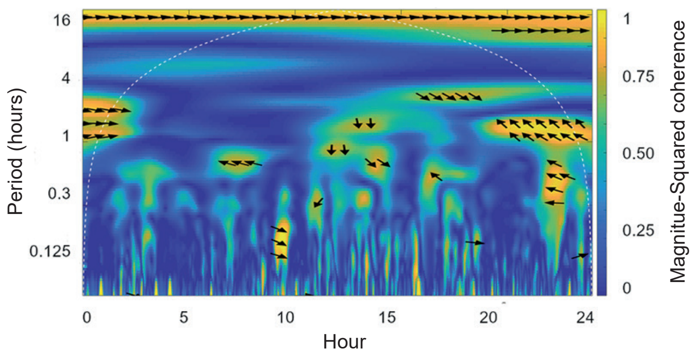

Fig. 10. Wavelet coherence of household 1 and household 2 load profiles during weekend, winter

factors are almost constant. Such information may infringe the privacy of residents.

Short-term large active power consumption is marked as scale factors with small values (Figs. 7 and 8, scale values from 5 to 29). Long-term larger active power consumption is marked as scale factors medium and high values (Figs. 7 and 8, scale values from 29 to 53).

Dark blue colour represents small active power demand, while orange and red colours represent medium and large demand. Different appliances make different shapes in wavelets scalograms. Shapes depend on chosen mother wavelet, type of appliances and their usage mode. The more demand for energy, the larger and wider stripes.

Regardless of used appliances, both mother wavelets can present accurate load profiles in a colourful way. In comparison to Meyer's wavelet, Morlet wavelet is slightly more sensitive to the abrupt demand changes (for example around 1150th minute, Fig. 8) which is visible through a yellow and green blue stripe for scales from 53 to 61.

Figure 9 shows the wavelet coherence of the household 1 and household 2 load profiles during a working day, while Fig. 10 shows wavelet coherence during a weekend. The horizontal axis shows the hours of the day, while the vertical axis shows the periods of time in hours.

The white outline illustrates the cone of the influence. Apart from the results of this area, both households power demands were strongly coherent during a working day from 0 am to 10 am and around 9 pm. Figure 9 shows that the coherence in the morning lasts for about 4 hours. Also, during evening peak hours, the coherence is high for only about half an hour (yellow stripe on Fig. 9 around 9 pm). This may be due to the use of appliances with a short operating time mode.

Figure 10 shows that during the weekend, coherence occurs randomly through a whole day. The strongest coherence can be observed after 8 pm, while the duration is approximately one hour.

The analysis of Figs. 9 and 10 shows that residents of both households use electricity in a coherent way for longer periods on evening during the weekend than in the working day.

In other time intervals, the coherence was low and was presented by dark blue colour. This can be the result of a strong differentiation of load profiles of a particular household appliances. It is worth noting that in the vicinity of yellow stripe the arrows are present.

4. Discussion and conclusions

In the author's opinion, the usage of Morlet wavelet is slightly more suitable than Meyer's wavelet for this kind of research. This is due to the fact, that the shape of the Morlet mother wavelet can better match the shape of a household profile. In case of both household load profile scalograms, it can be seen that with the increase of scale, scale coefficients were wider.

Though, they were still near the point in time at which they occurred. Sudden changes in active power demand were associated with many scale parameters.

However, one need to be careful in interpreting the results of wavelet coherence due to the presence of the cone of influence. Edge effects may be present at the beginning and the end of measured time series (it can be seen in Figs. 9 and 10). Fortunately, around the midnight there is a small activity in electricity usage in a majority of households. For this reason, the edge effects need to be taken into consideration while studying the wavelet coherence results.

The aim of this paper was to demonstrate the usage of continuous wavelet transform and wavelet coherence for residential consumer load profile analysis. The results obtained for each household were presented and discussed.

Wavelet coherence studies have a chance to provide the power system operator wider additional information about residential consumers, for example, whether one of the analysed values is in any way convergent with the other ones. Similar studies can be applied to find a potential relationship between load profiles and energy prices or temperature. The correlation between two different load profiles depends on many factors. One of them may be the kind of appliances and their usage patterns (time of use and operation mode). The choice of a mother wavelet also affects the interpretation of the results. It is very important to choose a proper mother wavelet for further analysis. Morlet or Meyer seem to be the best.

The related studies [16] and [18] highlighted the fact that wavelets are a promising tool for time series analysis in power engineering. Research of this type may be valuable for a power system operator. Wavelet analysis, similar to one presented in this paper, can be applied with some extensions for high accuracy load forecasting, analysis of fluctuations in generation and load (power balance studies), network planning, arranging load shedding scheme in obedience to the peak and valley time in load profiles. Companies selling electricity can use wavelets to exploit electricity consumption patterns, tariff design or customer clustering.

Acknowledgements. Most of the article was based on selected part of Author’s Ph. D. dissertation [21].

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