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PROJECT OF PLANETARY GEAR WITH THE SAME GEOMETRY OF A SATELLITE WHEEL

The solution of planetary gear with simplifying technology using the same geometry of the sun gear and the central gear is already known. The authors decided to change this concept i.e. to design a planetary gear with the same geometry of satellite wheels, which cooperate with a sun gear and a central gear with different number of teeth. The structural solution of elements of the gear is analyzed taking advantage of computational technique. Geometrical dimensions are described for the sake of teeth correction. Calculations and structural solutions of this kind of transmission are shown in the article.

1. Simlpifying manufacturing technology

The aim of this article is to present design procedure of involute planetary gear train with external gears and the same geometry of satellite wheels. The planetary gear is characterized by a simplifying technology. Satellite wheels are determined by a simple technological manufacturing process, and consequently the cost of productions can be reduced.

One can use a typical gear hobbing machine to manufacture the presented planetary gear. Then, the manufactured external gears can be subjected to carbonizing, and also hardening and tempering. It is possible to obtain a better dimensional accuracy thanks to grinding process (it is impossible for internal gears). Therefore, after heating treatment, the construction of the planetary gear is characterized by a higher load-carrying capacity, at the same time the gearbox can be manufactured in the simplifying technology.

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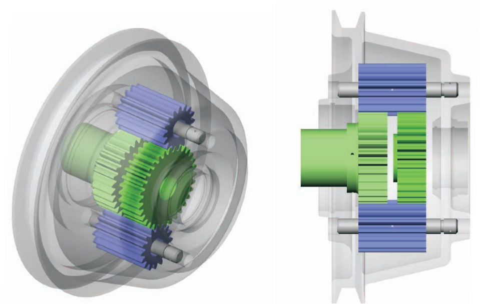

Fig. 1. Characteristic of construction

2. Construction of gearbox

2.1. Geometric design procedures

To make mounting of a planetary gearbox possible, it is necessary to take into consideration the assembly conditions. The basic mechanical configuration consist of two co-operating pairs of gears with the same distance of rotation axis, additionally $z_2 = z_3$. The important thing is the condition of satellite wheels coaxiality, as shown in Fig.2. This condition can be described by equation (0.1).

$$a_{w'} = a_{w''} \tag{0.1}$$

Where: $a_{w'}$ and $a_{w''}$ present the real dimensions of the co-operation gears z_1, z_2 and z_3, z_4.

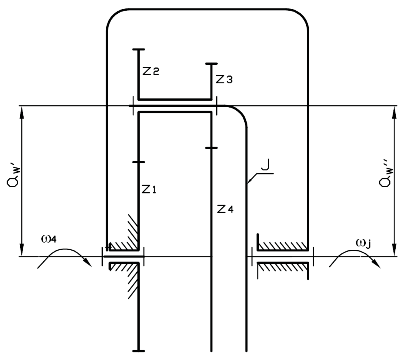

Fig. 2. Kinematic diagram

The toothed wheels must be modified, because of condition (0.2)

$$(z_1 + z_2) \cdot m = (z_3 + z_4) \cdot m, \tag{0.2}$$

$$z_2 = z_3, z_1 \neq z_4. \tag{0.3}$$

To satisfy condition (0.1), the rack-cutter must be moved during the cutting process.
The number of teeth and modules should be selected to meet condition (0.4) for reduction gears.

$$(z_1 + z_2) \cdot m < (z_3 + z_4) \cdot m, \tag{0.4}$$

$$i = \frac{\omega_j}{\omega_4} = 1 - \frac{z_1 \cdot z_3}{z_2 \cdot z_4} \tag{0.5}$$

Thanks to the application of the presented design procedures, it is possible to eliminate the undercutting of teeth, and to ensure that the pitch circle is between the inner and the outer circles of the face-gear. It is important not to exceed the limit of addendum coefficient modification during teeth modification. The thickness of teeth on the outer circle must be greater than the required minimal one. The cutting process is shown in Fig.3 and Fig.4.

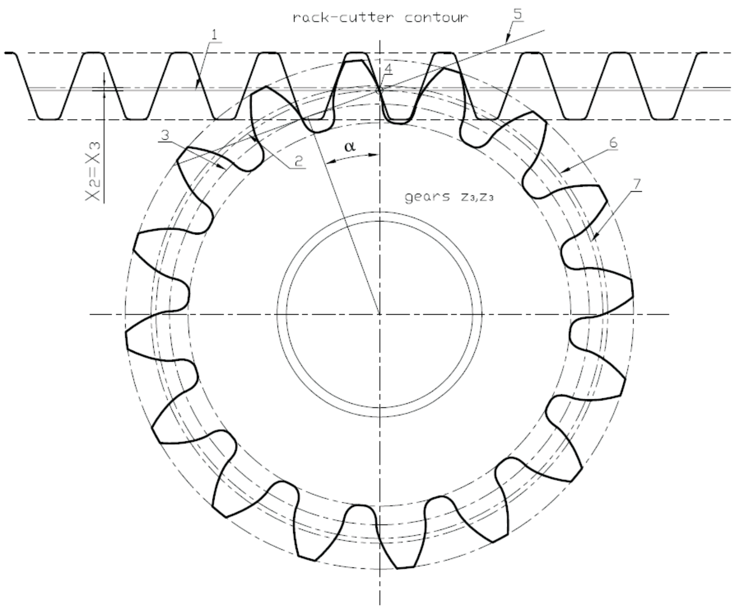

Fig. 3. Rack-cutter setting during the cutting process of satellite wheels: z_2, z_3.
1) pitch line of rack (tool), 2,7) pitch circle, 3) base circle,
4) pitch point, 5) contact line, 6) generating circle

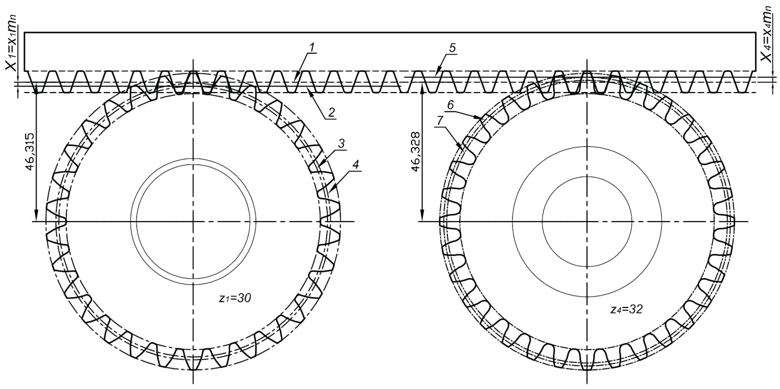

Fig. 4. Teeth correction: sun wheel and central wheel, 1) pitch line of tool, 2) addendum modification value for z_1, 3) pitch circle z_1, 4) generating circle z_1, 5) addendum modification value for z_4, 6) pitch circle z_4, 7) generating circle z_4

The second assembly condition is the correct distance between addendum circles of satellite wheels. Equation (0.6) presents the condition for the construction of the analyzed planetary gear.

$$(z_2 + z_1)\sin\frac{\pi}{s} - z_2 > 2y + 1 \tag{0.6}$$

Where: y – addendum coefficient. For normal teeth, $y = 1$.
Equation (0.7) is necessary to ensure the cooperation of teeth of planetary gear without collisions. The third essential condition is uniformity of array of satellite wheels.

$$(z_4 - z_1)/s \in N \tag{0.7}$$

Where: s – number of satellite wheels, N – natural number.
The result of correct calculations is the possibility of assembling the planetary gear. It is necessary to set precisely the sun wheel, the central wheel and the satellite wheels with angle orientation of $2\pi/s$. Thanks to an adequate arrangement of satellite wheels, there are no vibrations, which is the result of zero resultant force.

2.2. An example of gearbox project

As shown in Fig.5, the housing – 1) works as a carrier. In Placed in the housing, there is are placed the satellite bolts – 2) the satellite wheels z_2 , z_3 are the uniform gear wheel. The result of reeling of the satellite by the central gear is rotation of the sun gear, which is jointed to the driven shaft.

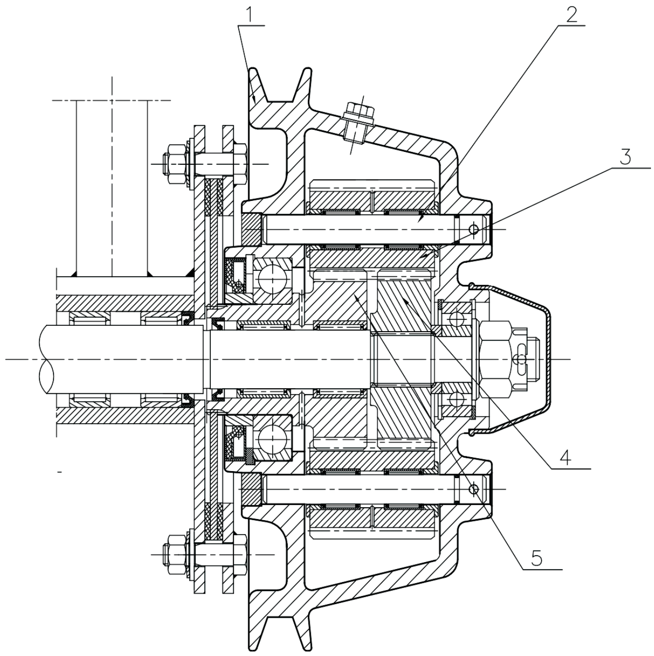

Fig. 5. Project of planetary gear with simplifying technology, 1) carrier-housing, 2) satellite bolts, 3) satellite wheels z_2, z_3, 4) sun gear z_4, 5) central gear z_1

2.3. Constraint meshing equation and its solution

The main objective for this solution is to take right variable values: spur gear and the module $m=3$. Number of teeth: central gear $z_1=30$, satellite wheel of the same geometry $z_2 = z_3=16$, sun gear $z_4=32$. Face width of the toothed wheels z_1, z_4 : $b_1=21$[mm], face width of the toothed wheels z_2/z_3 : $b_2=70$[mm], pressure angle $\alpha = 20°$, d_a – diameter of the addendum circle, d_b – base circle diameter, p_b – base pitch, $\alpha_{w'}$, $\alpha_{w''}$ – pressure angles of cooperating toothed wheels.

Superscripts (') determine data for toothed wheels z_1/z_2, and superscripts (") determine data for toothed wheels z_3/z_4, number of satellite s=2.
The axis distances before correction were ($a_{0'}$) , ($a_{0''}$)

$$a_{0'} = \frac{d_1 + d_2}{2}, \; a_{0'} = 69 \; [mm], \tag{0.8}$$

$$a_{0''} = \frac{d_3 + d_4}{2}, \; a_{0''} = 72 \; [mm], \tag{0.9}$$

The modules of the generating circles ($m_{w'}$, $m_{w''}$), under assumptions $a_{w'} = a_{w''} = a = 70,5$ $[mm]$:

$$m_{w'} = 2 \cdot \frac{a}{z_1 + z_2}, \; m_{w'} = 3.065 \; [mm] \tag{0.10}$$

$$m_{w''} = 2 \cdot \frac{a}{z_3 + z_4}, \; m_{w''} = 2.937 \; [mm] \tag{0.11}$$

The sum of addendum modification coefficients after enumerating the pressure angles ($\alpha_{w'}$) and ($\alpha_{w''}$):

$$c_1 = \frac{(z_1 + z_2) \cdot [(\tan \alpha_{w'} = \alpha_{w'}) - \tan \alpha_t + \alpha_t]}{2 \cdot \tan \alpha_t}, \quad c_1 = 0.538 \tag{0.12}$$

$$c_2 = \frac{(z_3 + z_4) \cdot [(\tan \alpha_{w''} = \alpha_{w''}) - \tan (\alpha_t) + \alpha_t]}{2 \cdot \tan (\alpha_t)}, \quad c_2 = -0.457 \tag{0.13}$$

The addendum modification values are:

$$\begin{gathered} X_1 = 1.3147[mm], X_2 = 0.3 \; [mm], \\ X_3 = 0.3[mm], X_4 = -1.6721 \; [mm]. \end{gathered} \tag{0.14}$$

2.4. Geometric dimensions of engagement of mating involute spur gears z_1/z_2:

Circular pitch – (p), base pitch – (p_b), generating pitch – (p_w):

$$p = 9.424 \; [mm], \;\; p_b = 8.856 \; [mm], \;\; p_w = 9.629 \; [mm] \tag{0.15}$$

Tip clearance – (c):

$$c = 0.75 \tag{0.16}$$

Circular thickness – (s_1, s_2):

$$s_1 = \left(\frac{\pi}{2} + 2 \cdot x_1 \cdot \tan \alpha_t\right) \cdot m_t, s_1 = 5.669 \; [mm] \tag{0.17}$$

$$s_2 = \left(\frac{\pi}{2} + 2 \cdot x_2 \cdot \tan \alpha_t\right) \cdot m_t, s_2 = 4.930 \; [mm] \tag{0.18}$$

The axis distance before correction ($a_{0'}$) and after correction ($a_{w'}$):

$$a_{0'} = 69[mm], a_{w'} = 70.5 \; [mm] \tag{0.19}$$

The height of the tooth (h) after enumerating the radiuses of generating circles ($r_{w1'}, r_{w2'}$), and the diameters of root circles (d_{f1} , d_{f2}):

$$h = 2 \cdot y_t \cdot m_t + c - k \cdot m, h = 6.635\ [mm] \tag{0.20}$$

Tooth contact ratio z_1/z_2 :

$$\varepsilon = \frac{\left(\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2}\right) \cdot 0.5 - a_{w'} \cdot \ \sin \alpha_{w'}}{p_b}, \varepsilon = 1.428 \tag{0.21}$$

2.5. Geometric dimensions of engagement of mating involute spur gears z_3/z_4:

Circular pitch – (p), base pitch – (p_b), generating pitch – (p_w):

$$p = 9.424[mm],\ p_b = 8.856[mm],\ p_w = 9.228[mm] \tag{0.22}$$

Tip clearance – (c):

$$c = 0.75 \tag{0.23}$$

Circular thickness – (s_3, s_4):

$$s_3 = \left(\frac{\pi}{2} + 2 \cdot x_3 \cdot \ \tan \alpha_t\right) \cdot m_t,\ s_3 = 4.930[mm] \tag{0.24}$$

$$s_4 = \left(\frac{\pi}{2} + 2 \cdot x_4 \cdot \ \tan \alpha_t\right) \cdot m_t,\ s_4 = 3.495[mm] \tag{0.25}$$

The axis distance before correction ($a_{0''}$) and after correction ($a_{w''}$):

$$a_{0''} = 72[mm],\ a_{w''} = 70.5[mm] \tag{0.26}$$

The height of the tooth (h) after enumerating the radiuses of generating circles ($r_{w1'}, r_{w2'}$), and the diameters of root circles (d_{f1}, d_{f2}):

$$h = 2 \cdot y_t \cdot m_t + c - k \cdot m,\ h = 6.622[mm] \tag{0.27}$$

Tooth contact ratio z_1/z_2 :

$$\varepsilon = \frac{\left(\sqrt{d_{a3}^2 - d_{b3}^2} + \sqrt{d_{a4}^2 - d_{b4}^2}\right) \cdot 0.5 - a_{w''} \cdot \ \sin \alpha_{w''}}{p_b}, \varepsilon = 1.692 \tag{0.28}$$

Thanks to the introduced teeth correction, it was possible to meet the specified conditions, including the condition of coaxiality of wheels.

$$a_{w'} = \frac{z_1 + z_2}{2} \cdot m_{w'}, \; a_{w'} = 70.5, \tag{0.29}$$

$$a_{w''} \frac{z_3 + z_4}{2} \cdot m_{w''}, \; a_{w''} = 70.5. \tag{0.30}$$

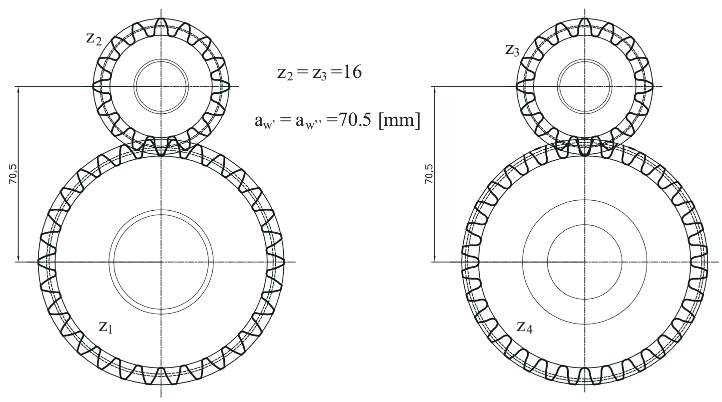

Fig. 6. Mating of toothed wheels

It is shown in equations (0.21) and (0.28) that the created toothed wheels can cooperate with mesh continuity. The obtained tooth contact ratio is ε = 1.428 for wheels z_1/z_2, and ε = 1.692 for wheels z_3/z_4. It allows for regular mating.

3. Conclusion

In this study, the authors assessed the possibility of manufacturing a planetary gear with simplifying technology at a lower cost of production. The presented construction is another structural solution with the same geometry of gear wheels. The satellite wheels z_2 and z_3 can be manufactured in a pocket as a uniform gear wheel. The selection of optimal geometrical parameters makes it possible to create a mechanism, which can be manufactured on industrial scale. It constitutes an alternative for expensive internal planetary gears. Some difficulty can appear in mounting the planetary gear, but thanks to lower manufacturing costs, the presented construction is more economical. It can be used in many industrial machines.

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Projekt przekładni planetarnej o identycznej geometrii satelitów

S t r e s z c z e n i e

Istnieje już rozwiązanie przekładni planetarnej o uproszczonej technologii [2], wykorzystujące taką samą geometrią kół centralnego i słonecznego. Zdecydowano się na zmianę koncepcji tzn. zaprojektowania przekładni o satelitach stanowiących wspólne koła zębate o jednolitej geometrii, współpracujących z kołem centralnym i słonecznym o różnej ilości zębów. Wykorzystując techniki obliczeniowe analizowane będzie rozwiązanie konstrukcyjne elementów przekładni. Wymiary geometryczne będą określone za względu na obszar rozwiązywalności korekcji zębów. Zaprezentowano obliczenia i przykładowe rozwiązanie konstrukcyjne tego typu przekładni.