T H E A R C H I V E O F M E C H A N I C A L E N G I N E E R I N G

VOL. LVI 2009 Number 4

DOI: 10.24425/ame.2009.132108

Key words: mechanics of solids, experimental methods, elastic-plastic states

BARBARA KOZŁOWSKA *

RESIDUAL STRESSES AND PLASTIC ZONES' BOUNDARIES IN TWO-DIMENSIONAL ELEMENTS

In the paper, author presents the analysis of the elastic-plastic residual stresses and the boundaries of plastic zones in two-dimensional model with central circular hole. The experimental testing was carried out by photoelastic coating method. The duralumin model was loaded within the overelastic range by uniformly distributed tensile stresses. For various levels of loading, the photographs of isochromatic pattern were taken. The residual stresses along the axis of symmetry perpendicular to the stretching direction were calculated by the characteristics method, using multisectional schematization of $\sigma - \varepsilon$ relation for the material. The boundaries of plastic zones in the loaded model were obtained on the basis of the Treska-Coulomb yield condition directly from the isochromatic pattern. The analysis and discussion of the test results is presented.

1. Introduction

The elastic-plastic strain and stress analysis is of great importance. The economical trend towards building more lightweight and cheaper constructions forces the designers to accept the partial material plastifying in the constructional elements during exploitation.

Numerical methods, widely applied nowadays, always contain some inaccuracy caused by the necessity of modelling a real object (especially nonlinear problems) and still need a final experimental verification.

In the elastic-plastic analysis, there are many problems associated with the modelling of constructional material: the character of strain hardening, conversion from elastic to plastic state, etc. In such cases, experimental methods seem to be very useful, especially these methods which give information

* *Warsaw University of Technology, Faculty of Mechatronical Engineering, ul. Św. Andrzeja Boboli 8, 02-525 Warsaw, POLAND, tel. (0)22 234 83 02; E-mail: B.Kozlowska @mchtr.pw.edu.pl*

about the real construction under working conditions without any model simplification.

One of the problems, important for proper exploitation of the construction working in the elastic-plastic range, is the analysis of residual stresses which remain in an unloaded constructional element after a part of the material has previously been plasified.

Residual stresses, which exist in some parts of constructional elements not subjected to the external loading, may be caused by different operations, among others, by loading beyond the yield point of material. Residual stresses may be introduced purposely to increase strength of the structure, but they may also be produced by non-controled external processes, which can cause negative effects including construction failure. The presence of the plastic zones only in some part of the element creates, after relieving, the residual stresses in the whole element's cross-section.

One of the experimental methods, which can be applied to elastic-plastic states analysis, is photoelastic coating method. It is the method which gives information about the deformation of the real object in the whole tested area (not only at several points) and can be used to investigate objects of complex shapes and loaded in different ways (dynamically and statically) also at various temperatures (−20°C÷50°C).

2. Application of photoelastic coating method to elastic-plastic analysis

The photoelastic coating method is based on the effect of the optical birefringence which occurs in some transparent materials under loading. The thin layer of birefringent material is bonded integrally to the surface of the analyzed object, and when the object is loaded, the surface strains are transmitted to the coating. When viewed through a reflection polariscope, the strained coating exhibits two families of fringe patterns: isoclinic fringes providing information about directions of principal strains, and isochromatic fringes supplying information about difference of principal strains. These fringes can be photographed and used to determine strain tensor components. Because determination of directions of principal strains by means of isoclinic fringes is labour-consuming and not precise, for strain separation the analytical methods taking advantage of the isochromatic pattern only are usually used (for example method of characteristics).

The method of photoelastic coating can be applied to the elastic-plastic states analysis due to the assumption of the linear relation between the photoelastic effect and strain in the birefringent material in a wide range. In the range of strain, where the material of the tested element is in the plastic state, the characteristic of photoelastic coating material is still linear. Therefore, one can assume that the strain constant of the photoelastic coating may be applied to analysis of plastified areas, as well as to these remaining in the elastic state.

The additional advantage of photoelastic coating method is good visualisation of the material plastifying process and the possibility of quick and easy determination of the plastic zone boundary.

3. Experimental testing

The investigation of the distribution of residual stresses created by plastic zones around stress concentrators was performed on the model of stretched strip weakened by central circular hole (Fig. 1). The photoelastic coating method was applied.

The model was made of duralumin sheet 4 mm thick. After mechanical working and special surface preparation (polishing and etching), the model was covered on both sides (to avoid bending effect) with the photoelastic coating made of epoxy resin. Finally, the model was subjected to the finishing machining (the hole) in the way which allowed to avoid creating stresses in the result of cutting.

The model's material characteristic (determined on the basis of repeatedly-taken uniaxial tensile tests) is shown on Fig. 2.

The strain constant of the photoelastic coating was determined experimentally, and its value was: $\mathbf{f} = 1.109 \cdot 10^{-3}$ 1/fringe order.

The model was loaded at its ends with a uniformly distributed tensile stresses **p**. As the measure of the intensity of loading, the 'loading factor's was accepted, calculated as a ratio of the tensile stresses at the cross-section weakened by the hole on the axis of symmetry **x** perpendicular to the stretching direction in relation to the offset yield strength $\mathbf{R_{0.2}}$ = 320 MPa obtained from the material characteristic.

The loading of the model was increased step by step within the over-elastic range of the material. At each level of loading, the photographs of isochromatic pattern were taken twice: for both dark- and light-field polariscope.

For further analysis of elastic-plastic states, the isochromatic patterns obtained for the selected loading levels (at which visible zones of the plastified material near the hole appeared) were taken into account. Fig. 3 and Fig. 4 present the pictures of isochromatic fringes taken for a dark-field polariscope for loading levels: s = 0.537, s = 0.596, s = 0.703 and s = 0.781.

Because of the symmetry of the model and loading, only one half of the isochromatic pattern is considered.

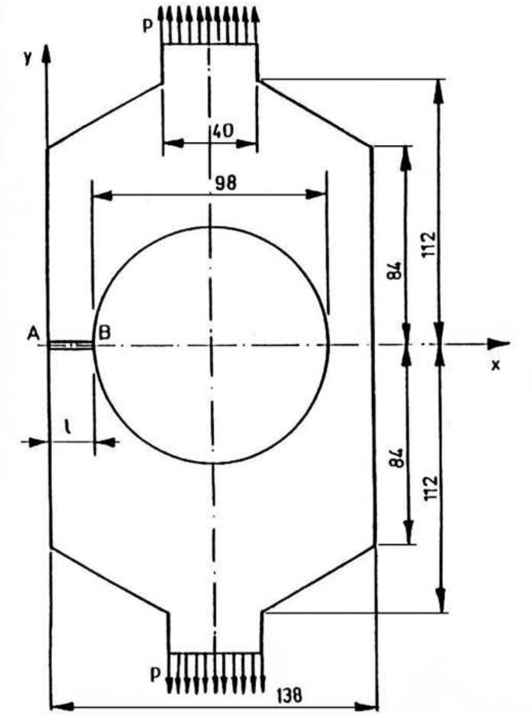

Fig. 1. Model of constructional element

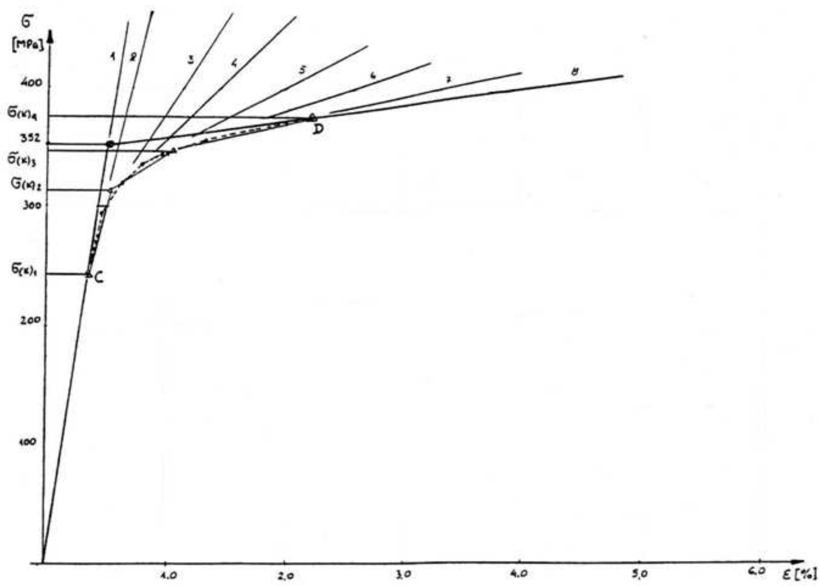

Fig. 2. Material characteristic

a) b)

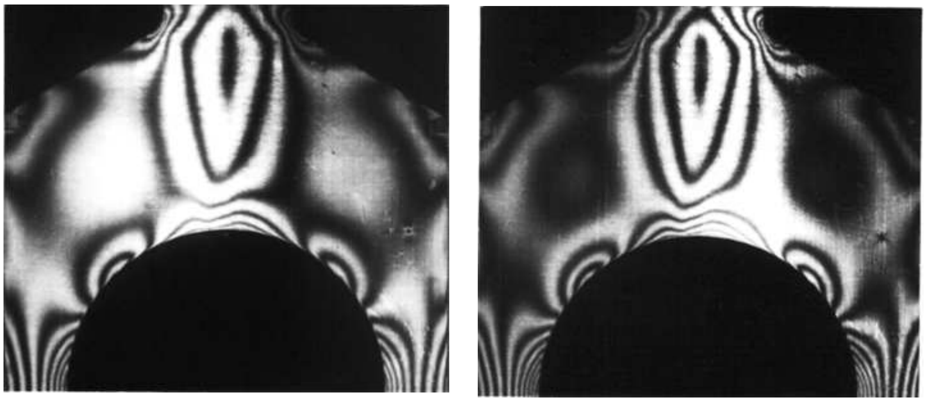

Fig. 3. Isochromatic pattern for loading level: a) s = 0.537, b) s = 0.596

a) b)

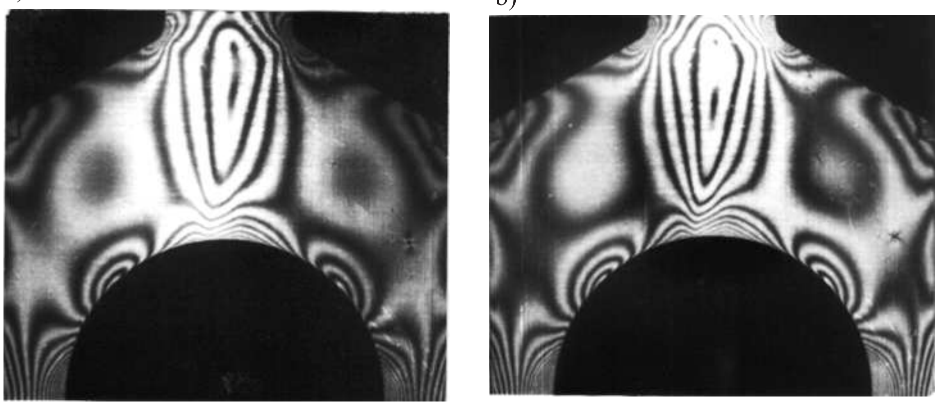

Fig. 4. Isochromatic pattern for loading level: a) s = 0.703, b) s = 0.781

4. Analysis of residual stresses and boundaries of plastic zones

4.1. Calculation of residual stresses

It is very difficult to determine the stress tensor components in every point of the model for the residual stresses when the structure is unloaded and the forces are balanced within the cross section.

For small plastic strain, the residual stresses can be determined as the difference between the elastic-plastic stresses and the stresses which would occur at the same level of loading if the material characteristic reminds elastic. It is so because relieving of the material is held on the way parallel to the linear part of the $\boldsymbol{\sigma} - \boldsymbol{\varepsilon}$ relation.

Separation of strain components on the basis of results obtained from the method of photoelastic coating (isochromatic fringe – the principal strain difference) was performed using the analytical method of characteristics [2]. This method makes use of the relations between the strain and stress tensor components (Hooke's law), and its extension for the overelastic range of the material requires creating analogous relations for nonlinear part of the characteristic. For derivation of formulas describing the mentioned relations, the geometrical model of real $\boldsymbol{\sigma} - \boldsymbol{\varepsilon}$ curve was considered [3], [4]. In this model, the experimentally obtained material characteristic is replaced by **n** line segments (Fig. 2). These relations are also used to calculate stress components for nonelastic range of material in the analyzed model.

An 8-sectional model of material characteristic was assumed to determine distribution of the elastic-plastic stress components $\boldsymbol{\sigma_y}$ and $\boldsymbol{\sigma_x}$ on the axis of symmetry x perpendicular to the stretching direction (Fig.1). This model was quite sufficient for precise representation of the real $\boldsymbol{\sigma} - \boldsymbol{\varepsilon}$ relation of the material.

The distribution of elastic stress components $\boldsymbol{\sigma_y}$ and $\boldsymbol{\sigma_x}$ on the axis of symmetry **x** was determined based on the picture of the isochromatic pattern taken for the loading level (**s** = 0.3125) when plastic deformation did not yet appeared.

The stresses which would occur at one of the higher loading levels (if the material characteristic reminded elastic) were obtained by multiplying the calculated elastic stresses by the ratio of higher loading (creating partial material plastification) to elastic loading (for **s** = 0.3125). After substractig the stress calculated in such way from the real elastic-plastic stresses existing in the model, the distribution of residual stresses was obtained.

4.2. Determination of plastic zones' boundaries

The boundaries of plastic zones can be easily determined using only the isochromatic pattern in such cases when signs of the principal stress components can be predicted before their separation. When the principal stress components have different signs or if they are of the same sign, but one of them is much greater than the other, the order of isochromatic fringe which corresponds to the boundary of the plastic zone can be calculated using the Treska-Coulomb yield condition [1]. Applying the Hooke's law and Wertheim's law can be obtained

$$m_{gr} = \frac{1+\nu}{E \cdot f}\sigma_{pl} \tag{1}$$

where: σ_{pl} – yield point;
ν – Poisson ratio;

E – modulus of elasticity;
f – strain constant of the photoelastic coating.

Using the above formula, on the basis of properties of the photoelastic coating material and the model material, was calculated the value of boundary isochromatic fringe order: $\mathbf{m_{gr}}$ = 5.43. For the yield point $\boldsymbol{\sigma_{pl}}$ of model's material, the offset yield strength $\mathbf{R_{0.2}}$ = 320 MPa was assumed.

4.3. Plastic zones and residual stresses in the model of constructional element

On the basis of the pictures of isochromatic pattern taken for a dark-field polariscope at selected loading levels: s = 0.537, s = 0.596, s = 0.703 i s = 0.781 (Fig. 3 and Fig. 4) were determined the plastic zones boundaries by the boundary isochromatic fringe method.

For the corresponding loadig levels, the residual stresses were calculated on the axis of symmetry perpendicular to the loading direction using the 8-sectional schematization of model's material characteristic.

The plastic zones under loading and the residual stresses after releasing are shown in Fig. 5 and Fig. 6. For better visualization, only a part of the picture containing the analyzed region of the model was considered.

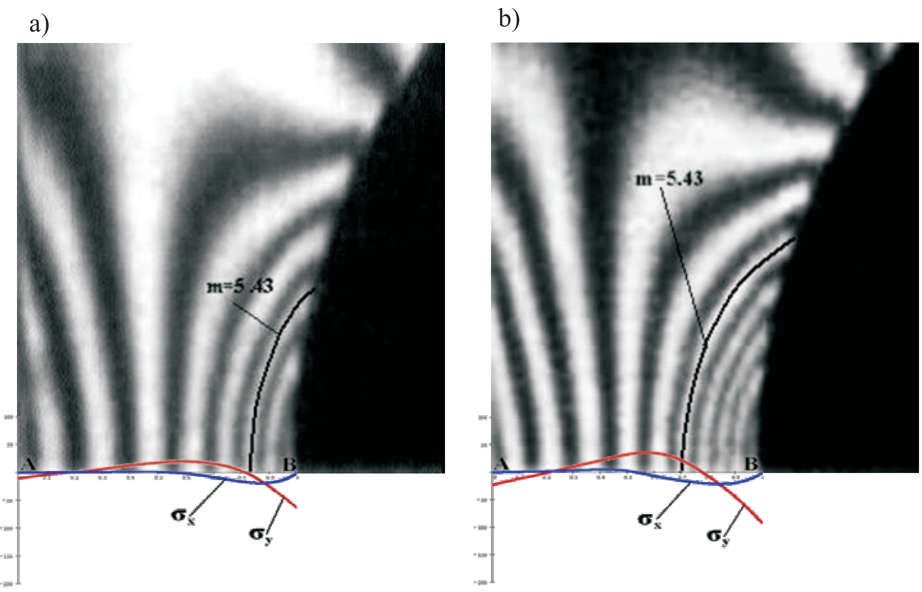

Fig. 5. Boundary of plastic zone and residual stresses distribution $\boldsymbol{\sigma_y}$ and $\boldsymbol{\sigma_x}$ on the axis of symmetry for loading level: a) s = 0.573, b) s = 0.596

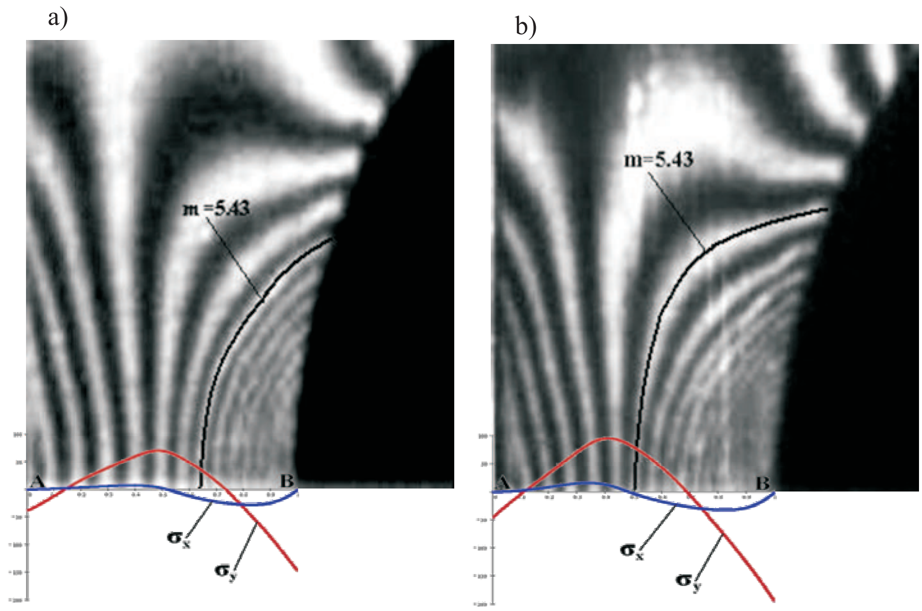

Fig. 6. Boundary of plastic zone and residual stresses distribution σ_y and σ_x on the axis of symmetry for loading level: a) s = 0.703, b) s = 0.781

The distributions of residual stresses σ_y and σ_x on the axis of symmetry **x** (section A-B) for different loadings are shownin Fig. 7 and Fig. 8.

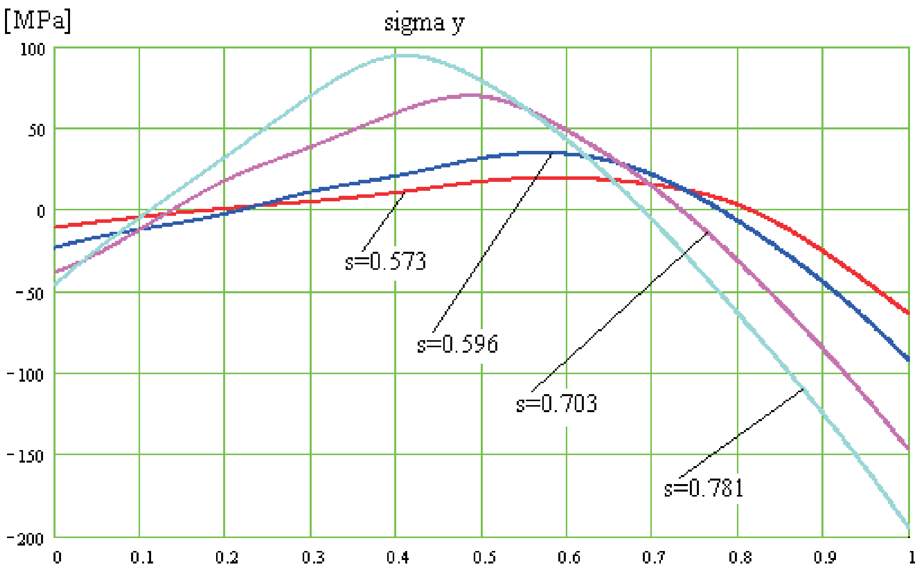

Fig. 7. Residual stresses σ_y distribution on **x** axis of symmetry (section **A-B**) for loading levels: s = 0.537, s = 0.596, s = 0.703, s = 0.781

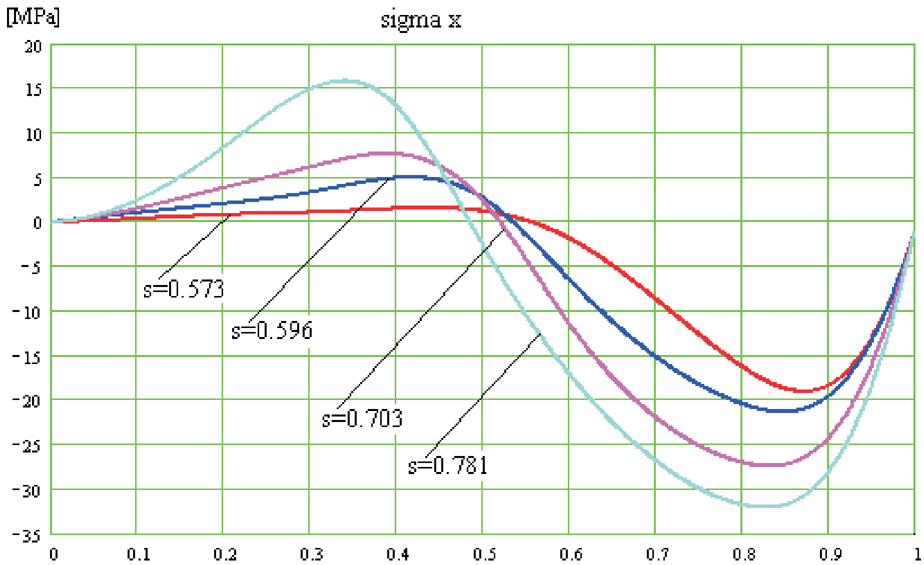

Fig. 8. Residual stresses σ_x distribution on **x** axis of symmetry (section **A-B**) for loading levels: s = 0.537, s = 0.596, s = 0.703, s = 0.781

5. Conclusions

The knowledge of the process of formation and propagation of elastic-plastic stresses under increasing loading allows proper designing constructional elements.

After loading an object over the yield point of the material, releasing and consecutive loading (in the same way) to a higher level, the object's material will behave like an elastic material. The phenomenon of increasing the elastic limit (Baushinger effect) is widely applied in engineering. For example effects of stress concentration introduced by holes and notches in constructional elements are reduced by residual stresses created as a result of material plastifying after repeated loading because they will be added to the stresses caused by external loading.

In the paper, the author presents the analysis of residual stresses created in a stretched model of a constructional element with central circular hole after loading the element over yield point. The residual stresses were determined based on the pictures of isochromatic pattern in the model taken at different (overelastic) loading levels. The stress component values were calculatied by means of a computer program, created by the author, using the method of characteristics for strain separation on the axis of symmetry **x**. In order to derive the relations between strain and stress components, 8-sectional schematization of model's material characteristic was assumed.

As it was shown by the analyzed model, the residual stresses $\boldsymbol{\sigma_y}$ and $\boldsymbol{\sigma_x}$ on the axis of symmetry **x**, caused by the plastified part of the material in the vicinity of the hole, determined for selected loading levels (s = 0.537, s = 0.596, s = 0.703 i s = 0.781), change their values and distribution in the function of the increasing loading (Fig. 7, Fig. 8).

The verification of correctness of the residual stresses distribution was performed by checking equilibrium of $\boldsymbol{\sigma_y}$ stresses on the axis of symmetry **x** perpendicular to the stretching direction. When $\boldsymbol{\sigma_y}$ stresses acting in the direction of axis y (the stretching direction) are in balance, the resultant force is equal zero in the cross-section weakened by the hole. For all of the considered loading levels, the error evaluated with respect to quilibrium condition did not exceed a few percents (the highest value was 8% for the lowest loading level s = 0.537).

It is worth noticing that plastification of relatively small areas of the tested element generates, after relieving, residual stresses in the whole element's cross-section. These stresses reach high values in the plastified zones and nearby. In the remaining (much greater) part of the cross-section, they are small, but sufficient to secure the equilibrium of residual forces.

Taking into account the obtained results, it can be concluded that the photoelastic coating method applied to the residual stresses analysis in stretched elements with rapid cross-section changes makes it possible to determine (in a relatively quick and easy way), the boundaries of plastified zones and to calculate stress tensor components. Therefore, it can be used for the evaluation the possibilities of improving the working conditions of constructional elements weakened by stress concentrators.

Manuscript received by Editorial Board, May 20, 2009;
final version, November 16, 2009.

REFERENCES

[1] Życzkowski M.: Obciążenia złożone w teorii plastyczności, PWN, Warszawa 1975.
[2] Szczepiński W.: Method of characteristics in computation of the experimental stress analysis, Bull. de l' Acad. Polonaisa des Sc., Vol. XII. No. 112, 1964.
[3] Kapkowski J., Kozłowska B.: Elastic-plastic strain analysis by photoelastic coating method, Journal of Theoretical and Applied Mechanics, No.3,Vol.31,Warszawa 1993.
[4] Kozłowska B.: Zastosowanie metody elastooptycznej warstwy powierzchniowej do analizy stanów sprężysto-plastycznych, Praca doktorska, Warszawa 1996.

Naprężenia własne i granice stref plastycznych w elementach dwuwymiarowych

S t r e s z c z e n i e

W pracy została przedstawiona analiza naprężeń własnych i granic stref plastycznych na przykładzie płaskiego modelu elementu konstrukcyjnego z otworem kołowym. Model wykonany z duraluminium został obciążony równomiernie rozłożonymi na końcach naprężeniami rozciągającymi wywołującymi częściowe uplastycznienie materiału. Do badań doświadczalnych zastosowano metodę pokryć optycznie czynnych. Na wybranych poziomach obciążenia w zakresie sprężysto-plastycznym zostały zarejestrowane obrazy izochrom, które posłużyły do wyznaczenia granic obszarów uplastycznionych oraz określenia wartości naprężeń własnych na osi symetrii prostopadłej do kierunku rozciągania. Granice obszarów plastycznych zostały wyznaczone metodą izochromy granicznej. Do rozdzielenia odkształceń wykorzystano metodę charakterystyk, a naprężenia zostały wyznaczone na podstawie przyjętego wieloodcinkowego modelu zależności $\sigma - \varepsilon$ dla materiału elementu. W pracy została przeprowadzona dyskusja otrzymanych wyników.