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A model of gas flow with friction in a slotted seal

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Abstract The paper discusses thermodynamic phenomena accompanying the flow of gas in a slotted seal. The analysis of the gas flow has been described based on an irreversible adiabatic transformation. A model based on the equation of total enthalpy balance has been proposed. The iterative process of the model aims at obtaining such a gas temperature distribution that will fulfill the continuity equation. The model allows for dissipation of the kinetic energy into friction heat by making use of the Blasius equation to determine the friction coefficient. Within the works, experimental research has been performed of the gas flow in a slotted seal of slot height 2 mm. Based on the experimental data, the equation of local friction coefficient was modified with a correction parameter. This parameter was described with the function of pressure ratio to obtain a mass flow of the value from the experiment. The reason for taking up of this problem is the absence of high accuracy models for calculating the gas flow in slotted seals. The proposed model allows an accurate determination of the mass flow in a slotted seal based on the geometry and gas initial and final parameters.

Keywords: Slotted seals; Annular slot; Friction coefficient; Gas flow

Nomenclature

A	–	flow area, m^2
$\mathbf{B}$	–	model matrix

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$\mathbf{\Delta T}$	–	temperature difference vector
$\Delta\dot{\mathbf{m}}$	–	mass flow difference vector
C_e	–	correction parameter of friction coefficient c_f
c_f	–	friction coefficient
c_p	–	specific heat, J/kg K
R	–	gas constant, J/kg K
Re	–	Reynolds number
T	–	temperature, K
O	–	wetted circumference for the gas flow, m
L	–	length of the seal, m
h	–	specific enthalpy, J/kg
HS	–	slot height, m
Δp	–	pressure drop, Pa
$\dot{m}$	–	mass flow, kg/s
p	–	pressure, Pa
q	–	heat of friction, J/kg
u	–	velocity, m/s
x	–	linear dimension alongside the gas flow, m

Greek symbols

δ	–	calculations accuracy
κ	–	isentropic exponent
λ	–	friction coefficient
Δ	–	increment of a given parameter
ν	–	gas kinematic viscosity, m^2/s
ρ	–	gas density, m^3/kg
τ	–	static tension, Pa

Subscripts and superscripts

1	–	value at the inlet to the seal
2	–	value at the outlet of the seal
e	–	value obtained in the experiment
i	–	value for the ith element
it	–	number of iterations
m	–	value obtained from the model
max	–	maximum value
n	–	number of elements
s	–	value referred to the isentropic transformation
T	–	transpose
x	–	distance from the beginning of the seal
$(\bar{\cdot})$	–	averaged value

1 Introduction

Slotted seals are applied in steam turbines to minimize steam leakage in the regulating valve spindles. They are also applied in compressors and combustion engines in piston cylinder assemblies. Such type of seal significantly influences the efficiency of many machines. The influence of the slot height on the efficiency of a piston engine was analyzed in [6]. Presented method enable the determination of the rate of incompressible liquid leakage through a slotted seal [2,8–12]. These methods are based on the Bernoulli equation with regard to experimentally obtained coefficients of of linear and local resistance. The results of the described methods have been presented for different geometries of the seal and pressure drop for both stationary and dynamic (rotating shaft) states. Paper [7] includes the analysis of gas flow in a short labyrinth seal based on experimental research using one dimensional theory. In the description of the mass flow, the authors of this paper, using one-dimensional theory, applied the gas flow model for a short pipe. Paper [1] includes the model allowing for the influence of tangential tensions in a two disc, one-sided seal geometry. The experimental research on the determination of coefficients of friction losses for short segments of one-sided seals have been presented in [14,16].

Equations for laminar and turbulent flows of compressible and incompressible liquids in slotted seals have been described in [15]. However, they provide mass flows far from those obtained in the experiments performed by the authors. In works related to slotted seals there is a deficit of methods providing satisfactory accuracy of calculations applicable in the flow of compressible liquids.

The thermodynamic parameters of gas flowing through any type of seal change along its length [3–5], which also includes the discussed slotted seal. Pressure, temperature, density and gas velocity are changed in the first place. This paper presents a model of gas flow in a slotted seal based on the friction coefficient obtained in the experimental research.

2 Calculation model

The flow of gas in a slotted seal is triggered by the pressure gradient. The mass flow of gas is reduced by the flow resistance generated by the tangential tensions between the gas and the seal walls [7,13,15]. The relation can be described with a set of equations in which, due to the gas flow, averaged values were applied referred to the conditions at the inlet and outlet of the seal

$$\left\{ \begin{array}{c} \Delta p A = \tau O L \\ \tau = c_f \bar{\rho} \frac{\bar{u}^2}{2} \end{array} \right. , \tag{1}$$

where the average value of the gas density is determined by the formula $\bar{\rho} = \frac{p_1+p_2}{R(T_1+T_2)}$. Assuming that the gas flow in the seal segment is realized according to the irreversible adiabatic process we can write the relation between kinetic energy, static enthalpy, work of friction and heat of friction [13]

$$d\left(\frac{u^2}{2}\right) + dh + dL_f - dq_f = 0\ , \qquad (2)$$

where $dL_f = \frac{c_{f,i}u_i^2}{HS}dx$, $dq_f = dT_f c_p$, $dL_f = dq_f$, and the subscript i denotes value for the ith element.

Considering Eqs. (1) and (2) for the flowing gas in ith portion of the length of $x_{i+1} - x_i$ (Fig. 1.) we can determine the increase in gas temperature by $\Delta T_{f,i}$ resulting from the friction heat generated by dissipation of kinetic energy

$$\Delta T_{f,i} = c_{f,i}\frac{u_i^2}{c_p}\frac{x_{i+1} - x_i}{HS}. \qquad (3)$$

This heat leads to an increase in the temperature and a reduction of the gas density that directly influences the velocity and flow resistance. Under measurement conditions, it is difficult to exclude or estimate the flow of heat between the seal in the walls and the decompressing gas. Without a doubt it has an effect on the final temperature of gas. The said heat flow together with the kinetic energy dissipated into friction heat influence the drop of the gas pressure Δp in the segment of the seal. The model presented in the paper is based on the assumption of constant total enthalpy. In the model, a local friction coefficient was taken into account according to the Blasius equation for a turbulent gas flow over flat surface [16]

$$c_{f,i} = \frac{0.0592}{\mathrm{Re}_{x,i}^{0.2}}\ , \qquad (4)$$

where $\mathrm{Re}_{x,i} = \frac{u_i x}{\nu_i}$ – local Reynolds number.

The seal of diameter D, length L and height of slot HS is divided in to n numbers of elements. We know the initial and final parameters of the medium: p_1, T_1, p_{n+1}. In the initial phase of the calculations the final temperature is assumed as $T_{n+1} = T_{2s}$, which results from the isentropic transformation and a linear temperature drop. The distribution of gas temperature in the iterative process is corrected by the increment of temperature (3) resulting from the friction work (2).

The flowing gas is decompressed and its temperature varies longitudinally. In the model, the following has been assumed:

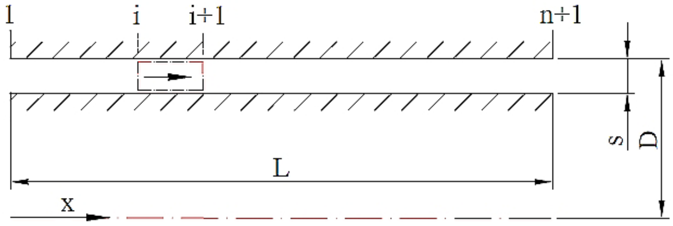

Figure 1: Geometry of the slotted seal.

- linear distribution of pressure,
- flow of gas with constant total enthalpy,
- mass flow of gas is limited by friction as per Eq. (4).

The presented model is applied in calculations of subsonic velocities of gas.

In the model, the continuity equation for the ith and $i+1$ cross-sections was taken into account

$$\dot{m}_i = A\rho_i u_i \ , \tag{5}$$

$$\dot{m}_{i+1} = A\rho_{i+1} u_{i+1} \ , \tag{6}$$

as well as the equation of state

$$\frac{p_i}{\rho_i} = RT_i \ , \tag{7}$$

$$\frac{p_{i+1}}{\rho_{i+1}} = RT_{i+1} \ , \tag{8}$$

and condition of conservation of mass flow between the calculation nodes:

$$\dot{m}_i = \dot{m}_{i+1} \ . \tag{9}$$

The equation of total enthalpy for the calculation cell between the ith and $i+1$ cross-sections has been written as follows:

$$T_i c_p + \frac{u_i^2}{2} = T_{i+1} c_p + \Delta T_{i+1} c_p + \frac{u_{i+1}^2}{2} = \\ = T_{i+1} c_p + \frac{u_{i+1}^2}{2}\left(1 + c_{f,i}\frac{x_{i+1} - x_i}{HS}\right) \ . \tag{10}$$

In the main model described by Eq. (10) the flow resistance triggered by the friction was taken into account causing dissipation of kinetic energy into heat dq_f (2).

Equations (5)–(10) written for $n+1$ cross-sections have an unknown mass flow, velocity in each cross-section and density as well as temperature in cross sections 2 to n. The mass flow resulting from Eqs. (5)–(10) can be written as

$$\dot{m}_i = \left[\frac{2A^2 c_p p_i^2 p_{i+1}^2 \left(T_i - T_{i+1}\right)}{R^2 \left[\left(1 + c_{f,i} \frac{x_{i+1}-x_i}{HS}\right) p_i^2 T_{i+1}^2 - p_{i+1}^2 T_i^2 \right]} \right]^{0.5} . \tag{11}$$

Assuming the constants

$$C_1 = 2A^2 c_p p_i^2 p_{i+1}^2 , \tag{12}$$

$$C_2 = R^2 \left(1 + c_{f,i} \frac{x_{i+1} - x_i}{HS}\right) p_i^2 , \tag{13}$$

$$C_3 = R^2 p_{i+1}^2 , \tag{14}$$

Eq. (11) takes the form

$$\dot{m}_i = \left[\frac{C_1 \left(T_i - T_{i+1}\right)}{C_2 T_{i+1}^2 - C_3 T_i^2} \right]^{0.5} . \tag{15}$$

Complete differential of the function of mass flow describes the change of the mass flow of gas flowing through the ith element depending on the temperature change at the beginning and end of a given element. The change was determined in the following way:

$$d\dot{m}_i = \frac{\partial \dot{m}_i}{\partial T_i} dT_i + \frac{\partial \dot{m}_i}{\partial T_{i+1}} dT_{i+1} . \tag{16}$$

The above equation transformed to finite increments can be written as follows:

$$\Delta\dot{m}_i = \frac{\partial \dot{m}_i}{\partial T_i} \Delta T_i + \frac{\partial \dot{m}_i}{\partial T_{i+1}} \Delta T_{i+1} , \tag{17}$$

adopting notation (15) of the mass flow equation, the partial derivatives in equation (17) have the following form:

$$\frac{\partial \dot{m}_i}{\partial T_i} = 0.5 \left[\frac{C_1 \left(T_i - T_{i+1}\right)}{C_2 T_{i+1}^2 - C_3 T_i^2} \right]^{-0.5} \times \frac{C_1 C_2 T_{i+1}^2 + C_1 C_3 \left(T_i^2 - 2T_i T_{i+1}\right)}{\left(C_2 T_{i+1}^2 - C_3 T_i^2\right)^2} , \tag{18}$$

$$\frac{\partial \dot{m}_i}{\partial T_{i+1}} = 0.5 \left[\frac{C_1 \left(T_i - T_{i+1} \right)}{C_2 T_{i+1}^2 - C_3 T_i^2} \right]^{-0.5} \times \frac{C_1 C_2 \left(T_{i+1}^2 - 2 T_i T_{i+1} \right) + C_1 C_3 T_i^2}{\left(C_2 T_{i+1}^2 - C_3 T_i^2 \right)^2} \ . \tag{19}$$

The vector of finite increments of mass flows can be described with the expression

$$\Delta \dot{m}_i - \Delta \dot{m}_{i+1} = \frac{\partial \dot{m}_i}{\partial T_i} \Delta T_i + \left(\frac{\partial \dot{m}_i}{\partial T_{i+1}} - \frac{\partial \dot{m}_{i+1}}{\partial T_{i+1}} \right) \Delta T_{i+1} - \frac{\partial \dot{m}_{i+1}}{\partial T_{i+2}} \Delta T_{i+2} \tag{20}$$

for $i = 1, \ldots, n-1$.

The presented method enables a notation of a matrix equation

$$\mathbf{B} \Delta T = \Delta \dot{m} \ . \tag{21}$$

Based on the right sides of the (n-1) set of Eqs. (17) matrix **B** was created

$$\mathbf{B} = \begin{bmatrix}
\left(\frac{\partial \dot{m}_1}{\partial T_2} - \frac{\partial \dot{m}_2}{\partial T_2} \right) & -\frac{\partial \dot{m}_2}{\partial T_3} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\
\frac{\partial \dot{m}_2}{\partial T_2} & \left(\frac{\partial \dot{m}_2}{\partial T_3} - \frac{\partial \dot{m}_3}{\partial T_3} \right) & -\frac{\partial \dot{m}_3}{\partial T_4} & 0 & \cdots & \cdots & \cdots & 0 \\
0 & \frac{\partial \dot{m}_3}{\partial T_3} & \left(\frac{\partial \dot{m}_3}{\partial T_4} - \frac{\partial \dot{m}_4}{\partial T_4} \right) & -\frac{\partial \dot{m}_4}{\partial T_5} & 0 & \cdots & \cdots & 0 \\
\vdots & 0 & \ddots & \ddots & \ddots & 0 & \cdots & 0 \\
\vdots & \vdots & 0 & \ddots & \ddots & \ddots & 0 & 0 \\
\vdots & \vdots & \vdots & 0 & \ddots & \ddots & \ddots & 0 \\
0 & \vdots & \vdots & \vdots & 0 & \frac{\partial \dot{m}_{n-2}}{\partial T_{n-2}} & \left(\frac{\partial \dot{m}_{n-2}}{\partial T_{n-1}} - \frac{\partial \dot{m}_{n-1}}{\partial T_{n-1}} \right) & -\frac{\partial \dot{m}_{n-2}}{\partial T_{n-1}} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial \dot{m}_{n-1}}{\partial T_{n-1}} & \left(\frac{\partial \dot{m}_{n-1}}{\partial T_n} - \frac{\partial \dot{m}_n}{\partial T_n} \right)
\end{bmatrix} \tag{22}$$

and vector $\mathbf{\Delta T}$, whose elements are defined in the following manner

$$\mathbf{\Delta T} = \begin{bmatrix} \Delta T_1 & \Delta T_2 & \ldots & \Delta T_{n-1} \end{bmatrix}^T \ . \tag{23}$$

The vector of differences of the mass flows was obtained from Eq. (20) based on the assumed drop of pressure and calculated drop of temperature

$$\mathbf{\Delta \dot{m}} = \begin{bmatrix} \Delta \dot{m}_1 & \Delta \dot{m}_2 & \ldots & \Delta \dot{m}_{n-1} \end{bmatrix}^T \ , \tag{24}$$

where

$$\Delta \dot{m}_i = \dot{m}_i - \dot{m}_{i+1} = \left[\frac{C_1 \left(T_i - T_{i+1} \right)}{C_2 T_{i+1}^2 - C_3 T_i^2} \right]^{0.5} - \left[\frac{C_1 \left(T_{i+1} - T_{i+2} \right)}{C_2 T_{i+2}^2 - C_3 T_{i+1}^2} \right]^{0.5} \tag{25}$$

for $i = 1, \ldots, n-1$. The superscript T denotes transpose of the vector.

By multiplying Eq. (21) by the matrix inversed to matrix $\mathbf{B}$ we obtain an unknown vector of temperature changes $\Delta\mathbf{T}$. The initially assumed linear gas temperature distribution is corrected iteratively by vector $\Delta\mathbf{T}$. This gives the correction of vector $\Delta\dot{\mathbf{m}}$. This is a quickly convergent process aiming at preservation of the continuity of flow, hence obtaining very small components of the vector of mass flow differences $\Delta\dot{\mathbf{m}} \to 0$.

The end of the iterative calculations occurs when the maximum value of the component of the vector of mass flow differences, referred to the average value of the mass flow, obtained for $n+1$ cross-sections in a given iteration is smaller or equal to the assumed calculation accuracy $\delta = 10^{-5}$:

$$\frac{|\Delta\dot{\mathbf{m}}_{\max,it}|}{\bar{\dot{m}}_{it}} \leq \delta \; . \tag{26}$$

The first part of the iterative calculations was preformed for a linear distribution of gas temperature. This distribution continues from the initial temperature obtained from the measurement through the final temperature resulting from the isentropic transformation.

In the next phase of calculations, based on the obtained vectors $\mathbf{u}$, $\boldsymbol{\rho}$, $\mathbf{T}$, temperature vector $\mathbf{T}$ is enlarged by the temperature increment vector resulting from friction $\Delta\mathrm{T}_f$ as per Eq. (3)

$$\mathbf{T} = \mathbf{T} + \Delta\mathbf{T}_f \; . \tag{27}$$

Taking into account Eq. (27) results in a change of the gas temperature distribution in the cross-sections from 2 to $n+1$. Then, based on the initial temperature T_1 and corrected final temperature T_{n+1} the iterative model aims at such a temperature distribution that will ensure a continuity of flow with condition (26). The source code of the presented calculation algorithm has been written in Fortran.

3 Comparison of experimental data with the model calculation results

The experimental research was performed on a test stand described in [3]. The tests were preformed for the seal of diameter $D = 0.15$ m, length $L = 0.2$ m and slot height $HS = 0.002$ m (Fig. 1). The data obtained in the experiment have been included in Tab. 1.

Table 1: Gas thermodynamic parameters and mass flows obtained in the experimental research and in the model.

No.	p_1 [Pa]	T_1 [K]	p_1/p_2 [–]	$\dot{m}_e$[kg/s]	$\dot{m}_m$[kg/s]
1	148122	295.9	1.54	0.2554	0.1820
2	144556	295.7	1.50	0.2458	0.1676
3	139989	295.4	1.45	0.2329	0.1478
4	135842	295.2	1.40	0.2203	0.1246
5	131625	294.8	1.35	0.2073	0.0948
6	127835	294.5	1.30	0.1931	0.0466
7	123969	294.2	1.25	0.1790	–
8	119905	293.7	1.20	0.1599	–
9	115823	293.3	1.15	0.1395	–

Including the friction in the model using the Blasius Eq. (4) results in an elevated friction-based flow resistance. As an effect, a much smaller gas mass flow was obtained in the model compared to the experiment (Tab. 2, Fig. 2).

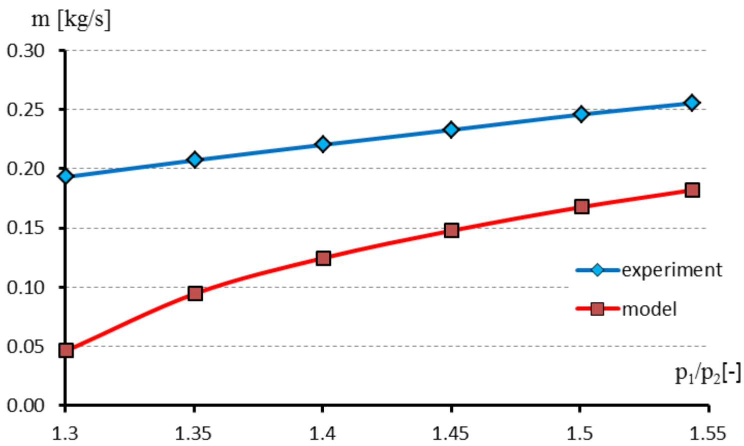

Figure 2: The mass flow obtained in the experiment and in the model including friction as in Eq. (4).

Besides, the application of Eq. (4) results in a situation when the range of applicability of the model is limited by small ratios of pressure $p_1/p_2 \geq 1.3$. This limitation results from conditions $T_i - T_{i+1} > 0$ in Eq. (11). In order for the model to work properly, the temperature of gas flowing in the seal must be decreasing. Hence, the gas temperature drop following the decompression must be greater than the temperature increment caused by friction $\Delta T_{f,i}$.

The lines representing the values of local friction coefficients $c_{f,i}$ for different pressure ratios are similar to parallel (Fig. 3). We may, thus apply, in Eq. (4), a constant correction parameter on the entire length of the seal causing a reduction of the friction effect.

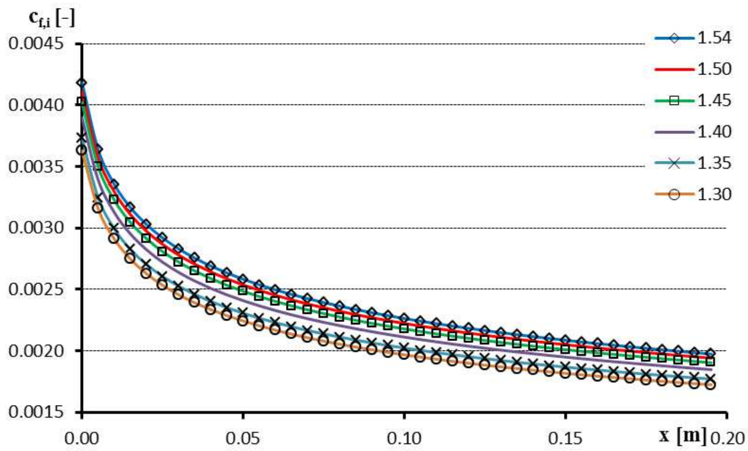

Figure 3: Local values of the friction coefficient on the seal length for the analyzed pressure drops p_1/p_2.

Based on the experimental data, the equation was corrected by friction coefficient (4) adding parameter C_e in the numerator of equation

$$c_{f,i} = \frac{0.0592 + C_e}{\mathrm{Re}_{x,i}^{0.2}} . \tag{28}$$

Parameter C_e was determined numerically so that the mass flow obtained from the model was equal to the value obtained in the experiment. The variability of C_e (Tab. 2.) as a function of pressure ratio has been shown in Fig. 4. The correction parameter of the local friction coefficient (28) was approximated by quadratic function

$$C_e = -0.0186 \left(\frac{p_1}{p_2}\right)^2 + 0.084 \left(\frac{p_1}{p_2}\right) - 0.105 . \tag{29}$$

Table 2: Values of the parameter correcting friction coefficient $c_{f,i}$ in Eq. (28) for condition $\dot{m}_{\rm m} = \dot{m}_{\rm e}$.

No.	p_1/p_2 [–]	C_e [–]
1	1.54	-1.97E-02
2	1.50	-2.07E-02
3	1.45	-2.19E-02
4	1.40	-2.34E-02
5	1.35	-2.54E-02
6	1.30	-2.68E-02
7	1.25	-2.95E-02
8	1.20	-3.09E-02
9	1.15	-3.24E-02

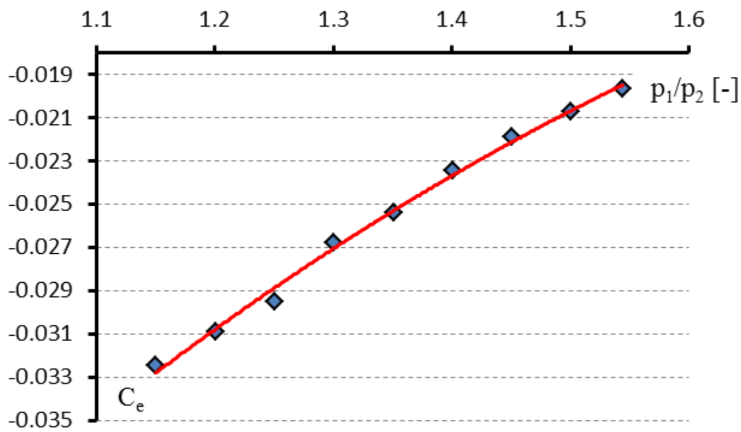

Figure 4: Parameter C_e correcting the local friction coefficient (28) as a function of pressure ratio p_1/p_2.

For selected pressure ratios p_1/p_2 in Fig. 5 a change in the gas velocity was presented as a function of length of the seal obtained from the calculation model. For the greatest pressure ratio p_1/p_2 equal to 1.54 the gas velocity clearly increases (Fig. 6.). High gas velocities in the seal may significantly intensify the heat transfer between the gas and the seal on the walls. For lower pressure ratios, the velocity increase follows the linear trend.

The change in the initial temperature of gas is a consequence of the experimental data. For parameter p_1/p_2 1.54 we can see the greatest drop of

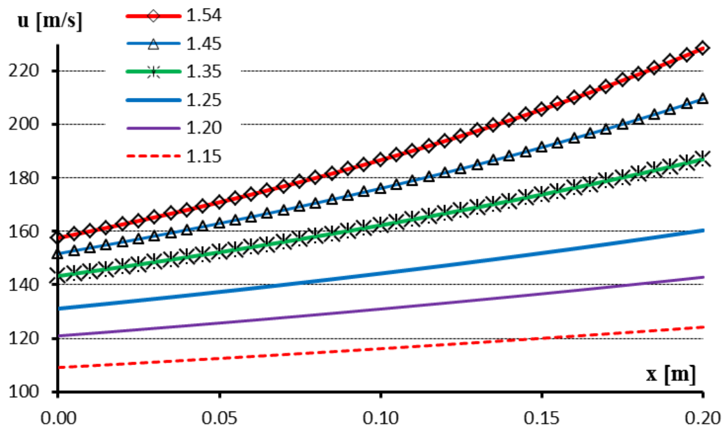

Figure 5: Distribution of the gas velocity on the length of the seal x for selected pressure ratios p_1/p_2.

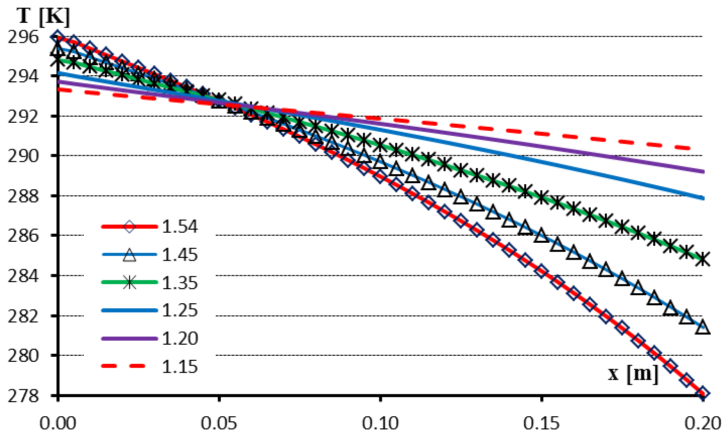

Figure 6: The change in the gas temperature on the length of the seal allowing for the friction heat according to Eqs. (10) and (28).

temperature resulting from the gas decompression. The lower the pressure ratio the lower is the drop of gas temperature (Fig. 6).

4 Conclusions

The presented calculation model based on the local friction coefficient, $c_{f,i}$, determined by the Blasius equation, gives the mass flow values that are much underestimated compared to the experimental data (Fig. 2). The introduction of the correction parameter, C_e, being a function of the pressure ratio in friction coefficient, $c_{f,i}$, enables obtaining of a accurate values of the mass flow through the model. The upper range of the pressure ratio, p_1/p_2, was limited by the measurement capability of the test facility. The proposed model allows for consideration of the kinetic energy dissipation in the form of heat, which allows the inclusion of the gas temperature increment. The model allows determining of the distributions of temperature, velocity and other gas thermodynamic parameters with a high level of accuracy. For significant differences of the seal wall and gas temperatures as well as high gas velocities, the heat transfer from the walls to the gas may significantly influence the leakage rate of the working medium in the slotted seal.

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