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Location of generating units most affecting the angular stability of the power system based on the analysis of instantaneous power waveforms

PIOTR PRUSKI and STEFAN PASZEK

In the paper, the results of investigations on the location of generating units most affecting the angular stability of a large power system (PS) are presented. For their location, the eigenvalues of the PS model state matrix associated with electromechanical phenomena (electromechanical eigenvalues) were used. The eigenvalues were calculated on the basis of the analysis of the disturbance waveforms of instantaneous power of the generating units operating in the PS. The used method of calculating eigenvalues consists in approximation of the disturbance waveforms of generating units by the waveforms being the superposition of modal components. The parameters of these components depend on the sought eigenvalues and their participation factors. The objective function was defined as the mean square error between the approximated and approximating waveforms. To minimize it, a hybrid algorithm, being a combination of genetic and gradient algorithms, was used. In the instantaneous power waveforms of generating units most affecting the PS angular stability, the least damped or undamped modal components dominate. They are related to eigenvalues with the largest values of real parts. The impact of individual modal components on the disturbance waveforms of subsequent generating units was determined with the use of participation factors and correlation coefficients of electromechanical eigenvalues.

Key words: power system, modal analysis, electromechanical eigenvalues, transient states, angular stability

1. Introduction

The power system (PS) is a large nonlinear dynamic system for the production, transmission and distribution of electrical energy. Its stability is associated with transient states and regulation processes occurring in PS [6, 23]. Generally, the

P. Pruski (corresponding author – E-mail: piotr.pruski@polsl.pl) and S. Paszek (E-mail: stefan.paszek@polsl.pl) are with Faculty of Electrical Engineering, Silesian University of Technology, Akademicka 10, 44-100 Gliwice, Poland.

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PS stability is understood as its ability to maintain a specific operating state after a disturbance. Taking into account electrical quantities important for the PS state, one can distinguish angular stability, frequency stability and voltage stability [2, 10]. The angular stability is associated with maintaining the synchronism of all synchronous generators operating in PS generating units. The loss of synchronism of synchronous generators is identified with the loss of PS angular stability.

The angular stability is directly related to electromechanical phenomena. Disturbances occurring during the operation of PS cause the appearance of slow, oscillatory changes in the angular speed of synchronous generators, i.e. electromechanical swings. These swings also appear in the waveforms of power angles and instantaneous power of synchronous generators in various places of the system, including generating units. In some generating units, electromechanical swings, among others the waveforms of instantaneous power of generators, can be weakly damped, and they can even increase, which can lead to a loss of the PS angular stability, and thus to a lack of power supply to loads over a large area.

Due to the above, it is necessary to locate the generating units that are most at risk. They can be called critical generating units from the point of view of a possible loss of the PS angular stability. In critical units, particular care must be taken to ensure that various means to improve angular stability, including well-made optimization or polyopimization of the parameters of power system stabilizers operating in voltage control systems of synchronous generators, are properly used. Other means which may be undertaken include: stabilizers operating in turbine governors, appropriate FACTS devices located close to the critical units, use of modern anti-swing automation [1, 5, 8, 10, 20].

This paper presents the method of determining the location of these critical generating units based on the analysis of transient waveforms of instantaneous power of synchronous generators. Instantaneous power waveforms can easily be determined based on the current and voltage waveforms of the synchronous generator stator expressed in phase coordinates or axial (in the longitudinal axis d and transverse axis q) components. To link the equations in these two coordinate systems, Park transformation is used [10, 14].

Investigations leading to the location of PS generating units most affecting the angular stability were carried out when assuming the occurrence of small steady state disturbances for which the linearized form of the system state equations applies.

2. The linearized PS model

The PS mathematical model is obtained by combining the mathematical models of generating units with the mathematical model of the power network. The mathematical model of the generating unit includes models of the following components: a generator (most often a synchronous generator, together with its excitation system and power system stabilizer) and a turbine with a governor.

When investigating electromechanical phenomena occurring in PS, it is usually assumed that the dynamic phenomena are associated with generating units, and the power network is described by algebraic equations, linking currents and voltages of generating units.

Assuming that the power system is a large system with lumped constants, its state equations and output equations can be given by the general relationships [10, 13]:

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t), \tag{1a}$$

$$\boldsymbol{y} = \boldsymbol{g}\,(\boldsymbol{x}, \boldsymbol{u}, t)\,, \tag{1b}$$

where: $\boldsymbol{x}$, $\boldsymbol{u}$, $\boldsymbol{y}$ – m_{PS}-, p_{PS}- and n_{PS}-dimensional vectors of state, input and output, respectively, $\boldsymbol{f}, \boldsymbol{g}$ – m_{PS}- and n_{PS}-dimensional functions of many variables, respectively, t – time.

In the state vector $\boldsymbol{x}$, there are fluxes, currents, angular speeds of rotors and power angles of generators, the quantities associated with the turbine models and various control systems in generating units.

Equations (1a) and (1b) must be supplemented with the inequalities describing the operation of limiters in the control systems of generating units operating in PS:

$$D_{\max j} \leqslant d_j\,(\boldsymbol{x}, \boldsymbol{u}, t) \leqslant D_{\min j}, \qquad j = 1,\ 2,\ \ldots,\ q_{\mathrm{PS}}\,, \tag{1c}$$

where: d_j – j-th element of a multidimensional function of many variables, $D_{\max j}$, $D_{\min j}$ – maximum and minimum values in the limiters for generating unit control, q_{PS} – number of limiters.

Assuming additionally that in the analyzed period of time (about a dozen or so minutes), the system does not change its properties in time, i.e. it is a stationary system, the PS model linearized at the steady state operating point can be presented in the form of the following state equations and output equations [3, 7, 13, 17–19]:

$$\Delta\dot{\boldsymbol{x}} = \boldsymbol{A}\Delta\boldsymbol{x} + \boldsymbol{B}\Delta\boldsymbol{u}, \tag{2a}$$

$$\Delta\boldsymbol{y} = \boldsymbol{C}\Delta\boldsymbol{x} + \boldsymbol{D}\Delta\boldsymbol{u}, \tag{2b}$$

where: Δ denotes the deviation from the steady values of the respective quantities, $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$ and $\boldsymbol{D}$ – state matrix and other matrices of specific dimensions and constant coefficients depending on the models and parameters of PS elements [17–19].

The linearized PS model, determined by dependencies (2), is a good approximation of reality in the case of system analysis for relatively small inputs or disturbances (e.g. for several percent changes in the value of the voltage regulator reference voltage of synchronous generators, low-current short circuits and disconnections in transmission lines).

According to the Hartman-Grobman theorem, the nonlinear PS model (1) can be replaced at a steady operating point by the linearized model (2) only if the linearized model is a hyperbolic system i.e. all eigenvalues of the state matrix have non-zero real parts [4, 11, 12, 23]. This assumption was made in the investigations presented in this paper.

The waveforms of output quantities in PS can be calculated by numerically solving equations (1) or (2). The solution can also be obtained on the basis of calculating eigenvalues and eigenvectors of the state matrix $\boldsymbol{A}$. The waveform of each output quantity is a superposition of modal components dependent on the eigenvalues and eigenvectors of matrix $\boldsymbol{A}$ [17–19].

The eigenvalues λ_h ($h = 1, 2, \ldots, m_{\mathrm{PS}}$) are the roots of the characteristic equation:

$$\det\left[\boldsymbol{A} - \lambda \boldsymbol{I}\right] = 0, \tag{3}$$

where: $\boldsymbol{I}$ – unit matrix.

The right-hand $\boldsymbol{V}_h$ and left-hand eigenvector $\boldsymbol{W}_h$ can be assigned to each eigenvalue λ_h. The state matrices of even large power systems have only distinct eigenvalues in practice. This assumption was made further in this paper. For distinct eigenvalues, the eigenvectors meet the following relationships:

$$\boldsymbol{A}\,\boldsymbol{V} = \boldsymbol{V}\,\boldsymbol{\Lambda}, \tag{4a}$$

$$\boldsymbol{W}^T\boldsymbol{A} = \boldsymbol{\Lambda}\,\boldsymbol{W}^T, \tag{4b}$$

where: $\boldsymbol{V}$, $\boldsymbol{W}$ – right-side and left-side modal matrices, the columns of which are, respectively, h-th right-side and left-side eigenvectors corresponding to the h-th eigenvalues, $\boldsymbol{\Lambda}$ – diagonal matrix [3], whose main diagonal consists of the state matrix eigenvalues.

After normalization of eigenvectors ($\boldsymbol{W}_h^T\boldsymbol{V}_h = 1$) [3], one obtains:

$$\boldsymbol{V}^{-1} = \boldsymbol{W}^T. \tag{4c}$$

Then, on the basis of (4), one can write:

$$\boldsymbol{A} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^{-1}. \tag{5}$$

Defining a modal state vector $\boldsymbol{z}$ as:

$$\boldsymbol{z} = \boldsymbol{V}^{-1}\boldsymbol{x}, \tag{6}$$

equations (2) can be written in the form [10]:

$$\frac{\mathrm{d}\Delta\boldsymbol{z}}{\mathrm{d}t} = \boldsymbol{\Lambda}\,\Delta\boldsymbol{z} + \boldsymbol{V}^{-1}\boldsymbol{B}\,\Delta\boldsymbol{u}, \tag{7a}$$

$$\Delta \boldsymbol{x} = \boldsymbol{V} \Delta z, \tag{7b}$$

$$\Delta \boldsymbol{y} = \boldsymbol{C}\,\boldsymbol{V} \Delta z, \tag{7c}$$

assuming that $\boldsymbol{D} = \boldsymbol{0}$ (which is often true for the PS analysis).

Equations (7a) are independent of each other, with each equation having only one coordinate of the state vector and its first derivative, which makes them easier to solve.

Assuming the zero initial condition for the state vector $\Delta \boldsymbol{x}$, one obtains a relationship determining the h-th coordinate of the modal vector in the form [3, 19]:

$$\Delta z = \int_{t_0}^{t} \mathrm{e}^{\boldsymbol{\Lambda}(t-\tau)} \boldsymbol{V}^{-1} \boldsymbol{B} \Delta \boldsymbol{u}(\tau) \, \mathrm{d}\tau, \tag{8}$$

where: t_0 – time instant of occurrence of the input.

Although the expressions (7) and (8) include, among others, the complex conjugate elements, the coordinates of the state vector $\Delta \boldsymbol{x}$ and output vector $\Delta \boldsymbol{y}$ are real.

3. Electromechanical eigenvalues

The real part of eigenvalue is related to the damping or lack of damping corresponding to its modal component. The modal component is damped when the real part of the eigenvalue is negative (the larger the module, the greater the damping). In the case of a positive real part, the modal component is increasing, which means that the system is unstable. The imaginary part of eigenvalue is equal to the oscillation pulsation of the associated modal component. The zero imaginary part corresponds to the aperiodic modal component [17–19].

Modal components associated with electromechanical eigenvalues dominate in the waveforms of electromechanical quantities (instantaneous power P, rotor angular speed ω, generator power angle δ) of individual generating units operating in PS. They are complex eigenvalues in conjugate pairs with imaginary parts in the range (0.63÷12.6) rad/s, which corresponds to the oscillation frequency in the range (0.1÷2) Hz. In Fig. 1, the ranges of real and imaginary parts of the electromechanical eigenvalues are marked on the plane of complex numbers. It can be estimated that a satisfactorily fast decay of electromechanical oscillations in the system is obtained if the real parts of all eigenvalues are less than −0.3 (this is a conventional, estimated value). This value corresponds to the conventional time of transition of the system to the steady state $t_{st} < 13$ s [17–19]. The modal components associated with these eigenvalues are decisive for maintaining the PS angular stability.

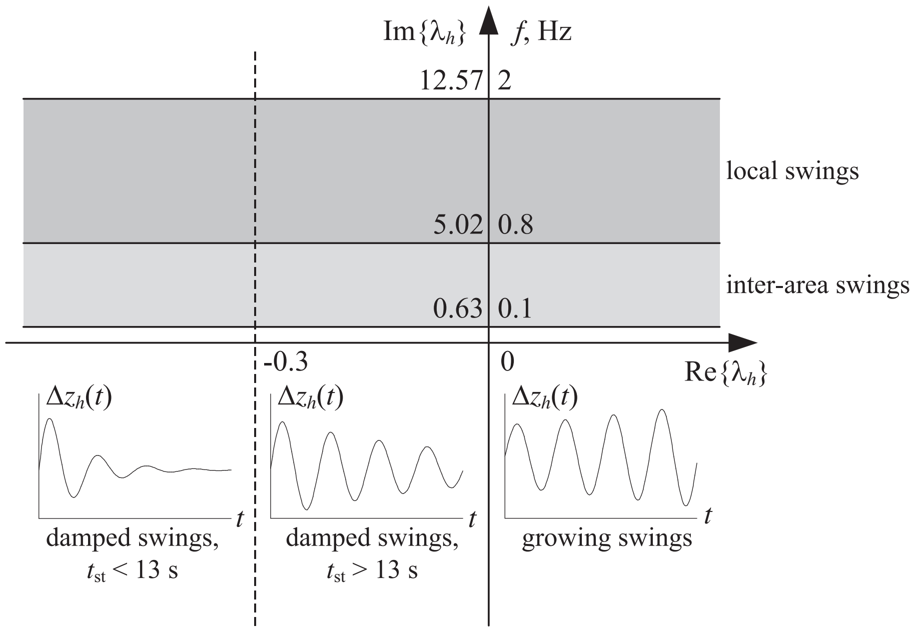

Figure 1: Influence of electromechanical eigenvalues on the swing damping in the power system

4. The classical concepts of control theory

Based on equations (2), it is possible to determine the response to the Dirac pulse and the system input-output transmittance in the form of Laplace operator [21]:

$$\boldsymbol{y}(t) = \boldsymbol{C}\,\mathrm{e}^{\boldsymbol{A}t}\boldsymbol{B}, \qquad \boldsymbol{Y}(s) = \boldsymbol{C}\,(s\boldsymbol{I}-\boldsymbol{A})^{-1}\,\boldsymbol{B}, \tag{9}$$

Expression (9) can be expanded into a series:

$$\boldsymbol{y}(t) = \sum_{k=0}^{\infty} \boldsymbol{Y}_k \frac{t^k}{k!}, \qquad \boldsymbol{Y}(s) = \sum_{k=1}^{\infty} \boldsymbol{Y}_{k-1} s^{-k}, \tag{10}$$

where

$$\boldsymbol{Y}_k = \boldsymbol{C}\boldsymbol{A}^k\boldsymbol{B} \tag{11}$$

are constant matrices, called *Markov parameters* in the literature [21].

The system of equations of the linear system (2), which gives specific answers to specific inputs, can be written using infinitely many sets of matrices $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}\}$. However, the same Markov parameters $\boldsymbol{Y}_k$, on the basis of which one can determine the system response to any input, correspond to each of these sets [21].

A system is said to be *completely state-controllable*, if any system state $\boldsymbol{x}(t_\text{f})$ can be achieved from any initial state $\boldsymbol{x}(t_0)$, in finite time, by applying the appropriate input. A system is completely controllable when for some input $\boldsymbol{u}(t)$ [13, 21]:

$$\boldsymbol{x}(t_0) = \mathrm{e}^{\boldsymbol{A}(t_0-t_\text{f})}\boldsymbol{x}(t_\text{f}) - \int\limits_{t_0}^{t_\text{f}} \mathrm{e}^{\boldsymbol{A}(t_0-\tau)}\boldsymbol{B}\,\boldsymbol{u}(\tau)\,\mathrm{d}\tau. \tag{12}$$

Since usually not all state variables are available for measurement, it can be written, according to the Ho-Kalman theorem [21], that the system is said to be *completely output–controllable* when the block matrix

$$\begin{bmatrix}\boldsymbol{CB} & \boldsymbol{CAB} & \boldsymbol{CA}^2\boldsymbol{B} & \cdots & \boldsymbol{CA}^{m-1}\boldsymbol{B}\end{bmatrix} \tag{13}$$

has the order equal to m.

A system is said to be *completely observable* if any change in the state vector $\boldsymbol{x}(t)$ causes a change in the output vector $\boldsymbol{y}(t)$. This occurs if and only if when [13, 21]

$$\begin{bmatrix}\boldsymbol{C}^T & \boldsymbol{A}^T\boldsymbol{C}^T & \cdots & \left(\boldsymbol{A}^T\right)^{m-1}\boldsymbol{C}^T\end{bmatrix} \tag{14}$$

has the order equal to m.

Therefore, based on the measurement data, a system of minimum order m that is fully controllable and observable should be created [3, 21, 22]. For this purpose, one can use the algorithm of Ho and Kalman [21] and determine the matrices:

$$\boldsymbol{A}_m = \boldsymbol{U}_m\,[\boldsymbol{J}\boldsymbol{P}(r\boldsymbol{S}_r)\boldsymbol{Q}\boldsymbol{J}]\,\boldsymbol{U}_m^T, \tag{15}$$

$$\boldsymbol{B} = \boldsymbol{U}_m\left[\boldsymbol{J}\boldsymbol{P}\boldsymbol{S}_r\boldsymbol{E}_n^T\right], \tag{16}$$

$$\boldsymbol{C} = \left[\boldsymbol{E}_p\boldsymbol{S}_r\boldsymbol{Q}\boldsymbol{J}\right]\boldsymbol{U}_m^T, \tag{17}$$

where: $\boldsymbol{I}_m$ – unit matrix of dimensions $m{\times}m$, $\boldsymbol{U}_m = \begin{bmatrix}\boldsymbol{I}_m & \boldsymbol{0}\end{bmatrix}$, $\boldsymbol{S}_r$ – Hankel matrix of dimensions $r \times r$, whose each element of the i-th row and j-th column is equal to Markov parameter $\boldsymbol{Y}_{i+j-2}$, r – degree of the least common polynomial denominator of $\boldsymbol{Y}(s)$, $\boldsymbol{E}_n = \begin{bmatrix}\boldsymbol{I}_n & \boldsymbol{0}_n & \boldsymbol{0}_n & \ldots & \boldsymbol{0}_n\end{bmatrix}$ – block matrix of dimensions $1 \times r$, $\boldsymbol{E}_p$ – matrix defined in a similar way, $\boldsymbol{P}$, $\boldsymbol{Q}$ – sought matrices satisfying the relationship:

$$\boldsymbol{P}\boldsymbol{S}_r\boldsymbol{Q} = \begin{bmatrix}\boldsymbol{I}_m & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix} = \boldsymbol{J} = \boldsymbol{U}_m^T\boldsymbol{U}_m\,. \tag{18}$$

In this paper, the method of reduction of the system model order consisting in the rejection of eigenvalues associated with the fast decaying modal components, not related to electromechanical phenomena, is used. The system obtained in this way is fully controllable and observable from the point of view of electromechanical phenomena.

The obtained reduced system should have eigenvalues and eigenvectors close to the eigenvalues and eigenvectors of the original system [3].

Another method of reducing the order of the system model is presented in [3]. This method uses the projection of the state vector $\boldsymbol{x}$ onto another smaller vector $\boldsymbol{x}_\mathrm{e}$:

$$\boldsymbol{x}_\mathrm{e} = \boldsymbol{C}_\mathrm{e}\boldsymbol{x}, \tag{19}$$

where $\boldsymbol{C}_\mathrm{e}$ is a properly selected, rectangular matrix.

5. Determination of the generating units most affecting the PS angular stability

In the case of an input in the form of a Dirac pulse of the j-th input variable $\Delta U_j(t) = \Delta U\delta(t)$, when assuming $\boldsymbol{D} = \boldsymbol{0}$ and occurrence of only distinct eigenvalues, the waveform of the i-th output variable for $t \geqslant t_0$ can be presented in the form [10]:

$$\begin{aligned}\Delta y_i(t) &= \int\limits_{t_0}^{t}\sum_{h=1}^{m_\mathrm{PS}}\boldsymbol{C}_i\boldsymbol{V}_h\mathrm{e}^{\lambda_h(t-\tau)}\boldsymbol{W}_h^T\boldsymbol{B}_j\Delta U\delta(\tau)\,\mathrm{d}\tau = \\ &= \sum_{h=1}^{m_\mathrm{PS}}\boldsymbol{C}_i\boldsymbol{V}_h\boldsymbol{W}_h^T\boldsymbol{B}_j\Delta U\int\limits_{t_0}^{t}\mathrm{e}^{\lambda_h(t-\tau)}\delta(\tau)\,\mathrm{d}\tau \\ &= \sum_{h=1}^{m_\mathrm{PS}}\boldsymbol{C}_i\boldsymbol{V}_h\boldsymbol{W}_h^T\boldsymbol{B}_j\Delta U\mathrm{e}^{\lambda_h(t-t_0)},\end{aligned} \tag{20}$$

where: $\boldsymbol{B}_j$ – j-th column of $\boldsymbol{B}$ matrix, $\boldsymbol{C}_i$ – i-th row of $\boldsymbol{C}$ matrix.

Defining the participation factor of the h-th eigenvalue in the i-th output quantity waveform as follows [17–19]:

$$F_{ih} = \boldsymbol{C}_i\boldsymbol{V}_h\boldsymbol{W}_h^T\boldsymbol{B}_j\Delta U, \tag{21}$$

the waveform of the i-th output quantity can be written in the form:

$$\Delta y_i(t) = \sum_{h=1}^{m_\mathrm{PS}} F_{ih}\,\mathrm{e}^{\lambda_h(t-t_0)}. \tag{22}$$

The system response to a short rectangular pulse with properly selected height and width is close to the response of this system to the Dirac pulse given by equation (20) [17–19].

The value of the participation factor (21) depends on the place of introducing the input into PS (the j-th column of matrix $\boldsymbol{B}$ corresponds to the j-th input variable). The larger the participation factor absolute value $|F_{ih}|$, the greater the influence of the modal component associated with the h-th eigenvalue on the waveform of the i-th output quantity.

Participation factors can also be approximately determined by analyzing the sensitivity of eigenvalues to a change in elements of the system state matrix caused by a small disturbance in PS [10]. This change can be included in the state matrix $\boldsymbol{A}$ instead of in the input vector $\Delta\boldsymbol{u}$. The sensitivity of eigenvalue λ_h to a change in any parameter q is expressed by the following equation [10]:

$$\frac{\partial \lambda_h}{\partial q} = \frac{\partial}{\partial q}\left(\boldsymbol{W}_h^T \boldsymbol{A} \boldsymbol{V}_h\right) = \boldsymbol{W}_h^T \frac{\partial \boldsymbol{A}}{\partial q} \boldsymbol{V}_h \,. \tag{23}$$

If a change in parameter q results in a change in only one element a_{kk} of the state matrix $\boldsymbol{A}$, one obtains:

$$\frac{\partial \lambda_h}{\partial q} = (W_{ih} V_{ih}) \frac{\partial a_{ii}}{\partial q} = p_{ih} \frac{\partial a_{ii}}{\partial q}, \tag{24}$$

where

$$p_{ih} = W_{ih} V_{ih} \tag{25}$$

determines the influence of the k-th state variable in the h-th modal component.

To distinguish the factor (25) from the participation factor (21), the former will be referred to as *correlation coefficient* further in the paper.

The responses to some small disturbances that appear automatically in PS (without introducing a disturbance), after some time since the occurrence of the disturbance, are similar in shape to the response to the Dirac pulse and can be calculated with satisfactory accuracy based on formula (22). In the case of such disturbances, in formula (22), instead of the participation factors there are the amplitudes of individual modal components which are approximately proportional to the correlation coefficients. This approximate proportionality between the respective participation factors and correlation coefficients also applies in the case of introducing a purposeful disturbance into PS.

The participation factors depend on the place of applying the input and the place of measuring the instantaneous power. In formula (21), there are submatrices $\boldsymbol{C}_i$ and $\boldsymbol{B}_j$. Only the elements of eigenvectors affect the correlation coefficients (25). Accordingly, these quantities are never the same and there are many more participation factors. However, if the modal component associated with the h-th eigenvalue has a significant impact on the power waveform in a specific generating unit, it is reflected in the relatively large values of the relevant participation factors and correlation coefficients, and thus these quantities are related to each other.

In the PS model, angular speeds of generating units are often state variables, and instantaneous powers of these units are output variables. These electromechanical quantities within the same generating unit are closely related. This is due to the fact that the generator angular speed is primarily influenced by the drive torque from the turbine and the load torque proportional to the instantaneous power produced by the generator.

In the case of investigating the PS angular stability and analyzing the waveforms of electromechanical quantities, the correlation coefficients p_{kh} between the angular speeds of generators operating in subsequent generating units (k-th variable in the state vector) and the individual (h-th) electromechanical eigenvalues are determined. Therefore, in formula (25), there is usually a product of the k-th components of the left-side and right-side eigenvectors of the h-th electromechanical eigenvalue.

Knowing the electromechanical eigenvalues of the PS state matrix, as well as their participation factors and/or correlation coefficients, one can determine which modal components have the greatest impact on individual waveforms of electromechanical quantities in PS generating units. It is also possible to determine the generating units in which the waveforms of electromechanical quantities containing dominant (with high amplitudes), weakly damped (or not damped) modal components associated with eigenvalues with the largest real parts (among others, larger than -0.3) occur. These are the generating units that have the greatest impact on the PS angular stability.

6. The method of calculations of eigenvalues using an optimization approach

Electromechanical eigenvalues and their participation factors can be calculated based on the analysis of deviations (from the steady value) of the waveforms of electromechanical quantities appearing in PS after small disturbances.

In the presented investigations, the calculations were carried out on the basis of the disturbance waveforms of instantaneous power of generating units appearing after the introduction of a short pulse disturbance into the voltage regulation system of one of the generators.

This is an example of conducting experimental modal analysis. It takes into account the disturbance waveforms occurring after purposeful introduction of a test disturbance into the system.

The calculation method used consists in approximation of the instantaneous power waveforms of individual units by the waveforms calculated on the basis of relationship (22). The latter waveforms are the superposition of modal components determined by the sought eigenvalues and their participation factors. Using these eigenvalues and their participation factors, one can directly define the parameters of the objective function (26). These parameters are iteratively selected to minimize the value of the objective function ε_i defined as a mean square error between the approximated and approximating waveforms [17–19]:

$$\begin{aligned}\varepsilon_i(\boldsymbol{\lambda},\boldsymbol{F}) &= \varepsilon_i(\mathrm{Re}\{\boldsymbol{\lambda}\},\ \mathrm{Im}\ \{\boldsymbol{\lambda}\},\ |\boldsymbol{F}_i|,\ \arg(\boldsymbol{F})) = \\ &= \sum_{l=1}^{N}\left(\Delta P_{i,l,\mathrm{a}} - \Delta P_{i,l,\mathrm{b}}\left(\mathrm{Re}\{\boldsymbol{\lambda}\},\ \mathrm{Im}\ \{\boldsymbol{\lambda}\},\ |\boldsymbol{F}_i|,\ \arg(\boldsymbol{F}_i)\right)\right)^2 = \\ &= \sum_{l=1}^{N}\left(\Delta P_{i,l,\mathrm{a}} - \sum_{h=1}^{m/2} 2\,|F_{ih}|\ e^{\mathrm{Re}\{\lambda_h\}(t-t_0)}\times\right. \\ &\qquad \left.\times \cos\left(\mathrm{Im}\ \{\lambda_h\}(t-t_0) + \arg(F_{ih})\right)\right)^2,\end{aligned} \tag{26}$$

where: $\boldsymbol{\lambda} = \begin{bmatrix}\lambda_1 & \cdots & \lambda_{m/2}\end{bmatrix}^T$ – vector of eigenvalues, $\boldsymbol{F}_i = \begin{bmatrix}F_{i,1} & \cdots & F_{i,m/2}\end{bmatrix}^T$ – vector of participation factors (of the i-th generating unit), ΔP_i – analyzed instantaneous power deviation waveform (of the i-th generating unit), l, N – waveform sample number and the number of waveform samples, $m/2$ – number of the pairs of electromechanical eigenvalues, the index "a" denotes the approximated waveform, the index "b" – the approximating waveform.

Only a few dominant modal components (with the highest amplitudes) are taken into account in the calculations of the approximating waveform. The vectors which are the arguments of the objective function are real. They consist of the real $Re\,\{\boldsymbol{\lambda}\}$ and imaginary $Im\,\{\boldsymbol{\lambda}\}$ parts of eigenvalues, as well as the absolute values $|\boldsymbol{F}_i|$ and arguments $\arg\,(\boldsymbol{F}_i)$ of participation factors.

The objective function ε_i has numerous local minima. Getting stuck in the local minimum results in obtaining incorrect calculation results.

The hybrid optimization algorithm (Fig. 2), which is a combination of genetic and gradient Newton algorithms was used to minimize the objective function

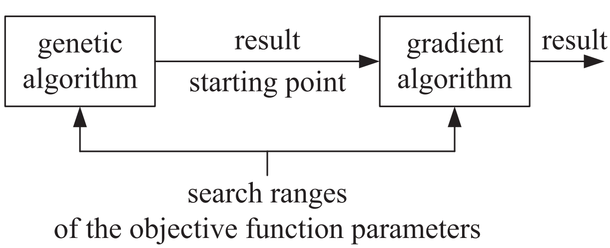

Figure 2: Functional diagram of the hybrid algorithm

[1, 9, 15, 17–19, 24]. The results of calculations of λ and $\boldsymbol{F}_i$ using the genetic algorithm are the starting point for the gradient algorithm. Serial combination of these algorithms eliminates their basic disadvantages. The genetic algorithm is a global optimization algorithm and is hardly susceptible to getting stuck in the minima of local objective functions. For the genetic algorithm, it is not required to specify the starting point, but only the search ranges for individual parameters of the function ε_i. This algorithm is used in the first stage to approximately determine the global minimum of the objective function. In the second stage, the gradient algorithm which converges faster and allows finding the global minimum of the objective function with greater accuracy is used.

To initially determine which eigenvalues significantly interfere in the instantaneous power of the analyzed unit, and to calculate the ranges of searching for the objective function parameters, the PS model developed on the basis of approximately known parameters of the generating units can be used. The PS model is always a simplified model and does not take into account all the phenomena occurring in it. It often contains only selected generating units of a relatively large impact on the PS operation.

When calculating the eigenvalues from relationship (26), to eliminate the effect of fast decaying modal components (not related to electromechanical phenomena), the instantaneous power waveforms are analyzed after some time t_p from the disturbance occurrence t_0. The time t_p is selected experimentally to achieve satisfactory calculation accuracy [17–19]. This method is similar to the method used in [3], but in this case it is not necessary to analyze the matrices in the linearized system equations (2). By experimental selection of the time t_p, it is possible to eliminate (to a satisfactory degree) the impact of fast disappearing modal components on the analyzed waveforms. After t_p, mainly electromechanical eigenvalues influence the electromechanical quantities of the system. In this way, the system order is significantly reduced, which makes the calculations of electromechanical eigenvalues easier.

To further reduce the risk of the optimization algorithm getting stuck in a local minimum of the objective function, calculations based on each instantaneous power waveform are performed repeatedly. The results with the objective function values higher than the specified limit value are rejected. The average values of the calculation results of the relevant parameters not rejected are assumed to be the final result of calculations of the individual parameters of the objective function based on the analyzed waveform [17–19].

Because the calculation accuracy is higher when the number of simultaneously optimized parameters is smaller, the calculations of eigenvalues are carried out in several stages. Initially, the eigenvalues related to the least damped modal components are calculated. Other eigenvalues do not affect the calculations because the modal components associated with them are sufficiently damped before the start of the appropriately selected time of the waveform analysis $t_0 + t_\mathrm{p}$.

In subsequent stages, the eigenvalues related to strongly damped modal components are calculated, when taking into account the knowledge of the previously calculated eigenvalues. The participation factors are calculated for all the eigenvalues and individual instantaneous power waveforms of the generating units [17–19].

7. Exemplary calculations

Exemplary calculations were carried out for the Polish Power System model. It is a nonlinear model developed in the Matlab/Simulink program environment, including 49 selected generating units operating in high and highest voltage networks and 8 equivalent generating units representing the impact of power systems of the neighboring countries. The PS model consists of 57 models of generating units, 519 load nodes and 1043 elements (transmission lines and transformers) in the transmission network model. The dimension of the state vector $\boldsymbol{x}$ for the analyzed PS is 843. The apparent powers of the generating units (the number of the generators in the unit multiplied by the power of a single generator is given in brackets) and the rated voltages of the PS nodes to which they are connected are listed in Table 1. The equivalent generating units are marked with an asterisk.

The following models of the generating unit components were taken into account in the PS model: the GENROU model of a synchronous generator [14, 16], the model of a static [14] or electromachine [14, 16] excitation system operating in the Polish Power System, the model of a steam turbine IEEEG1 [14, 16] or a water turbine HYGOV [14, 16] and, optionally, the model of a power system stabilizer PSS3B [14, 16]. The equivalent generating units were described by the simplified model of a synchronous generator GENCLS [16], neglecting influence of the excitation system, turbine, and power system stabilizer.

The eigenvalues of the state matrix of the PS model linearized at the operating point can be calculated directly in the Matlab/Simulink environment. The eigenvalues calculated in this way are called *original eigenvalues* in the paper.

In the calculations presented in this paper, the instantaneous power waveforms obtained from the simulations with the use of the nonlinear PS model are used as the input data, approximated during optimization calculations.

The comparison of the eigenvalues calculated on the basis of instantaneous power waveforms and the original eigenvalues was assumed to be a measure of the accuracy of calculations. The PS model state matrix has 56 electromechanical eigenvalues that were sorted descending by the real parts and numbered from λ_1 to λ_{56}. The original electromechanical eigenvalues of the PS state matrix associated with the least damped modal components and the absolute errors $\Delta\lambda$

Table 1: Generating units included in the PS model

Node	S, MVA	U, kV	Node	S, MVA	U, kV
ADA214	450 (3 · 150)	220	OST111	235 (1 · 235)	110
ADA124	150 (1 · 150)	110	PAT224	235 (1 · 235)	220
BLA123	275 (4 · 68.75)	110	PAT214	705 (3 · 235)	220
ZAP213	300 (2 · 150)	220	PAT114	470 (2 · 235)	110
ZAP223	300 (2 · 150)	220	PEL412	940 (4 · 235)	400
BYC233	300 (2 · 150)	220	PEL212	758.4 (3 · 252.8)	220
BYC223	505.6 (2 · 252.8)	220	PEL112	235 (1 · 235)	110
DBN113	426 (1 · 426)	110	ROG411	2556 (6 · 426)	400
DBN133	426 (1 · 426)	110	ROG211	852 (2 · 426)	220
HAL113	68.75 (1 · 68.75)	110	ROG221	1278 (3 · 426)	220
KRA414	705 (3 · 235)	400	SIE133	150 (1 · 150)	110
KRA214	505.6 (2 · 252.8)	220	SKA253	150 (1 · 150)	220
KON214	137.5 (2 · 68.75)	220	SKA113	206.25 (3 · 68.75)	110
KON224	150 (1 · 150)	220	STW112	150 (1 · 150)	110
KON114	68.75 (1 · 68.75)	110	STW122	300 (2 · 150)	110
KON124	300 (2 · 150)	110	WIE413	470 (2 · 235)	400
KOP213	470 (2 · 235)	220	WIE213	758.4 (3 · 252.8)	220
KOP123	235 (1 · 235)	110	WIE113	235 (1 · 235)	110
KOZ212	1410 (6 · 235)	220	WIE133	235 (1 · 235)	110
KOZ112	470 (2 · 235)	110	ZRC415	836 (4 · 209)	400
LAG213	300 (2 · 150)	220	PAK41W *	9450 (15 · 630)	400
LAG113	150 (1 · 150)	110	VIE21G *	6300 (10 · 630)	220
LAG133	300 (2 · 150)	110	HAG21G *	3780 (6 · 630)	220
LAZ123	150 (1 · 150)	110	HAG22G *	5040 (8 · 630)	220
MIK214	705 (3 · 235)	220	KIS41G *	11970 (19 · 630)	400
MIK224	705 (3 · 235)	220	NOS41C *	11970 (19 · 630)	400
MIK124	235 (1 · 235)	110	ALB41C *	6930 (11 · 630)	400
MIK414	235 (1 · 235)	400	LIS21C *	7560 (12 · 630)	220
OST211	470 (2 · 235)	220			

of calculating these eigenvalues based on the instantaneous power waveforms are presented in Table 2.

Table 2: Original eigenvalues of the PS state matrix and the absolute errors of their calculation based on the instantaneous power waveforms

h	1	2	3	4
λ_h, 1/s	$-0.0835 \pm j\,5.6278$	$-0.1710 \pm j\,4.9780$	$-0.4165 \pm j\,8.0932$	$-0.4488 \pm j\,6.6540$
$\Delta\lambda_h$, 1/s	$-0.0745 \pm j\,0.4093$	$-0.0340 \pm j\,0.1444$	$0.0138 \pm j\,0.0976$	$-0.0277 \pm j\,0.0088$
h	5	6	7	8
λ_h, 1/s	$-0.4788 \pm j\,7.6653$	$-0.4955 \pm j\,7.3005$	$-0.5713 \pm j\,8.5011$	$-0.5910 \pm j\,8.5763$
$\Delta\lambda_h$, 1/s	$0.0691 \pm j\,0.1107$	$0.0293 \mp j\,0.2115$	$-0.0818 \mp j\,0.0409$	$-0.0595 \mp j\,0.8201$
h	9	10	11	12
λ_h, 1/s	$-0.6372 \pm j\,8.3382$	$-0.6417 \pm j\,8.8039$	$-0.6723 \pm j\,8.6222$	$-0.7368 \pm j\,9.6011$
$\Delta\lambda_h$, 1/s	$0.0646 \pm j\,0.0754$	$-0.1158 \mp j\,0.0635$	$0.0731 \mp j\,0.0028$	$-0.0383 \pm j\,0.2604$

For example, Fig. 3 shows the waveforms of the instantaneous power deviation and the band of the approximating waveforms corresponding to the non-rejected calculation results. This band defines the range of changes in the instantaneous power, in which all the approximation waveforms corresponding to the individual calculation results are included.

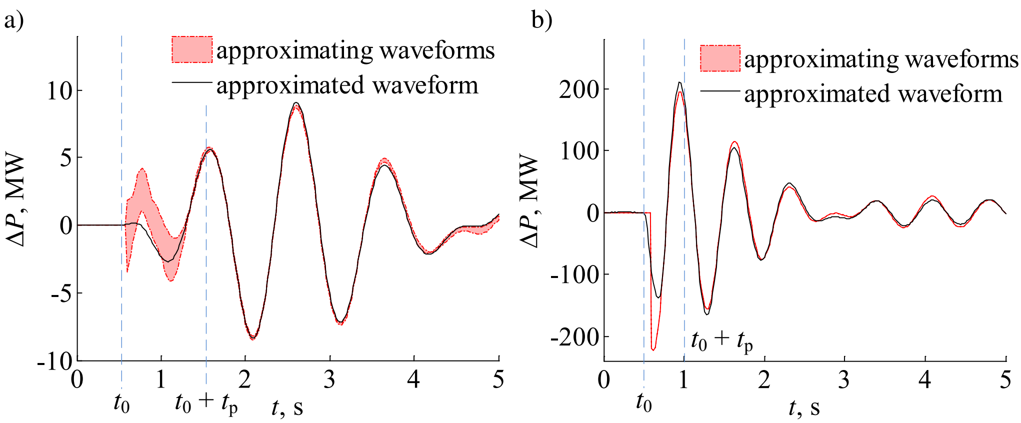

Figure 3: The instantaneous power deviation of: unit ZRC415 for the disturbance in unit PEL412 (a), unit ROG411 for the disturbance in unit ROG411 (b)

To find the generating units critical for the PS angular stability, the participation factors and correlation coefficients of the eigenvalues with the real parts greater than −0.3 and having a significant impact on the waveforms of the generating units physically occurring in PS were analyzed. These are the eigenvalues λ_1 and λ_2. Analyzing the participation factors of eigenvalues λ_1 and λ_2, the waveforms in which the modal components associated with these eigenvalues dominated were selected. The components whose absolute values of the participation factors were at least 70% of the largest absolute value of the participation factor of the electromechanical eigenvalue in the analyzed waveform were assumed as dominant. The equivalent generating units were not included in the investigations.

The relative absolute values of the correlation coefficients $|p|_{\text{rel}}$ for the generating units selected as described above are listed in Table 3.

These relative modules of the correlation coefficients of the eigenvalues and the generating units physically existing in the PS are determined as follows:

$$|p_{kh}|_{\text{rel}} = \frac{|p_{kh}|}{\max\limits_{k} \left(|p_{kh}|\right)} . \tag{27}$$

In formula (27), when calculating the correlation coefficients, the elements of eigenvectors related to the angular speed of the k-th generating unit of the h-th eigenvalue are used.

In order to locate the generating units critical for the angular stability of the analyzed PS (i.e. the generating units most influencing the angular stability) the following criteria were assumed:

1. The generating units having the largest absolute values of the correlation coefficients are successively selected (it was assumed that $|p|_{\text{rel}}$ was to be greater than 0.2) for the eigenvalues with the largest real parts (e.g. greater than -0.3).

2. The repeating of the same generating units for several eigenvalues in the above criterion increases its importance for the angular stability of PS.

3. The generating units for which at least one of the cases occurs:

   - a significant modal (dominant) component associated with the eigenvalue with the largest real parts for at least two other places of introducing the disturbance occurred in the instantaneous power waveforms of these units,
   - introduction of a disturbance in this unit caused the occurrence of significant components associated with the eigenvalue with the largest real parts in the instantaneous power waveforms of at least two other generating units,

   are successively selected.

The generating units meeting criterion 3 are marked with an asterisk in Table 3. They have the largest absolute values of the participation factors for the eigenvalues from Table 3.

Table 3: Correlation coefficients of eigenvalues λ_1 and λ_2

	Unit	$\|p\|_{rel}$	Unit	$\|p\|_{rel}$
λ_1	KRA214	1.0	ROG411	0.5862
	Unit	$\|p\|_{rel}$	Unit	$\|p\|_{rel}$
λ_2	ZRC415 *	1.0	KOZ112	0.1641
	KOZ212 *	0.4245	PEL412 *	0.1586
	ROG411 *	0.3769	OST211	0.1536
	ROG221 *	0.2748	PEL212	0.1351
	PAT214	0.2262	KON124 *	0.0949
	PAT114	0.1900	PAT224	0.0719
	ADA214 *	0.1825	ADA124	0.0681
	ROG211 *	0.1682	KON114 *	0.0275

On the basis of the conducted analyzes and specific criteria, the generating units were finally selected and ordered according to their significance for the angular stability of the analyzed PS. They are presented in Table 4.

Table 4: Generating units critical for the PS angular stability

No.	Unit	Reason
1	ROG411	large absolute values of the correlation coefficients for the eigenvalues λ_1 and λ_2 (equal to 0.5862 and 0.3769, respectively), meeting criterion 3 for λ_2
2	KRA214	the largest absolute value of the correlation coefficient (1.0) for the eigenvalue λ_1 associated with the least damped modes
3	ZRC415	the largest absolute value of the correlation coefficient (1.0) for the eigenvalue λ_2, meeting criterion 3 for λ_2
4	KOZ212	large absolute value of the correlation coefficient (0.4245) for the eigenvalue λ_2, meeting criterion 3 for λ_2
5	ROG221	large absolute value of correlation coefficient (0.2748) for eigenvalue λ_2, meeting criterion 3 for λ_2
6	PAT214	large absolute value of the correlation coefficient (0.2262) for the eigen-value λ_2
7	ADA214	meeting criterion 3 for λ_2
8	ROG211	meeting criterion 3 for λ_2
9	PEL412	meeting criterion 3 for λ_2
10	KON124	meeting criterion 3 for λ_2
11	KON114	meeting criterion 3 for λ_2

8. Conclusions

Based on the performed investigations one can draw the following conclusions:

- It is possible to find generating units critical for the PS angular stability on the basis of electromechanical calculations of the eigenvalues of the PS state matrix and determination in which waveforms of the generating units these eigenvalues significantly interfere. The criteria for selection of these generating units have been presented in this paper. These criteria are influenced by the participation factors and correlation coefficients of the electromechanical eigenvalues with the largest real parts.

- The applied method of calculating electromechanical eigenvalues assumes a reduction of the order of the model to eliminate satisfactorily the impact of eigenvalues not related to electromechanical phenomena. This reduction consists in proper selection of the initial time of the waveform analysis, on the basis of which the eigenvalues are calculated. The system thus obtained, described by a set of the calculated electromechanical eigenvalues and the participation factors associated with them, is completely controllable and observable from the point of view of electromechanical phenomena.

- Based on the analysis of the actual instantaneous power waveforms of generating units, it is possible to calculate the electromechanical eigenvalues and their participation factors with satisfactory accuracy. The calculation accuracy is usually best for the eigenvalues with the largest real parts, associated with the least damped modal components. This is advantageous for assessing the PS angular stability.

- Having got the PS model, it is possible to calculate the correlation coefficients for individual eigenvalues in the waveforms of subsequent generating units based on the eigenvectors of the state matrix of this model.

- The conducted investigations show that the results of determining the location of the units critical for the PS angular stability based on the analysis of the participation factors and correlation coefficients are partly the same. The agreement occurs primarily for the units in which individual eigenvalues have large correlation coefficient values.

- The advantage of the proposed method of location of generating units critical for the angular stability of PS is the use of the real measurement waveforms to calculate electromechanical eigenvalues and their participation factors. These waveforms can be measured relatively easily in power plants when introducing a simple test disturbance. One may also use the waveforms measured during small disturbances occurring naturally during PS operation. An additional advantage is the possibility to use the correlation coefficients calculated on the basis of the eigenvectors of the PS model state matrix. It is therefore possible to compare the location results obtained based on the measurements with those obtained on the basis of the PS model. The disadvantage of the proposed method is the lack of strictly defined criteria for the location of generating units critical for the angular stability of PS. However, these criteria may be specified in further investigations, which will also enable the accurate classification of the generating units most affecting the PS angular stability.

- In general, the greater the power of the unit into which the disturbance has been introduced, the greater the number of units in which power swings with significant amplitudes and different absolute values of participation factors of individual eigenvalues occur.

The location of critical generating units in PS and the calculations of electromechanical eigenvalues and their participation factors can also be carried out using the presented method based on the waveforms of other electromechanical quantities of generating units, namely: angular speed and power angle of synchronous generators. One can use electromechanical waveforms that appear after purposeful introduction of a disturbance in the form of a rectangular pulse (as in this paper) or a unit step, but also after occurring random disturbances such as short circuits and load changes on the load nodes.

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