Design of Sparse FIR Filters
with Low Group Delay

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Abstract—**The aim of the work is to present the method for designing sparse FIR filters with very low group delay and approximately linear-phase in the passband. Significant reduction of the group delay, e.g. several times in relation to the linear phase filter, may cause the occurrence of undesirable overshoot in the magnitude frequency response. The method proposed in this work consists of two stages. In the first stage, FIR filter with low group delay is designed using minimax constrained optimization that provides overshoot elimination. In the second stage, the same process is applied iteratively to reach sparse solution. Design examples demonstrate the effectiveness of the proposed method.**

Keywords—**digital filters, FIR filters design, sparse filters, reduced group delay filters**

I. INTRODUCTION

THE digital finite impulse response (FIR) filters are often applied because of their numerous advantages, including inherent stability, well elaborated design methods, low coefficient sensitivity and easiness of obtaining a linear-phase response. Moreover, non-recursive structures can be easily implemented in programmable systems. Significant drawbacks of linear-phase FIR filters are large group delay and higher number of multipliers and adders needed for filter realization, in particular when the order of the filter is large, e.g. for a narrow transition band filters. A low group delay is required in many applications, such as energy storage systems [1], audio systems [2,3,4] or active noise control systems [5].

In order to eliminate the above-mentioned drawbacks of FIR filters, numerous works are carried out which can be divided into two categories. The first category includes works aimed at reducing filter group delay, usually by limiting the linear-phase requirement to the passband only [3,4,6]. For the design of such filters, methods dedicated for the broader category of nonlinear-phase FIR filters design can be applied [7-9]. There are other methods based on block convolution [10] to filter long data sequences. If no fixed group delay is required standard methods for designing minimum-phase filters can also be used [11,12]. Unfortunately, as a consequence of reducing the filter's group delay, its impulse response is not symmetrical, which increases the number of multipliers needed in filter implementation. Usually, the order of such filter will also be larger compared to the linear-phase one. The works belonging to the second category focus on reducing the number of arithmetic operations and are generally based on limiting the number of non-zero coefficients to obtain low-complexity and low-power FIR filters. This approach is called a sparse FIR filter technique and has been intensively developed recently.

The intuitive approach to obtain sparse impulse response of FIR filter is to minimize the l_0-norm of the filter coefficients. Unfortunately, the l_0-norm minimization problems are nonconvex, thus it can be approximated by l_1-norm formulation. Recently, a number of efficient l_1-norm minimization based sparse filter design algorithms have been proposed [13-19]. Another solutions that allow to search for zero-coefficient locations based on heuristic methods, such as branch-and-bound algorithms [20,21] or genetic algorithms [22] were also developed. Linear programming can also be used for sparse FIR filters design. Known algorithms bring the smallest coefficients to zero or iteratively nullify the coefficient which results in the minimum increase of approximation error as long as the design specifications are still met [23,24].

The aforementioned sparse FIR filter design algorithms require a filter order. When designing, a higher than required order is usually assumed to ensure that the filter meets the design specifications during the optimization process that forces zero coefficients. In order to improve the performance of the filter joint optimization algorithm can be used that simultaneously optimizes the order of the filter and coefficient sparsity [25,26].

It should be mentioned that historically other methods based on cascaded form were used to reduce the number of non-zero filter coefficients. Such methods can be called indirect methods, e.g. frequency-response masking technique. These types of methods also include those based on interpolated FIR filters dedicated to narrowband filters. Both approaches are still being developed [27,28].

This paper presents the study of designing a sparse FIR filter with low group delay and near linear-phase in passband. It has been shown that in some cases the magnitude frequency response can show a large overshoot. The design algorithm has been proposed that consists of two stages. In the first stage, FIR filter with low group delay is designed using minimax constrained optimization, while in the second stage the same process with successive thinning algorithm [23] is applied iteratively to search sparse solution.

The structure of this paper is as follows: in the second section the problem of designing filters with prescribed, reduced group delay and a linear-phase in the passband is formulated. The third section presents the concept of designing FIR filters free from undesirable overshoot in the magnitude frequency response that appears when a low group delay is forced. The same section also includes the proposition how to obtain a sparse filter from a previously designed filter with reduced group delay. Some examples showing the advantages of the proposed method are presented in section four. Finally, conclusions are given.

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II. PROBLEM FORMULATION

The digital FIR filters considered in the paper are described by the following frequency response $H(\mathbf{b},\omega)$:

$$H(\mathbf{b},\omega) = \sum_{k=0}^{N-1} b_k e^{-j\omega k} = \mathbf{b}^T \mathbf{c} \qquad (1)$$

where $\mathbf{b}$ is a vector of N-dimensional real-valued coefficients:

$$\mathbf{b}^T = \begin{bmatrix} b_0 & b_1 & b_2 & \cdots b_{N-1} \end{bmatrix}$$

and vector $\mathbf{c}$ contains exponential terms of (1):

$$\mathbf{c} = \begin{bmatrix} 1 & e^{-j\omega} & e^{-j2\omega} & \cdots e^{-j(N-1)\omega} \end{bmatrix}^T$$

The aim of design is to determine $\mathbf{b}$ so that the resulting filter $H(\omega)$ approximates desired complex-valued frequency response function $D(\omega)$ and has low group delay, nearly linear phase in the passband and sparse coefficients. To achieve linear-phase, $D(\omega)$ for lowpass filter must have a form:

$$D(\omega) = \begin{cases} e^{-j\tau\omega} & 0 \le \omega \le \omega_p \\ 0 & \omega_s \le \omega \le \pi \end{cases}$$

where: ω_p, ω_s are edges of passband and stopband, respectively. Group delay τ should be significantly smaller than $(N-1)/2$ to consider it as low.

III. DESIGN METHOD

The algorithm proposed in this paper consists of two stages. In the first stage, optimization is carried out to obtain the best approximation of the desired frequency response $D(\omega)$ in minimax sense over a frequency region $\Omega = \{\omega: 0 \le \omega \le \pi\}$. In the second stage, the same process is applied iteratively to reach sparse solution.

A. First stage

The design problem of the first stage can be formulated as finding vector $\mathbf{b}$ by solving constrained optimization problem:

$$\begin{aligned} &\underset{\mathbf{b},\delta}{\text{minimize}} \; \delta \\ &\text{subject to} \; W(\omega)\big|H(\mathbf{b},\omega) - D(\omega)\big| \le \delta \quad \text{for } \omega \in \Omega \end{aligned} \qquad (2)$$

where $W(\omega) \ge 0$ is a known weighting function and δ is an upper bound of approximation error that is minimized. The value of error δ depends obviously on the filter length N which must be determined before optimization. Usually the solution of the optimization problem (2) is carried out excluding the transition band by assuming $W(\omega)=0$ for $\omega_p \le \omega \le \omega_s$. It is possible to force error value in one of the bands (e.g. in stopband) by modifying the problem (2) in the following way:

$$\begin{aligned} &\underset{\mathbf{b},\delta}{\text{minimize}} \; \delta \\ &\text{subject to} \; \big|H(\mathbf{b},\omega) - D(\omega)\big| \le \delta \quad \text{for } \omega \in \Omega_p \\ &\qquad\qquad \big|H(\mathbf{b},\omega)\big| \le A \quad \text{for } \omega \in \Omega_s \end{aligned} \qquad (3)$$

where: A is maximum stopband error and Ω_p, Ω_s are subsets of Ω containing frequencies in the passband and stopband, respectively.

The conducted experiments showed that forcing a low group delay can cause the appearance of significant overshoot of the magnitude frequency response in designed filter transition band. It has been found that the overshoot value depends on the filter length N to group delay τ ratio, with the overshoot increasing as the ratio increases. This situation is shown in Fig. 1, which presents the magnitude frequency responses of three filters with different transition band widths, i.e. 0.2π, 0.1π, 0.05π (in normalized radian frequency). Other parameters common for all three filters were as follows: $A=-40$ dB, $\tau = 7$ (in samples), and $\omega_s = 0.5\pi$. The filter lengths depended on the transition band width and were N=21, 55, 121.
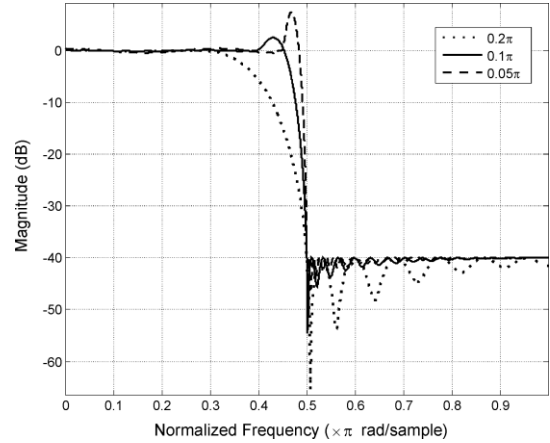

Fig. 1 Magnitude response overshoot for three filters with different width of transition band

For the filter with N=121, the ratio N/τ is over 17, so in this case the group delay should be considered very low.

The solution of the described problem is to perform optimization in transition band as well. However, it is difficult to predict the shape of magnitude in the transition band. Thus the easiest way is to limit the value of maximum gain in the transition band and modify the optimization algorithm (3) by adding another constraint:

$$\begin{aligned} &\underset{\mathbf{b},\delta}{\text{minimize}} \; \delta \\ &\text{subject to} \; \big|H(\mathbf{b},\omega) - D(\omega)\big| \le \delta \quad \text{for } \omega \in \Omega_p \\ &\qquad\qquad \big|H(\mathbf{b},\omega)\big| \le A \quad \text{for } \omega \in \Omega_s \\ &\qquad\qquad \max(\big|H(\mathbf{b},\omega)\big|) \le \gamma \quad \text{for } \omega_p \le \omega \le \omega_s \end{aligned} \qquad (4)$$
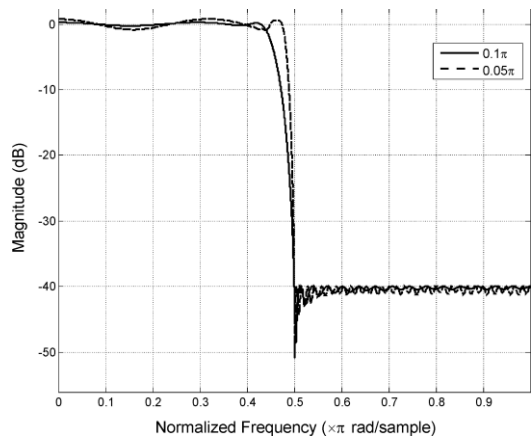
where γ is the maximum gain in the transition band.

Fig. 2 Magnitude response of two filters with transition band widths 0.1π and 0.05π for γ=1.02 γ=1.07 respectively, after overshoot elimination by (4).

Figure 2 shows the result of introducing the new constraint to design the same filters as before for transition band widths 0.1π and 0.05π. Unwanted overshoot has been removed, but the additional constraint caused increase of the error δ. Also, the group delay approximation error may change. Both cases are shown in Fig. 3. The problem (4) describes the optimization for lowpass filters. In a similar way, an optimization problem for other types of filters can be constructed by changing the desired function $D(\omega)$ and the ranges of ω in which the constraints apply.
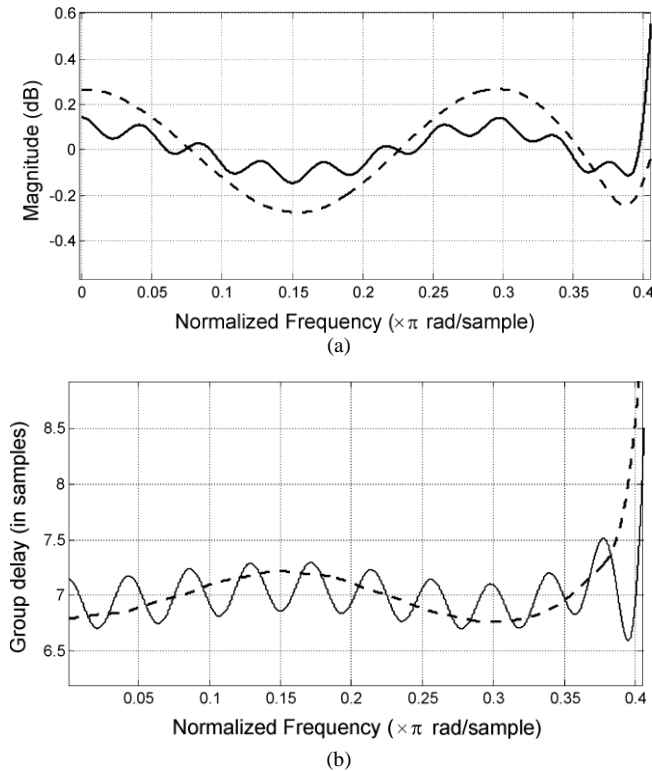

(a)

(b)

Fig. 3 Magnitude response in the passband (a) and group delay response (b) for filter with transition bandwidth 0.1π, before (solid line) and after (dashed line) overshoot elimination by (4).

B. Second stage

The goal of the second stage of the proposed algorithm is to obtain a sparse filter based on the solution obtained from the first stage. Of the filter design methods mentioned in the introduction, the successive thinning algorithm [23] is the easiest to adapt to optimization (4). It involves searching for the smallest coefficient, replacing it with zero, and repeating the optimization to obtain the remaining coefficients. This process is performed iteratively as long as the design specifications are still met. The application of the presented algorithm requires modification of optimization problem (4) and introduction of an additional vector $\mathbf{m}$, which has the same dimension as $\mathbf{b}$ and allows forcing zero instead of the coefficient b_k that has the smallest absolute value. Initially, all elements of the vector $\mathbf{m}$ are set to one. If the smallest element $\mathbf{b}$ has the number k, then m_k is changed to zero and the vector $\mathbf{b}$ in (4) is replaced by $\mathbf{m} \bullet \mathbf{b}$. The operator $\bullet$ denotes multiplication of two vectors element by element (Hadamard product). The detailed description of the proposed algorithm is presented in Table I. The ε denote the maximum permissible error in bandpass.

TABLE I
STEPS OF THE PROPOSED ALGORITHM

Step number	Description
1	Input N and desired filter specifications: ω_p, ω_s, τ, A, ε, γ
2	Set iteration index $i=0$ and $\mathbf{m}=\mathbf{1}$
3	Solve (4) and set $\mathbf{b}^i=\mathbf{b}$
4	If $\delta < \varepsilon$ go to step 5 otherwise go to step 1 (no solution , new specifications)
5	Search smallest $|b_k|\neq0$, save index k and set $m_k=0$
6	Replace $\mathbf{b}=\mathbf{m}\bullet\mathbf{b}^i$ and solve (4)
7	If $\delta > \varepsilon$ or i reaches maximum go to step 8 otherwise set $i = i +1$, $\mathbf{b}^i=\mathbf{b}$ and go to step 5
8	Output $\mathbf{b}^i$

IV. DESIGN EXAMPLES

Application of the proposed approach (4) for filter design will be shown in the examples given below. Examples 1 and 4 come from the literature and best match the problem considered in this work. The first example only shows the advantage of the first stage of the proposed algorithm. Calculations have been implemented in MATLAB using CVX package for specifying and solving convex programs [29].

A. Example 1

The first example is taken from [3] where Filter #3 has specification: edges of passband and stopband frequency 5 kHz and 6 kHz respectively, stopband attenuation 60.13 dB, filter length 140, sampling rate 45 kHz. In order to compare with the presented method, the same specification was set for $A=-60.13$ dB, $N=140$, $\omega_s=0.222\pi$, $\omega_p =0.267\pi$ (after conversion to normalized radian frequency). Additionally, the following specific parameters for algorithm (4) were assumed: $\tau =34$ (in samples), $\gamma=1.0055$. The result of optimization is presented in Fig. 4 which shows magnitude frequency response (Fig. 4a) and impulse response (Fig. 4b). The performance of the filter presented in Fig 4 and the similar one with $A=-80$ dB ($N=155$) obtained by the method proposed and those of [3] are summarized in Table II.

TABLE II
PERFORMANCE COMPARISON OF THE FILTERS IN EXAMPLE 1, (MPR – MAXIMUM PASSBAND RIPPLE)

Method	A [dB]	N	MPR[dB]	τ [ms]	τ deviation [ms]
Proposed	−60	140	0.055	0.756	0.051 0.011[a]
[3]	−60	140	0.058	1.087	0.033
Proposed	−80	155	0.094	0.933	0.1 0.013[a]
[3]	−80	155	0.100	1.400	0.060

[a] in the range from 0 to 4.91 kHz

In both cases, the method (4) provides a significant reduction in group delay while maintaining the remaining parameters of filters. It is worth noting that the new method allows further reduction of the group delay, but this will increase the error δ in the passband (Fig. 5). It is interesting that for the Filter #2 from

[3] it was not possible to reduce the group delay, because despite assuming a smaller τ, a filter with the same delay as in [3] was obtained. The explanation of this fact is very simple: the Filter #2 is minimum-phase so no further reduction is possible.
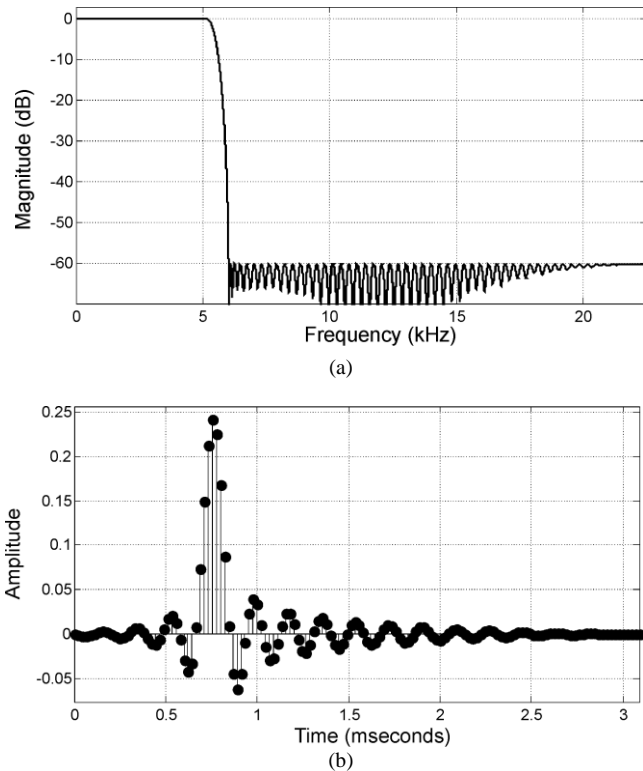

(a)

(b)

Fig. 4 Magnitude response (a) and impulse response (b) of the filter in Example 1
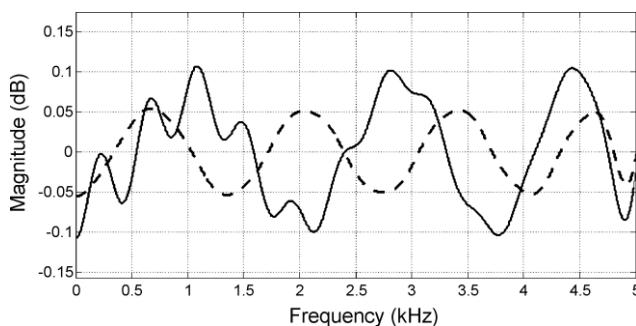

Fig. 5 Magnitude responses in passband for filter in Example 1 with A=60 dB, τ=0.756 ms (dashed line) and τ=0.644 ms (solid line).

B. Example 2

Design a bandpass FIR filter with the given parameters: lower passband edge 0.3π, upper passband edge 0.5π, lower stopband edge 0.2π, upper stopband edge 0.6π (all frequencies in rad/sample), minimum attenuation in the stopband 50 dB, constant group delay in the passband τ=10 samples, N=71, γ=1.01. The magnitude response and impulse response of the designed filter are shown in Fig. 6. As can be seen, the design requirements for the considered filter have been fulfilled. The maximum error of group delay equals 2 samples, but in the frequency range from 0.31π up to 0.49π it is smaller than 0.5 sample, and increases only at the ends of the passband. Figure 6a shows also the effect of magnitude overshoot elimination.
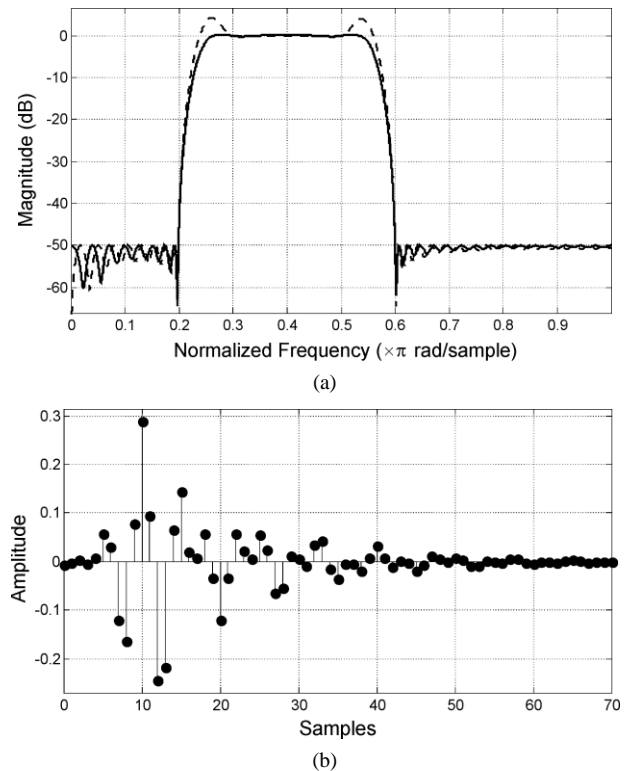

(a)

(b)

Fig. 6 Magnitude response (a) and impulse response (b) of the filter in Example 2. Dashed line shows magnitude response without overshoot elimination constraint.

Application of the second stage of the proposed algorithm (according to Table I with ε=0.035) allows to replace with zero 16 of 71 coefficients with a slight increase in approximation error in the passband and almost unchanged deviation of the group delay (Fig. 7).
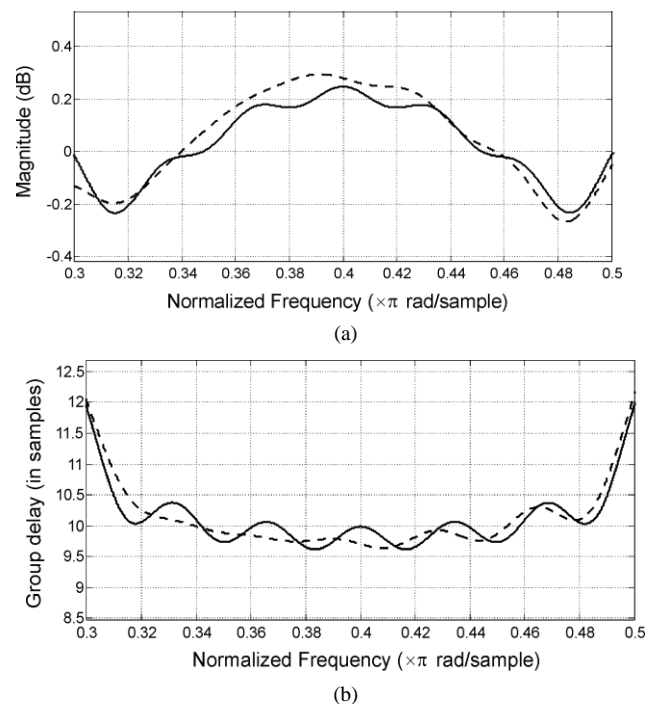

(a)

(b)

Fig. 7 Frequency responses of the filter in Example 2, (a) magnitude response in passband and (b) group delay response obtained after first stage (solid line) and second stage (dashed line) of proposed algorithm

A minimum-phase filter with the same magnitude frequency response has also been designed using the method from [12] to compare with the filter obtained using the proposed algorithm. The minimum-phase filter has a slightly smaller delay of impulse response (8 samples versus 10 obtained by proposed algorithm) and a smaller filter length (50), but the group delay deviation is 5 samples (versus 2) thus the phase response shows a visible non-linearity (Fig. 8).
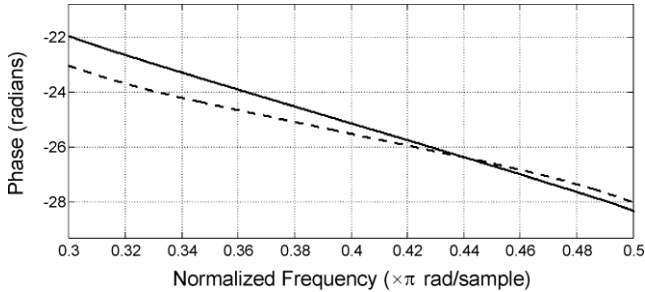

Fig. 8 Phase frequency response in passband for filter design in Example 2 (solid line) and minimum-phase filter (dashed line)

C. Example 3

The third example comes from [21], where a lowpass filter was designed with reduced, constant in the passband, group delay of 20 samples and other specifications as follows: filter order 80, passband edge 0.5π, stopband edge 0.55π. The filter presented in [26] was obtained as a result of least-squares optimization and minimum attenuation in stopband of about 35 dB was reached. In the example under consideration, the more stringent requirements for group delay and attenuation were assumed, i.e. $\tau=15$ samples and $A=-40$ dB. Figure 9 shows the frequency responses of designed sparse filter by the proposed algorithm. It has 13 zero coefficients as opposed to 10 of filter from [21]. Moreover, the approximation error in the passband is smaller and equals 0.0334 versus over 0.04 in [21].

D. Example 4

The fourth example is taken from [4] where a lowpass filter was designed with low group delay $\tau=6$ (in samples), passband edge frequency 0.4π, stopband edge frequency 0.6π, filter order $N=24$, stopband attenuation 50 dB, passband ripple 0.5 dB. The transition band is so wide that when designing a filter with these specifications the constraint preventing overshoot of the magnitude response is not necessary. The example shows that the application of the presented method allows to obtain a filter with low group delay for wide transition band, and parameters comparable to filters designed with other methods. The magnitude frequency response of the designed filter is shown in Fig. 10 (dashed line). As can be seen, the considered filter fulfils the imposed design requirements. The solid line in Fig.10 demonstrates magnitude response for sparse filter with 5 coefficients equal to zero. In this case, 20% reduction of the multiplication operation during the filter implementation can be obtained. Figure 11 presents group delay response of the designed filter in two cases. For sparse solution (solid line) approximation error of group delay is smaller than 0.4 sample (in the frequency range from 0 up to 0.375π). This error is slightly greater than for non-sparse filter.
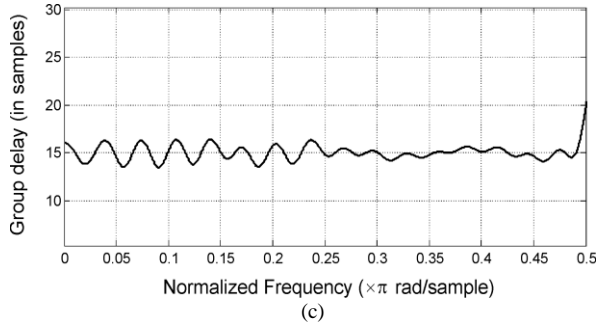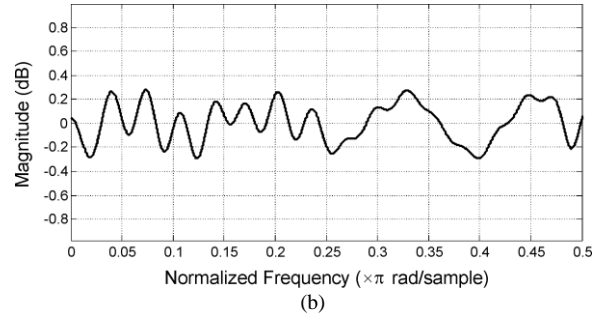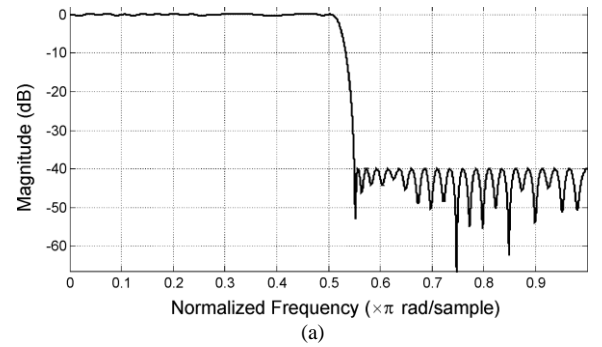

Fig. 9. Frequency responses of the filter in Example3, (a) magnitude response, (b) magnitude response in passband and (c) group delay response
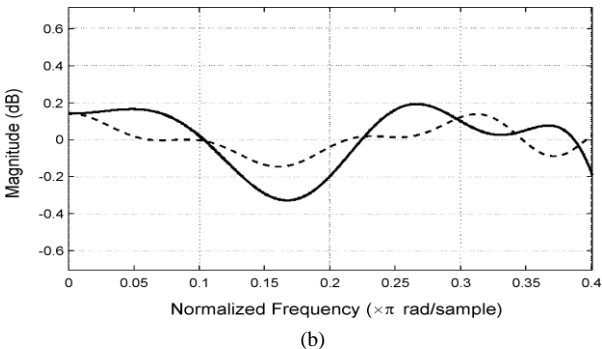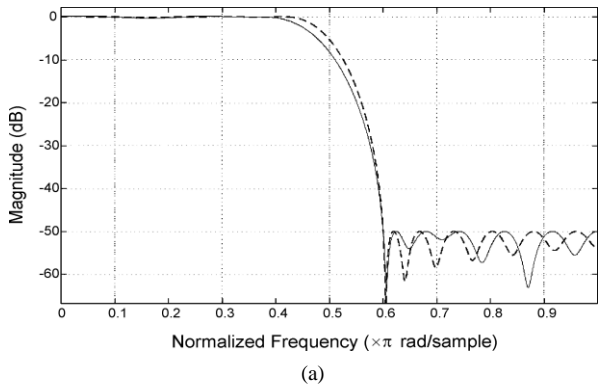

Fig. 10. Frequency responses of the filter in Example 4, (a) magnitude responses, (b) magnitude responses in passband
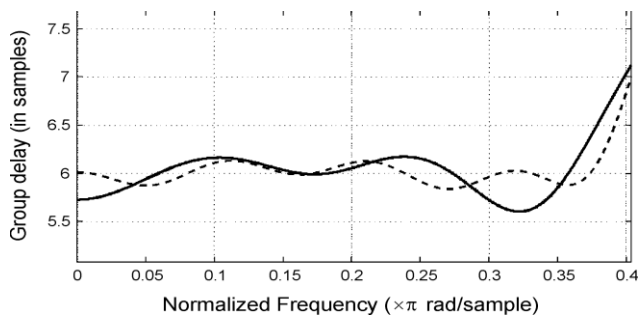
Fig. 11. Group delay responses of the filter in Example 4 for sparse (solid line) and non-sparse (dashed line) solution

CONCLUSION

In the paper, the sparse FIR filters with very low group delay and approximately linear-phase in the passband has been proposed. It has been shown that significant reduction of the group delay can lead to unwanted overshoot of the magnitude frequency response in the transition band. A method of eliminating this overshoot was proposed, thanks to which it was possible to further reduce the group delay while maintaining approximately linear-phase property. The combination of this method with the successive thinning algorithm has resulted in a sparse filter. The filters designed with the proposed method are shown to have better performances than that obtained by the method in [3,21]. Research has demonstrated that the constraint for overshoot elimination can increase the approximation error of magnitude frequency response in passband.

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