BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 68, No. 4, 2020
DOI: 10.24425/bpasts.2020.134177

Energy-based spectrum sensing with copulas for cognitive radios

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Abstract. In this study, an energy-based spectrum sensing method combined with copula theory is proposed for cognitive radio systems. In the proposed spectrum sensing model, cognitive radio users first make their own local spectrum decision with energy-based spectrum sensing. Then, they forward their decision to the fusion center. In the fusion center, this decision is compared with the threshold value determined by copula theory and global spectrum decision is made. The test statistic at the fusion center were obtained with the Neyman Pearson approach. Thus, the fusion rule was created for the fusion center and necessary simulation studies were performed. According to the results of the simulation studies, the proposed detection method showed better results than the traditional energy based detection method.

Key words: energy-based sensing, cognitive radio, copula theory, Neyman Pearson, spectrum management.

1. Introduction

There are two main reasons for the spectrum scarcity problem in recent years. One of them is the increase in the data sizes transmitted in wireless communication systems and consequently the increase of the bandwidth required for communication [1]. The other is the increase of the number of applications in wireless communication systems [2]. Thereby, measurements show that the radio frequency spectrum is idle for most of the time [3]. This is due to fixed spectrum assignment policies [4]. According to fixed spectrum assignment methods, the spectrum region assigned to a particular user is not made available to another user, even if that user is not in the spectrum. In order to solve this problem, firstly, instead of fixed spectrum allocation, dynamic spectrum assignment policies should be adopted which is exactly the main purpose of cognitive radio (CR) systems [5]. A CR system continuously scans the spectrum environment in which it is located, opening the received empty spectrum regions to secondary users. Secondary user means an opportunistic user who is not licensed for a particular spectrum region. Thus, idle spectrum gaps are evaluated. In CR systems, spectrum detection is the initial step. In the literature, different methods for spectrum detection have been proposed. The best known of these methods is Energy Detection (ED), due to the calculation cost [6]. In addition, eigenvalue-based detection and cyclostationary based detection are used in the literature [7]. The choice of the method to be used is made according to the technical characteristics of the signal to be detected (modulation, OFDM, etc.). For example, if the signal to be detected is fully known, cyclostationary detection is the most successful detection method [8]. In addition to these methods, different methods have been proposed to improve the detection performance at high noise levels and under noise uncertainty factor. Cooperative detection [9–10], fuzz tests [11], some clustering algorithms [12] have been used in the literature for spectrum detection.

In this study, a copula theory combined with energy based detection is proposed for spectrum detection in CR systems. In the proposed detection model, CR users transmit their local decisions to the fusion center, and the fusion center makes global decisions. Therefore, the fusion rule at the fusion center was obtained and the necessary simulation studies and theoretical expressions were confirmed.

The rest of the paper is organized as follows. In Section 2, proposed spectrum sensing model and Copula theory are provided. The sensing algorithms and their theorical analysis are presented in Section 3. Simulation results based on randomly generated signals are given in Section 4. Conclusions are drawn in Section 5.

In this study, bold letters $(\mathbf{x})$ and italics (x) represent matrices and vectors, respectively.

2. Spectrum sensing model

The main purpose of spectrum sensing is to identify the empty spectrum bands and allocate them to secondary users. The secondary user represents the radio user using an empty spectrum band as an opportunist. The Primary Base Station (PBS) user is the radio user who has the legal right to use a specific spectrum band. According to the detection model to be used in this study (Fig. 1), first of all, each CR user creates their own local decision to detect the spectrum. Local decisions are sent to the fusion center, where a global spectrum decision is made. Then the Cognitive Radio Base Station (CRBS) communicates with CR users. PBS is passive at this time.

According to the detection and radar theory, the detection of a communication signal embedded in noise is expressed by binary hypothesis testing. Binary hypothesis testing for detection theory and radar systems is given below [13].

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Manuscript submitted 2020-01-06, revised 2020-04-25, initially accepted for publication 2020-05-30, published in August 2020

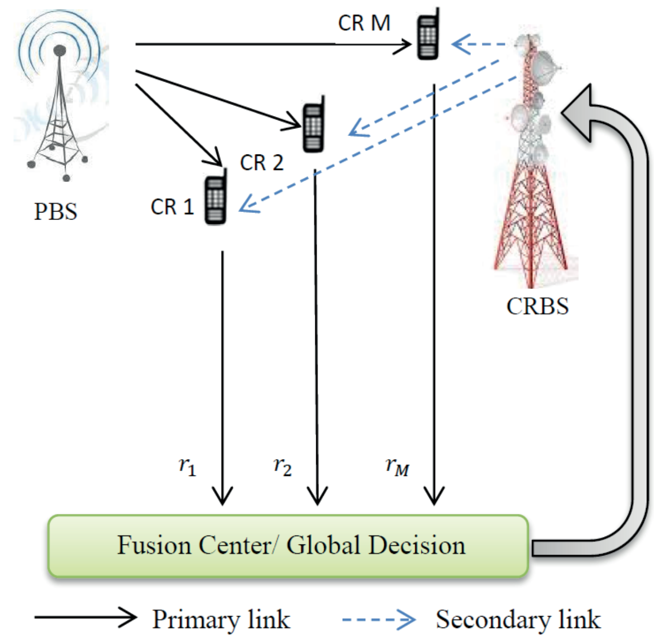

Fig. 1. Proposed detection scenario

$$H_0 \rightarrow y(n) = \eta(n) \tag{1}$$

$$H_1 \rightarrow y(n) = s(n) + \eta(n), \tag{2}$$

where $y(n)$ are samples of transmitted signals that include the effects of wireless communication channels such as multipath fading and path loss, $s(n)$ is the primary user signal, $\eta(n)$ is the White Gaussian noise, which is i.i.d., having mean zero and variance σ_η^2, namely $\eta(n) \sim CN(0, \sigma_\eta^2)$. In this study, we assume that the detection model is given in Fig. 1. Let us assume that the users of CR users have their own local spectrum decision. Assuming the CR users observations to be continuous variables, let the Probability Density Function (PDF) of the observations received by the i.th CR users at the n.th time instant under the H_1 and H_0 hypotheses be $f(y_{in} | H_1)$ and $f(y_{in} | H_0)$, respectively, where $i = 1, \ldots, M$ and $n = 1, \ldots, N$. However, no information is available about the common probability distribution functions of the CR observations. Thus, according to signal samples from CR users, the local decision can be expressed mathematically as follows.

$$r_i = \begin{cases} 1 & if \qquad TS_i < \zeta_i \\ 0 & if \qquad TS_i > \zeta_i \end{cases}, \tag{3}$$

where TS_i and ζ_i are the test statistic and the threshold at the i.th CR user respectively, r_i is i.th user's local spectrum decision. According to the proposed detection model, each CR user transmits their local decision to the global decision center. These decisions are combined in the global decision-making center. The problem of spectrum sensing consists of determining optimal fusion test statistic to achieve global decision using local thresholds at CR users. Individual and global decision thresholds in CR users are derived according to Neyman Pearson (NP) detection theorem. When determining global and individual threshold values, we assume that CR user observations (and therefore CR users decisions) are conditionally independent and identically distributed (IID).

2.1. Copula theory and spectrum sensing. To find the optimal test statistic in the fusion center, for simplicity, let us assume that there are two CR users in the detection model (i = 1, 2 and M = 2) [14]. Let $\mathbf{r} = [r_1, r_2]^T$, where r_1 and r_2 show the local decision of each CR user. So r_1 and r_2 as vector $r_1 = [r_{11}, \ldots, r_{1N}]$ and $r_2 = [r_{21}, \ldots, r_{2N}]$. Then the optimal test statistic in the fusion center is defined by the Likelihood Ratio (LR) function as follows [15].

$$\delta(\boldsymbol{r}) = \frac{P(r_1, r_2 | H_1)}{P(r_1, r_2 | H_0)}, \tag{4}$$

where $P(r_1, r_2 | H_p)$ is the joint probability distribution function of the CR users under the p.th hypothesis, $p = 0, 1$. Using the assumption of temporal independence of BR users decisions, optimal fusion statistics are expressed as follows.

$$\delta(r) = \frac{\prod_{n=1}^{N} P(r_{1n}, r_{2n} | H_1)}{\prod_{n=1}^{N} P(r_{1n}, r_{2n} | H_0)}. \tag{5}$$

We assume that $Q_{jk} = \Pr(r_{1n} = j, r_{2n} = k | H_1)$ and $W_{jk} = \Pr(r_{1n} = j, r_{2n} = k | H_0)$ for all $j = 0, 1, k = 0, 1$. Thus, for the spectrum detection model with two CR users, the set of probabilities in the fusion center is defined as follows.

$$Q_{00} = \int_{-\infty}^{\zeta_1} \int_{-\infty}^{\zeta_2} f(r_{1n}, r_{2n} | H_1)\, dr_{1n} dr_{2n} \tag{6}$$

$$W_{00} = \int_{-\infty}^{\zeta_1} \int_{-\infty}^{\zeta_2} f(r_{1n}, r_{2n} | H_0)\, dr_{1n} dr_{2n} \tag{7}$$

Equations 6 and 7 are the logic values of the decisions made by CR users. Therefore, the Q and W functions define combined joint PDF's under the H_1 and H_0 hypotheses. But as can be seen, for the calculation, the joint PDF of the r_1 and r_2 CR users' local decisions under the hypotheses H_1 and H_0 are required. Since this information is not known a priori, the marginal PDFs of CR observations and the marginal PDFs of local users of CR users should be obtained before an optimal fusion rule can be obtained. Still, we want to emphasize that multivariate probability density functions are not for the marginal density functions of two variables. Namely, given the random marginal distributions, common distribution functions cannot be simply written. For this we need to use the Sklar's theorem.

2.2. Sklar's theorem. In statistics, the relationship between multivariate probability distribution functions and their marginal probability distribution functions is explained by Sklar's theorem [16]. Let an p-dimensional distribution function K with marginal distribution functions $K_1, \ldots, K_p$. Then there exists a copula C, for all $x_1, \ldots, x_m$. Namely

$$F(x_1, x_2, \ldots, x_N) = C\big(F_1(x_1), F_2(x_2), \ldots, F_N(x_N)\big), \tag{8}$$

where C is a standard p- dimensional copula. For absolutely continuous distributions F and $F_1, \ldots, F_p$ the joint PDF of random variables $x_1, x_2, \ldots, x_p$ can be derivated by differentiating both sides (1);

$$\begin{aligned} &f(x_1, x_2, \ldots, x_p) = \\ &= \prod_{n=1}^{N} f_n(x_n)\, c\big(F_1(x_1), F_2(x_2), \ldots, F_N(x_N)\big), \end{aligned} \tag{9}$$

where $f_n(x_n)$, are the marginal PDFs and c is denoted as the density of standard multivariate copula C that is given as follows,

$$\frac{\partial^L(C(u_1, \ldots, u_d))}{\partial u_1, \ldots, \partial u_d}, \tag{10}$$

where $u_1 = F_1(x_m)$ and $u = [u_1, \ldots, u_d]$.

The selection of copula functions is a key problem in the common statistics of random variables, since different copula functions can model different types. Therefore, families of multivariable standard copula functions with different properties are defined.

3. Proposed spectrum sensing method

According to the proposed detection model, firstly CR users form their own local decisions. In the proposed model, the local spectrum decision is the energy-based spectrum sensing method and mathematically expressed as follows.

$$\sum_{n=1}^{N} |y_i(n)|^2 \underset{H_1}{\overset{H_0}{\lessgtr}} F_{\chi_2^2}^{-1}(1 - P_{fa})\frac{\sigma_\eta^2}{2}, \tag{11}$$

where $F_{\chi_2^2}^{-1}$ and P_{fa} represent the distribution of Chi Square with 2 degrees of freedom and the limit value of the Probability of False Alarm (P_{fa}) determined by the Federal Communication Committee (FCC), respectively. Thus, local decisions (1 or 0) generated by CR users are transmitted to the global decision center. In addition $\sum_{n=1}^{N} |y_i(n)|^2$ and $F_{\chi_2^2}^{-1}(1 - P_{fa})\frac{\sigma_\eta^2}{2}$ represent the test statistic and threshold value at the i.th CR user respectively. So information received from CR users can be expressed as follows.

$$r_{1n} = \left\{ \begin{array}{ll} 0 & if \quad \sum_{n=1}^{N} |y_i(n)|^2 < F_{\chi_2^2}^{-1}(1 - P_{fa})\dfrac{\sigma_\eta^2}{2} \\ 1 & if \quad \sum_{n=1}^{N} |y_i(n)|^2 > F_{\chi_2^2}^{-1}(1 - P_{fa})\dfrac{\sigma_\eta^2}{2} \end{array} \right\} \tag{12}$$

Thus, the information received from each CR user (0 or 1) is sent to the fusion center. To find the optimal test statistics, assume that there are two CR users in the detection model. Thus, the combined probability distribution functions of CR users' local decisions are expressed as follows.

$$\begin{aligned} P\big(r_{1n}, r_{2n} \,|\, H_1\big) = {} & W_{00}^{(1-\mathbf{r}_{1n})(1-\mathbf{r}_{2n})} \\ & W_{10}^{\mathbf{r}_{1n}(1-\mathbf{r}_{2n})} W_{01}^{(1-\mathbf{r}_{1n})\mathbf{r}_{2n}} W_{00}^{\mathbf{r}_{1n}\mathbf{r}_{2n}} \end{aligned} \tag{13}$$

$$\begin{aligned} P\big(r_{1n}, r_{2n} \,|\, H_0\big) = {} & Q_{00}^{(1-\mathbf{r}_{1n})(1-\mathbf{r}_{2n})} \\ H_0 \quad & Q_{10}^{\mathbf{r}_{1n}(1-\mathbf{r}_{2n})} Q_{01}^{(1-\mathbf{r}_{1n})\mathbf{r}_{2n}} Q_{00}^{\mathbf{r}_{1n}\mathbf{r}_{2n}} \end{aligned} \tag{14}$$

Substituting (13) and (14) in (5), taking the log on both sides, the optimum test statistic at the fusion center is obtained.

$$\begin{aligned} log\,\zeta_g(r) = {} & C_1 \sum_{n=1}^{N} r_{1n} + C_2 \sum_{n=1}^{N} r_{2n} + \\ & + C_3 \sum_{n=1}^{N} r_{1n} r_{2n}, \end{aligned} \tag{15}$$

where, $\zeta_g(r)$ represent global threshold or fusion rule. In addition [14],

$$C_1 = log\,\frac{W_{00} P_{10}}{W_{10} P_{00}} \tag{16}$$

$$C_2 = log\,\frac{W_{00} P_{01}}{W_{01} P_{00}} \tag{17}$$

$$C_3 = log\,\frac{P_{00} P_{11} W_{10} W_{01}}{P_{01} P_{10} W_{11} P_{00}}. \tag{18}$$

The joint probability density of CR users observations with the parameters of copula can be created using Gauss (Normal) or Student-t copula functions. Common possibilities of sensor decisions of CR users can be obtained by solving the double integrals of the intensity of the signals received by the corresponding CRs. Thus optimal fusion rule is now given by

$$log\,\zeta_g(r) \underset{H_1}{\overset{H_0}{\lessgtr}} \psi, \tag{19}$$

where ψ is the global threshold at the fusion center. Performance measurements for detection in radar terminology are determined by Probability of Detection (P_d) and probability of False Alarm (P_{fa}). The optimal fusion test statistic denoted by $log\,\zeta_g(r)$ is asymptotically Gaussian. Therefore P_d and P_{fa} are identified by the following conditional possibilities.

$$P_d = P(H_1 \,|\, H_1) = Q\left(\frac{\psi - \mu_1}{\sigma_1}\right) \tag{20}$$

$$P_{fa} = P(H_1 \,|\, H_0) = Q\left(\frac{\psi - \mu_0}{\sigma_0}\right), \tag{21}$$

where μ_x and σ_x, the first- and second-order statistics are denoted by x.th hypothesis $x = 0, 1$.

Suppose that the number of CR users in the given detection model is M, global decision rule is defined as follows.

$$log\,\zeta_g(r) = \aleph^T \tau(r) = \sum_{n=1}^{N} \sum_{k=1}^{2^M-1} \aleph_k \psi_k, \tag{22}$$

where $\aleph^T$ represents the weight vector and see Appendix 1 for $\tau(r)$. The combined intensity of CR users observations with the knowledge of the copula parameters can be generated using Gaussian or Student-t copula. The common probabilities of sensor decisions that determine optimal test statistics can be obtained by solving the double integrals of the combined intensity of CR observations.

4. Simulation studies

In this section, we illustrate our proposed energy-based spectrum sensing with Copulas through numerical examples. According to the detection model, we produce random PBS signal by adding noise at different SNR values. We assume that the communication channel is Rayleigh channel. In addition, the OFDM technique is used and OFDM parameters are given in Appendix 2. Two different graph types are used for performance evaluation in simulation studies. One of them is the ROC curves and the other is SNR-P_d graphs. Also, copula function is used for normal and Chair-Varshney copula functions.

In Fig. 2, traditional energy-based spectrum sensing and proposed sensing model (energy-based with copula) performances are compared. Although the two methods show close detection performance, the proposed method is significantly more successful. As it is known, energy-based detection, which is one of the most common spectrum detection methods, is the most successful detection method when the noise variance is fully known. If the variance of noise is unknown, some estimation methods can be found, but detection errors can adversely affect the performance of the method. Therefore, the most negative aspect of the energy detection method is its vulnerability to noise uncertainty. However, the most reliable method in terms of calculation costs is ED. In addition, in Fig. 2, the number of samples is 100. Increasing the number of samples can provide a more successful detection performance, but with the increase in the number of samples, the detection time increases.

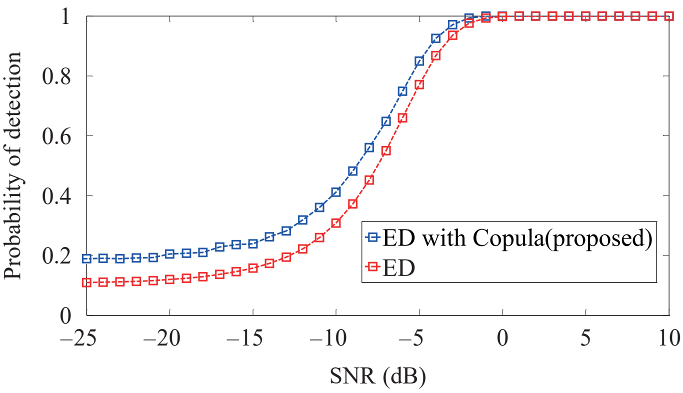

Fig. 2. ED and Prosed method sensing performances(normal copula), N = 100, M = 4

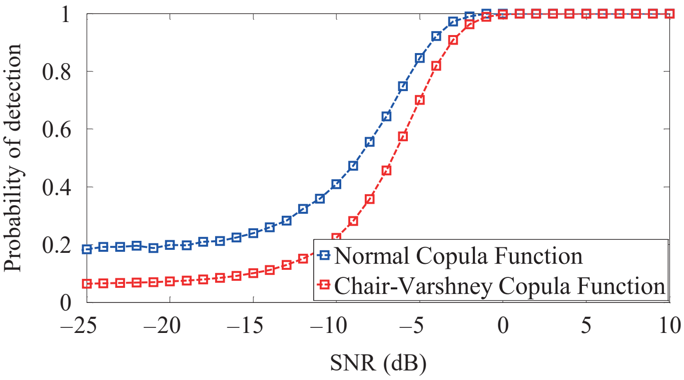

Fig. 3. ED and Prosed method sensing performances, N = 100, M = 4

If the CR users in the detection model are evaluated with the assumption that they are conditionally independent from each other, the simulation results observed (Chair-Varshney) are given in Fig. 3. As it is known, when the Chair-Varshney rule is applied, the processing cost decreases but this affects the detection results negatively. The chair rule shows that users of BR should be too far to be correlated with each other. but in practice this is not easy for CR systems. Also, as it is known, in order to make a spectrum decision in Energy based spectrum sensing, noise variance must be known in advance. In practice, noise variance is estimated by estimation methods. In this study, no estimation method was used, noise variance was used theoretically. Looking at Eq. 12, it is seen that noise variance is necessary to find the threshold value. It should be known that in ED based detectors, estimation errors may occur in estimating noise variance. This phenomenon is called noise uncertainty and leads to performance loss in ED based methods. However, this negative factor was not taken into consideration in this study.

But it is also known that the cost of calculation decreases with the chair rule. It should be kept in mind that when the number of CR users is 3 or more, the Chair Varshney rule will reduce the calculation cost and detection times.

Refer to Fig. 4 for the variation of the number of CR users and detection performances, wherein the y-axis shows the correct detection sequence when the proposed method is operated 100 times. For example, assuming M = 4, approximately 88 times of accurate detection were performed (with the Normal Copula function). Thus, referring to Fig. 4, the increase in the number of CR users positively improves the detection performance. The increase in the number of CR users in distributed detection systems improves the detection performance, but in practice it is difficult to increase the number of CR users.

ROC curves provide important information for detection performance in radar systems [17, 18]. In radar systems, the maximum value for the P_{fa} value has already been determined by certain organizations (for example Federal Communication Committee for CR systems). However, detection performance is

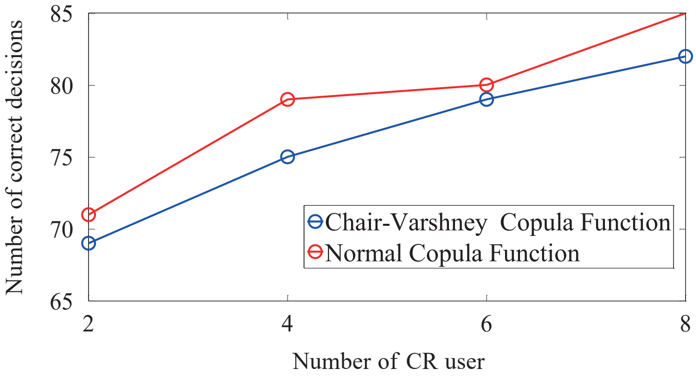

Fig. 4. CR users versus number of correct decision

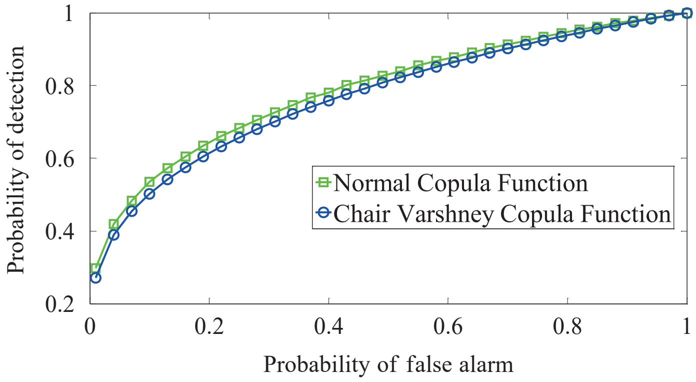

Fig. 5. ROC curves for proposed detection schemes

still important when the limit values are exceeded. ROC curves for the proposed detection method are given in Fig. 5.

One of the detector performance indicators in detection theory is the ROC curves. In ROC curves, P_d changes against P_{fa} are shown together. Normally, P_{fa} is predetermined by FCC, but ROC curves are used when detector performance is important when exceeding certain limits.

ROC curves for the proposed detection method are given in Fig. 5. As expected, a more successful detection performance was obtained with normal copula function. Figure 5 shows the detection performance of the proposed detector in the presence of 5 dB noise.

Table 1 gives the processing times of algorithms for traditional ED and the proposed method.

Table 1
Algorithm calculation times for different N, $M = 4$

	N = 100	N = 500	N = 5000
Classic ED	1.2 sec	1.4 sec	2.7sec
ED with Copulas (Normal Copula function)	1.8 sec	2.7 sec	5.1 sec

As can be seen from Table 1, the proposed method increases the detection time. However, when the proposed spectrum sensing model (Fig. 1) is analyzed, it is seen that this is an expected phenomenon. Because in traditional ED based detection, the decision is made only by CR users. However, in the proposed detection model, local decisions made by each CR user are forwarded to the fusion center for the global decision. The global spectrum decision is given here. Thus, the processing time is extended.

5. Conclusion

In this paper, we studied the problem of spectrum sensing for CR systems. We propose an energy-based detection model with Copulas for cognitive radios. In this study, spectrum sensing model, local decision and global decision rules are proposed. The theoretical results were confirmed by simulation studies. As a result of the simulation studies, the proposed sensing model showed remarkable success compared to traditional energy-based detection.

Appendix 1

Cyclic prefix length (us)	157
Effective symbol duration (ms)	2.51
Sub carrier bandwidth (Hz)	398.4375
Number of sub carriers	244

Appendix 2

$$\begin{aligned} &\tau_1, && k = 1 \\ &\tau_M, && k = M \\ &\tau_1\tau_2, && k = L+1 \\ &\tau_1\tau_3, && k = L+2 \\ &\tau_1\tau_2\tau_3, && k = L+1+\frac{M(M-1)}{2} \end{aligned}$$

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