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A novel predictive fuzzy adaptive controller for a two-mass drive system

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Abstract. In the paper, a novel control structure based on the fuzzy logic and model predictive control methodologies for an elastic two-mass drive system is proposed. In order to reduce the computational requirements of the classical MPC methodology, the multi parametric programming (MPT) approach is used. The robustness of the system is ensured by implementation of three MPT controllers generated for different operation points and a supervisory fuzzy system. The main goal of the fuzzy system is suitable shaping of the control signal. The effectiveness of the proposed approach is checked in simulation and experimental tests. In order to show the properties of the proposed control structure, a critical comparison with an adaptive classical MPC controller is carried out. Both control structures are tested taking into account the performance and possibility of real-time implementation.

Key words: two-mass system, torsional vibration, MPC, robust control.

1. Introduction

The growing demand for enhancing the productivity of industrial processes results in the necessity to develop more and more advanced control structures. Electrical drives are objects which play a very important role in the efficiency of whole processes. The drives should respond very dynamically and reach/follow the set speed or position command. However, there are different factors which can decrease the control performance of the drive. One of the most important factors is parameter changes inside electrical drives [1–4]. Especially due to the operation cycles the total inertia can vary in a wide range [3, 4]. Also the mechanical construction of the drive system can influence the performance significantly. The drive is connected to the load machine through a mechanical shaft with limited stiffness, which also reduces the characteristics of the system [5–14]. These two factors are evident for example in modern robot-arm drives [5, 6], servo-drives [7–9], and others. In order to challenge these problems, it is necessary to design special control structures, such as: classical PI controller supported by different additional feedbacks [10], resonance ratio control [11], sliding-mode control [12], fuzzy or adaptive control [13]. Recently some papers have proposed a relatively new methodology – model predictive control (MPC) [14, 15]. This control methodology is considered in the paper.

For the last 40 years MPC has become more and more popular. At the beginning, due to the high computational requirements necessary in real-time implementation, it was applied only in chemical and petrochemical industry, where time constants are relatively big [16, 17]. However, due to the progress of the computational power of microprocessors evident in recent decades, the MPC has become popular in various industrial branches including power electronics and drives. In this area two frameworks can be distinguished, namely the short- and long-horizon MPC. In the first approach only one or two steps ahead are predicted [18, 19], which makes this methodology relatively simple and easy to implement in a standard microprocessor. On the other hand, such short prediction decreases the performance of the system. Hence, the long horizon approach attracts attention of researchers. In the literature there are limited works regarding the real-time implementation of the long-horizon MPC in electrical drives, e.g. [14, 15, 20, 21]. The main reason is high-computation requirements of such an approach, so different methodologies which can decrease numerical effort are sought after.

The long-horizon MPC can be implemented in two ways. In the first approach, the classical methodology is used. It means that the minimisation of the cost function is done on-line, which results in high-calculation effort [16]. The computational power of the algorithm can be lowered by the implementation of multi-parametric programming and solving the control problem off-line [22, 23]. The control law is calculated as feedbacks from the system-state and gain matrix which depend on the operation region of the plant. Definitely, this approach does not involve high calculation effort. However, for a plant with changeable parameters, the parameters of the controller cannot be changed directly. So, the decrease of the performance index of the control structure is inevitable. Thus, a modification of the abovementioned algorithm is expected.

In the plant with changed parameters, robust MPC can be used. It relays the suitable tuning of the off-line MPC controller to obtain similar the transient of the controlled variable for the whole range of parameter(s) changes [15]. An example of the above solution can be the work in which the transients of the load speed are almost identical for different values of the me-

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chanical time constant. The drawback of this approach is the decrease in the system dynamics (for the small value of inertia), which may be unacceptable in some industrial processes.

The work investigates two approaches to long-horizon MPC applied to a complex drive system with changeable parameters, namely on-line MPC and fuzzy off-line MPC. In the on-line methodology the optimization problem is solved in every calculation step, hence it ensures the demanded responses of the plant for nominal and changed parameters. On the other hand, it is computationally demanding and unsuitable for practical implementation. The standard solution which ensures the reduction of computational complexity is the off-line version of MPC. However, the regions of the off-line controller are generated for the constant parameters of the plant. The changes of the plant parameters influence the responses of the system in a negative way. Therefore, other solutions are sought after. In the paper fuzzy off-line methodology is proposed, which is less computationally demanding than the standard version of the on-line MPC, yet it generates responses comparable to those of an on-line algorithm. The proposed fuzzy system is a simple one. It can be easily extended, however, as a result the computational complexity will also be increased.

2. MPC algorithm

In the linear version of MPC, the model of the plant is applied to predict the impact of future actions of the manipulated variables on the process output. Typically, the linear discrete-time state-space model is used.

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) \end{aligned} \tag{1}$$

where $\mathbf{x}(k) \in \Re^n$, $\mathbf{u}(k) \in \Re^m$, $\mathbf{y}(k) \in \Re^p$ are the system state, input and output vectors, respectively. $\mathbf{A} \in \Re^{n \times n}$, $\mathbf{B} \in \Re^{n \times m}$, $\mathbf{C} \in \Re^{p \times n}$ are matrices describing the dynamics of the plant, k – is a time step. At every sampling period k, the MPC algorithm solves the optimization problem specified as [17]:

$$\min_{\mathbf{u}_0^T, \ldots, \mathbf{u}_{N_c-1}^T} \left\{ \sum_{i=0}^{N} \mathbf{x}_{k+i|k}^{ref} \mathbf{Q} \mathbf{x}_{k+i|k}^T + \sum_{i=0}^{N_c-1} \mathbf{u}_{k+i|k}^{\mathbf{T}} \mathbf{R} \mathbf{u}_{k+i|k} \right\} \tag{2a}$$

$$\begin{aligned} &\mathbf{u}_{\min} \le \mathbf{u}_{k+i|k} \le \mathbf{u}_{\max} \quad i = 0, 1, \cdots, N_c - 1 \\ &\mathbf{x}_{\min} \le \mathbf{x}_{k+i|k} \le \mathbf{x}_{\max} \quad i = 0, 1, \cdots, N \\ &\mathbf{y}_{\min} \le \mathbf{y}_{k+i|k} \le \mathbf{y}_{\max} \quad i = 0, 1, \cdots, N \\ &\mathbf{x}_{k+i+1|k} = \mathbf{A}\mathbf{x}_{k+i|k} + \mathbf{B}\mathbf{u}_{k+i} \quad i \ge 0, \\ &\mathbf{y}_{k+i|k} = \mathbf{C}\mathbf{x}_{k+i|k} \quad i \ge 0, \\ &\mathbf{x}_{k|k} = \mathbf{x}(k), \end{aligned} \tag{2b}$$

where $\mathbf{Q} \ge 0$ and $\mathbf{R} > 0$ represent the weighting matrices, N, N_c are the prediction and control horizon, and $\mathbf{u}_{\min}$, $\mathbf{u}_{\max}$, $\mathbf{x}_{\min}$, $\mathbf{x}_{\max}$, $\mathbf{y}_{\min}$, and $\mathbf{y}_{\max}$ are the input, state and output constraints of the system respectively. In the system the following inequality is specified $N_c \le N$. Equation (2a, 2b) can be written in the form of a matrix using quadratic programming (QP) [17]:

$$J\big(\mathbf{U}, \mathbf{x}(k)\big) = \mathbf{X}^T \tilde{\mathbf{Q}} \mathbf{X} + \mathbf{U}^T \tilde{\mathbf{R}} \mathbf{U} \tag{3}$$

where $\mathbf{X}$ i $\mathbf{U}$ are predictive vectors of state and control variables. These matrices are as follows:

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}(k+1|k) \\ \vdots \\ \mathbf{x}(k+N|k) \end{bmatrix}, \quad \mathbf{U}(k) = \begin{bmatrix} \mathbf{x}(k|k) \\ \vdots \\ \mathbf{u}(k+N_u-1|k) \end{bmatrix}. \tag{4}$$

The matrices $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{R}}$ have the following form:

$$\tilde{\mathbf{Q}} = \operatorname{diag}\big(\mathbf{Q}, \ldots, \mathbf{Q}\big); \quad \tilde{\mathbf{R}} = \operatorname{diag}\big(\mathbf{R}, \ldots, \mathbf{R}\big). \tag{5}$$

Finally, the problem of optimal control using quadratic programming can be written as:

$$\begin{gathered} V\big(\mathbf{x}(k)\big) = \mathbf{x}(k)^T \mathbf{Y} \mathbf{x}(k) + \min_{\mathbf{U}^*} \left(\frac{1}{2} \mathbf{U}^T \mathbf{H} \mathbf{U} + 2\mathbf{x}^T(k) \mathbf{F} \mathbf{U} \right) \\ \text{subject to} \\ \mathbf{G}\mathbf{U} \le \mathbf{W} + \mathbf{E}\mathbf{x}(k) \end{gathered} \tag{6}$$

where: $\boldsymbol{H}$, $\boldsymbol{F}$, $\boldsymbol{Y}$, $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$ are defined as follows:

$$\mathbf{H} = \tilde{\mathbf{B}}^T \tilde{\mathbf{Q}} \tilde{\mathbf{B}} + \tilde{\mathbf{R}}, \;\; \mathbf{F} = \tilde{\mathbf{A}}^T \tilde{\mathbf{Q}} \tilde{\mathbf{B}}, \;\; \mathbf{Y} = \tilde{\mathbf{A}}^T \tilde{\mathbf{Q}} \tilde{\mathbf{A}} \tag{7}$$

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{I} & \mathbf{A} & \cdots & \mathbf{A}^{N_c} & \cdots & \mathbf{A}^N \end{bmatrix}^T \tag{8}$$

$$\mathbf{X}(k) = \begin{bmatrix} 0 & \cdots & 0 \\ \mathbf{B} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \mathbf{A}^{N_c} & \cdots & \mathbf{B} \\ \vdots & \vdots & \vdots \\ \mathbf{A}^{N-1}\mathbf{B} & \cdots & \sum_{i=0}^{N-N_c} \mathbf{A}^i \mathbf{B} \end{bmatrix}. \tag{9}$$

$\mathbf{U}^* = \begin{bmatrix} \mathbf{u}_0^* & \mathbf{u}_1^* & \cdots & \mathbf{u}_{N_c-1}^* \end{bmatrix}$ – is the optimal sequence of control signals. The implementation of an MPC controller necessitates on-line solving of problem (6) for a given state $\mathbf{x}(k)$ in a receding horizon. This means that, at time k, only the first element $\mathbf{u}_0^*$ of the optimal input sequence is applied to the object and the remaining control actions $\mathbf{u}_1^*, \ldots, \mathbf{u}_{N_c-1}^*$ are extruded. At the next sampling time the optimization procedure is repeated for the new measured or estimated state $\mathbf{x}(k+1)$. Let $\mathbf{U}^*$ be the minimizing sequence of (6). The on-line MPC algorithm can be implemented in a few steps as presented below:

Algorithm 1 (On-line MPC controller)
1) At time k, measure (or estimate) the current system state $\mathbf{x}(k)$
2) Solve (6) to obtain $\mathbf{U}^*$.
3) Apply the first element of $\mathbf{U}^*$ to the plant
4) Update $k \leftarrow k+1$ and return to step 1.

This strategy can be computationally demanding for systems with fast sampling requirements and thus greatly limiting the scope of applicability to systems with relatively slow dynamics. Alternatively, rather than using the initial state $\mathbf{x}(k)$ to "update" the optimization problem (6) at each time k, the idea is to treat the state vector as a parameter vector and then solve problem (6) off-line for all realizations of $\mathbf{x}(k)$ within a predefined set of states using multi-parametric programming [22–25]. In this strategy, the parameter space is subdivided into characteristic regions, where the optimizer is given as an explicit piecewise affine (PWA) function of the parameters:

$$\mathbf{u}_0^*(\mathbf{x}_k) = \mathbf{K}_r\mathbf{x}_k + \mathbf{g}_r, \qquad \forall \mathbf{x}_k \in \mathbf{P}_r, \tag{10}$$

where $\mathbf{P}_r$ are polyhedral sets defined as:

$$\mathbf{P}_r = \{\mathbf{x} \in \mathfrak{R}^n | \mathbf{H}_r\mathbf{x} \le \mathbf{d}_r\}, \qquad r = 1, \dots N_r. \tag{11}$$

The resulting explicit MPC controller is completely characterized by matrix.

Matrices $\boldsymbol{K}_r$ and $\boldsymbol{g}_r$ define the gains of the controller, $\boldsymbol{H}_r$ and $\boldsymbol{d}_r$ describe boundary multi- regions in state-space. The implementation of the control law (10) is simply executed according to the following algorithm.

Algorithm 2 (Explicit MPC controller). *Given* $\mathbf{K}_r$, $\mathbf{g}_r$, $\mathbf{H}_r$, $\mathbf{d}_r$, N_r:
1) At time k, measure (or estimate) the current system state $\mathbf{x}(k)$
2) Search active region $(\mathbf{P}_r)$:
   for $r = 1$ to N_r
   $\mathbf{H}_r \times \mathbf{x}(k) - \mathbf{d}_r \le 0$
3) Apply the corresponding r-th control law:
   $\mathbf{u}^*(k) = \mathbf{K}_r \cdot \mathbf{x}(k) + \mathbf{g}_r$
4) Update $k \leftarrow k+1$ and return to step 1.

The off-line algorithm is definitely faster and, for this reason, it should be applied in the system where the on-line version of MPC cannot be implemented due to the computational requirements.

3. Design of MPC strategy for two-mass drive

3.1. Model of a two-mass system. The model of the plant can be non-linear or linear, which determines the use of linear or non-linear MPC. However, in the literature regarding control there are few works considering the use of the non-linear MPC algorithm. For example in [27] a comparative study describing the application of linear and non-linear MPC to a wave energy absorber is demonstrated. The authors pointed out different application problems of non-linear MPC. For example, the solution of this algorithm is not convex, which means that the global optimum of the function could not be found. Additionally, the computational complexity of non-linear MPC is much higher than of the linear one. For these reasons in the paper [27] experimental validation is not considered, only a simulation study is included.

In order to use MPC, the model of the plant should be specified. There are different models which can be used to analyse two-inertia systems, e. g. poles models, model with distributed parameters, the Raleigh model, and the inertia shaft-free model. The choice of the suitable model depends on its purpose. In the case of the control system, the selected model cannot be computation demanding because it is used to determine the control signal. For this reason in this work the inertia-shaft free model is selected [10]:

$$\frac{\mathrm{d}}{\mathrm{dt}}\omega_1 = \frac{1}{T_1}\big(m_e - m_s - d(\omega_1 - \omega_2)\big) \tag{12a}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\omega_2 = \frac{1}{T_2}\big(m_s - m_L + d(\omega_1 - \omega_2)\big) \tag{12b}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}m_s = \frac{1}{T_c}(\omega_1 - \omega_2) \tag{12c}$$

where: ω_1, ω_2 – motor and load speeds, m_e, m_s, m_L – electromagnetic, shaft and load torques, T_1, T_2 – mechanical time constant of the driving motor and load machine, T_c – stiffness time constant, d – vibration damping factor.

The load torque includes two components: external disturbance torque and the friction of the load machine.

The abovementioned model can be presented in the form of state equations (13).

The nominal parameters of the considered system are: $T_1 = T_2 = 0.2$ s, $T_c = 1.2$ ms, $d = 0$.

The control structure for electrical drives consists of two major loops. The torque control loop includes a power converter, the electromagnetic part of the motor, current sensor(s) and a torque controller. The parameters of the controller are tuned to provide sufficiently fast torque control. Theoretically, the time constant of the electromagnetic part of the motor can be compensated, only the small time constant of the power converter determines the properties of the torque control structure. If the ratio of the mechanical time constant to the delay imparted by the torque control loop is greater than 40–50 then in the speed control loop this delay can be ignored without consequences. If the dynamics of the torque loop is too big (so the ratio of the mechanical time constant to the time constant of the torque loop is smaller than 40), two possible solutions can be applied, two possible solutions can be applied. Firstly, the dynamics of the torque control loop can be included in the model of the plant and then used in the MPC algorithm. In this case, the particular properties of the torque loop are not important because its dynamics is taken into account. However, this approach will increase the computational complexity of the control algorithm. Contrary to this approach the dynamics of the speed control loop can decrease. In this case the overall performance of the system is worsened but the computational complexity does not change. Other solutions are described in [6].

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{bmatrix}\omega_1(t)\\ \omega_2(t)\\ m_s(t)\end{bmatrix}}_{\mathbf{x}}=\underbrace{\begin{bmatrix}\frac{\mathrm{d}}{T_1} & -\frac{\mathrm{d}}{T_1} & -\frac{1}{T_1}\\ -\frac{\mathrm{d}}{T_2} & \frac{\mathrm{d}}{T_2} & \frac{1}{T_2}\\ \frac{1}{T_c} & -\frac{1}{T_c} & 0\end{bmatrix}}_{\mathbf{A}}\cdot\begin{bmatrix}\omega_1(t)\\ \omega_2(t)\\ m_s(t)\end{bmatrix}+ \tag{13}$$

$$+\underbrace{\begin{bmatrix}\frac{1}{T_1}\\ 0\\ 0\end{bmatrix}}_{\mathbf{B}}\cdot m_e(t)+\underbrace{\begin{bmatrix}0\\ -\frac{1}{T_2}\\ 0\end{bmatrix}}_{\mathbf{B_d}}\cdot m_L(t)$$

3.2. Classical MPC. The control structure shown in Fig. 1a is considered. Vector $\mathbf{y}$ is defined as follows:

$$\mathbf{y}=\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}\omega_2-\omega^{ref}\\ m_s-m_L\end{bmatrix}. \tag{14}$$

Vector $\mathbf{y}$ results in the minimization of the difference between the motor and reference speed (y_1). The impact of the load torque on the drive system is reduced by y_2 (14).

For the purpose of computing the explicit MPC control law and to guarantee offset-free control, the basic drive system model (12) needs to be augmented with additional state variables which take into account the effect of the load disturbance m_L and the reference speed ω^{ref}. This can be achieved by defining the following augmented model:

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{bmatrix}\mathbf{x}\\ m_L\\ \omega^{ref}\end{bmatrix}}_{\mathbf{x}_c}=\underbrace{\begin{bmatrix}\mathbf{A} & \mathbf{B_d} & 0\\ 0 & 0 & 0\\ 0 & 0 & \mathbf{A_\omega}\end{bmatrix}}_{\mathbf{A}_c}\cdot\begin{bmatrix}\mathbf{x}\\ m_L\\ \omega^{ref}\end{bmatrix}+\underbrace{\begin{bmatrix}\mathbf{B}\\ 0\\ 0\end{bmatrix}}_{\mathbf{B}_c}m_e \tag{15}$$

where: $\mathbf{x}_c$, $\mathbf{A}_c$, $\mathbf{B}_c$ are the new state-space vector and matrices describing the model used in predictive controller. In the considered problem the MPC controller requires information about the evolution of the load torque and the reference signal ω^{ref}.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}m_L(t)&=0\\ \frac{\mathrm{d}}{\mathrm{d}t}\omega^{ref}(t)&=A_\omega\omega^{ref}\end{aligned} \tag{16}$$

where A_ω is the coefficient describing the reference trajectory dynamics.

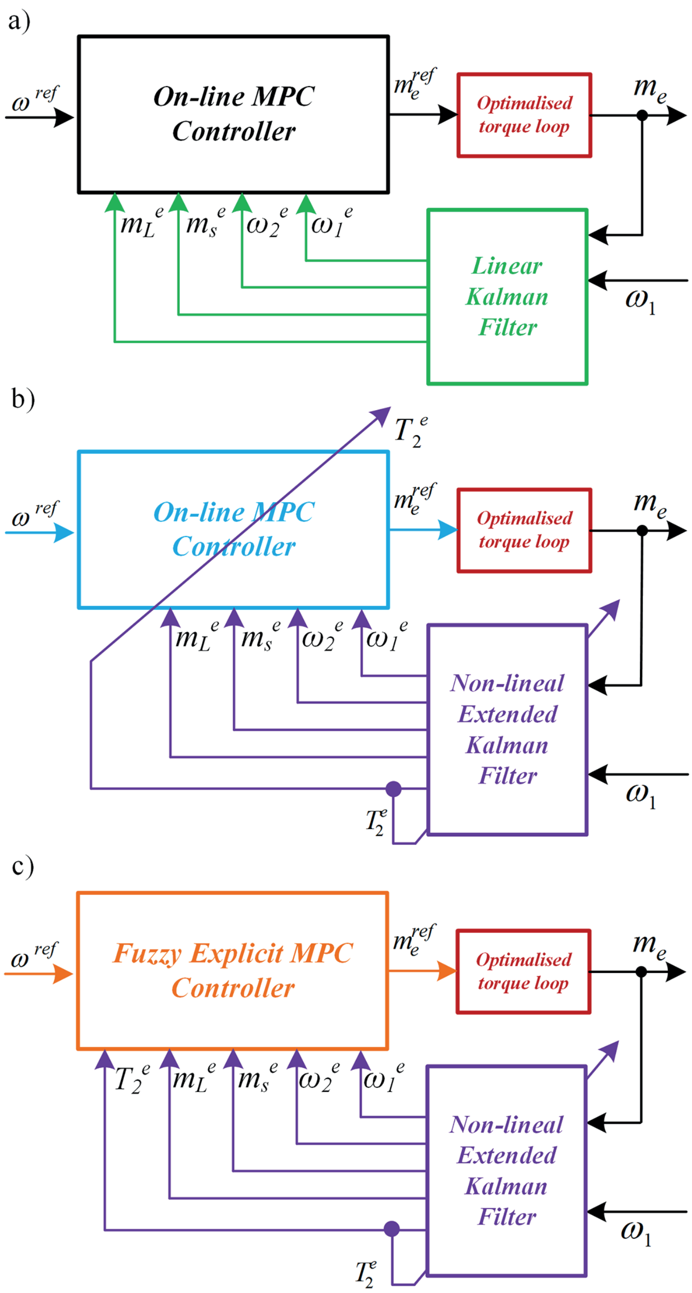

Fig. 1. Considered control structures: a) on-line MPC, b) on-line adaptive MPC, c) Fuzzy Explicit MPC

Since their future behaviour is usually unknown, the following assumption is made:

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega^{ref}(t)=0\rightarrow A_\omega=0. \tag{17}$$

It is assumed in the paper that the changes of the reference value are not known. For this reason, A_ω is equal to zero.

However, it should be stated that when the dynamics of the reference signal is known, better performance can be achieved.

The weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ have the following sizes: $\mathbf{Q} \in \mathfrak{R}^{2\times 2}$, $\mathbf{R} \in \mathfrak{R}^{1\times 1}$. Matrix $\mathbf{Q}$ is defined as follows:

$$\mathbf{Q} = \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix}. \tag{18}$$

While the primary goal of the MPC controller is to provide good load speed tracking performance, the control system must respect the physical and safety limitations of the electrical drive variables during operation. For the drive in question, the following constraints are considered: electromagnetic torque and shaft torque constraints:

$$-3 \le m_e \le 3 \tag{19a}$$

$$-1.5 \le m_s \le 1.5. \tag{19b}$$

Constraints (19) incorporate the physical limits of the drive power converter and are introduced in order to ensure that the produced motor torque (19a) does not exceed its maximum allowable value. The shaft torque constraint in (19b) places a limit on the admissible shaft twists that are likely to occur during the drive transients. The damping of torsional vibrations is essential for safety and reliable long-term operation. The magnitude of the oscillation cannot exceed the metal fatigue point in order to guarantee the maximal lifetime of the shaft.

The limitation of the shaft torque can be set according to the following formula [28, 29]:

$$m_s^{\max} = \left(\frac{T_2}{T_1 + T_2} \cdot m_e^{\max} \right). \tag{21}$$

Taking into account the abovementioned equation the control problem can be defined as follows:

$$\begin{gathered} \min_{m_{e0}^{ref},\, m_{e1}^{ref}} \left\{ \sum_{p=1}^{N} \begin{pmatrix} q_{11}\big(\omega_1(p) - \omega^{ref}(p)\big)^2 + \\ q_{22}\big(m_s(p) - m_L(p)\big)^2 \end{pmatrix} + \sum_{k=0}^{N_c-1} \big(R \cdot m_e(k)^2\big) \right\} \\ |m_e| \le 3 \\ |m_s| \le 1.5. \end{gathered} \tag{22}$$

There are no analytical formulas which can be used to determine the values of the $\mathbf{Q}$ matrix elements. Taking into account its form (4), it can be observed that MPC controller minimizes two opposite components. Component $(\omega^{ref} - \omega_1)$ shortens the transients time of the system. Contrary to this, component $(m_s + m_L)$ minimises the difference between torques, which eliminates the torsional vibrations but influences the dynamics in a negative way. The ratio between q_{11} and q_{22} determines system properties.

In the study, the state and control prediction horizons are set to $N = 8$, $N_c = 2$, respectively.

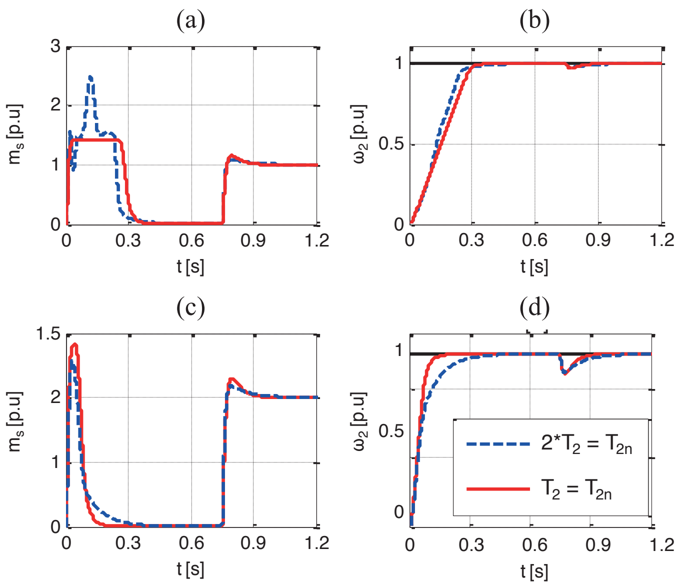

Fig. 2. Simulation study of the system states: shaft torque (a, c), and load speed (b, d) for different value of T_2 and different reference signal: nominal (a, b) and low 0.2 (c, d)

The transients of the system (simulation results) with a standard MPC controller for the nominal and increased value of the time constant of load machine T_2 (used in the model of the plant in MPC) are shown in Fig. 2. After start-up to the system the nominal load torque is applied at the time $t = 0.7$ s.

It can be concluded from the presented transients that the MPC controller does not work properly for the changed parameters of the plant. The limitations of the shaft torque are validated (Fig. 2 blue line), which can be dangerous in some applications. Therefore, other solutions are necessary.

3.3. On-line adaptive MPC. The basic solution for the plant with changeable parameters is to combine the adaptive control with the MPC methodology. In order to implement the adaptive MPC, the knowledge of the present value of the changeable parameter is necessary. The proposed idea of an adaptive on-line MPC (Fig. 1b) relies on the use of the estimated value of T_2^e provided by the Kalman Filter (described in appendix 1), to update the model used in the MPC methodology. It is required for the application of the on-line MPC, which needs to compute in every calculation step, and then in minimization of the cost function (2, 22).

It is computationally demanding and requires a very fast microprocessor. Additionally, in order to obtain the same shape of the load speed an additional change of matrix $\mathbf{Q}$ is proposed.

$$\mathbf{Q} = \begin{bmatrix} \frac{145}{T_2^{e^{-1}}} + 10 & 0 \\ 0 & 1.7 \end{bmatrix} \tag{23}$$

The following formula is set experimentally. For the system with the increased value of the mechanical time constant of the

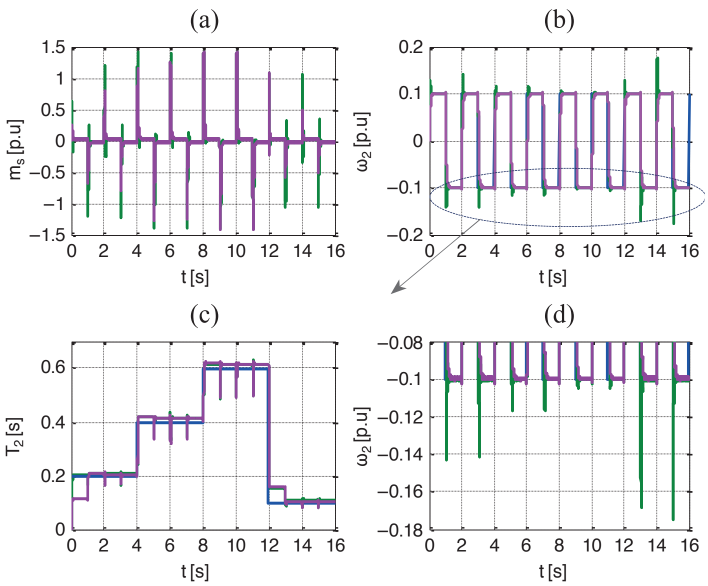

Fig. 3. Transients of the system with and without adaptation of **Q** matrix: a) torsional torque, load speeds (b, d), variation of T_2 (c) magenta/green line system with/without adaptation

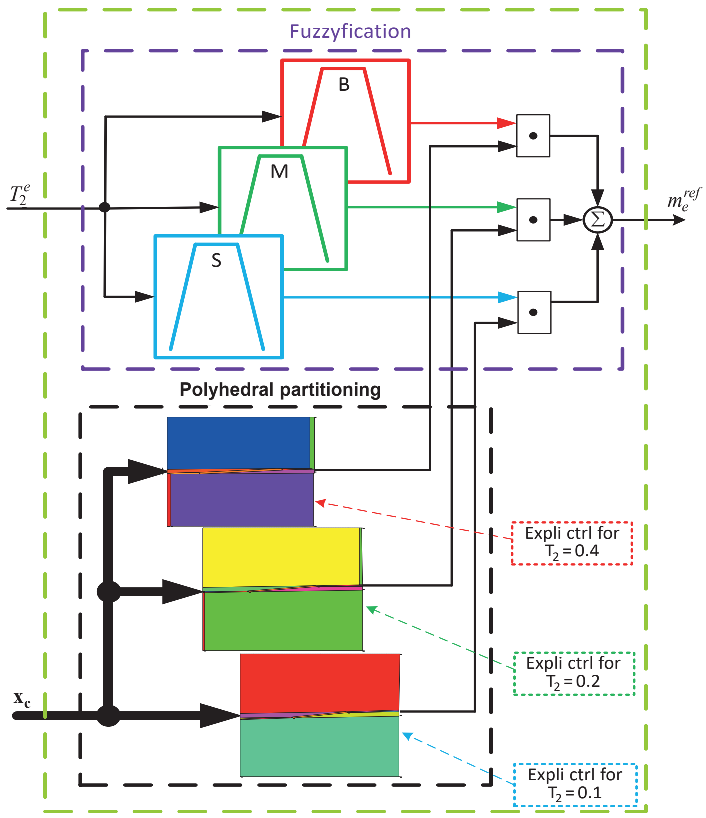

Fig. 4. The proposed structure of the controller

load machine, the weight responsible for the minimization of the component with control errors should be minimized.

In order to show the properties of the proposed adaptive online MPC controller, simulation tests are performed. Transients of the load speeds are shown in Fig. 3.

The magenta line indicates the system with the adaptation mechanism of **Q**, while the green line refers to a standard MPC. The variation of T_2 is shown in Fig. 3c. In both cases the model of the plant is updated in MPC. It can be concluded from the presented transients that changing matrix **Q** reduces the overshoot from the load speed almost completely despite the variation of T_2. The system in which only the model of the plant is updated cannot eliminate overshoot from transients.

3.4. Fuzzy explicit MPC controller. The general block diagram of the proposed Fuzzy Explicit MPC controller is shown in Fig. 1c. The internal structure of fuzzy MPC is presented in Fig. 4. Two main parts can be distinguished in the proposed concept. The first part consists of three explicit controllers generated for the different values of changeable parameter T_2. The fuzzy supervisory system is the second part of the proposed controller. It generates the resulting control signal as a fuzzy combination between three internal control signals of specific controllers.

Three internal controllers are designed for the following values of the time constant of the load machine: $T_2 = 0.5 \cdot T_{2N}$, $T_2 = 1 \cdot T_{2N}$, $T_2 = 2 \cdot T_{2N}$. It means that three explicit control laws are generated. The hypothetical fragments of polyhedral regions are shown in Fig. 5. Three different weight matrices are used in

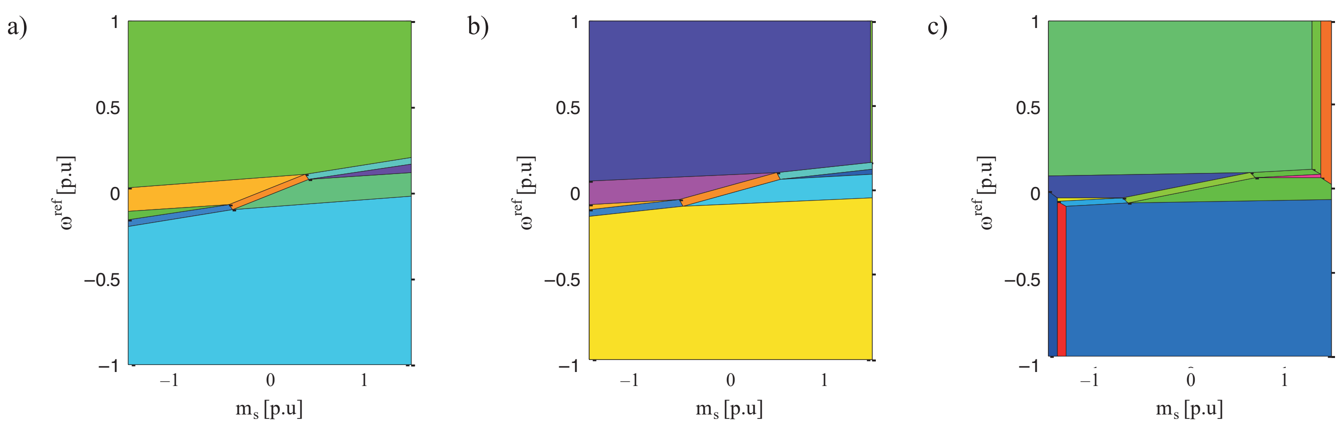

Fig. 5. Fragments of polyhydral regions between shaft torque and reference speed, a) $T_2 = 0.5T_{2N}$, b) $T_2 = T_{2N}$, c), $T_2 = 2T_{2N}$

the designing procedure in order to obtain the transients of the load speed similarly to the on-line adaptive MPC:

$\mathbf{Q}_0 = \mathrm{diag}(51; 3.8)$ for $T_2 = 0.5T_{2N}$,
$\mathbf{Q}_1 = \mathrm{diag}(71; 3.8)$ for $T_2 = T_{2N}$
$\mathbf{Q}_2 = \mathrm{diag}(101; 3.8)$ for $T_2 = 2T_{2N}$.

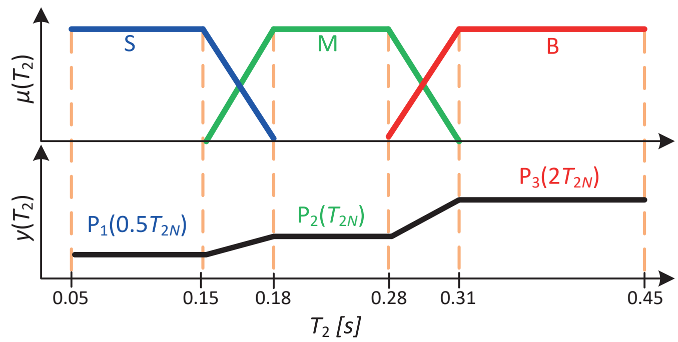

Fig. 6. Parameters of the fuzzy block

The different values of $\mathbf{Q}$ ensure similar transients of load machine for different value of parameter T_2.

In order to produce the resulting control signal the fuzzy system is applied. It is based on the trapezoidal membership function. The division of the space and resulting fuzzy system characteristics is shown in Fig. 6.

The presented supervisory fuzzy system has a simple structure. It can be further extended by using more fuzzy sets with different shapes, e.g. Gaussian, or even the inclusion of other input. All this modification will decrease the difference between the outputs of the on-line and off-line versions. However, it will also increase the computational complexity of the whole control strategy. For this reason only three membership functions are used in the fuzzy systems. As it will be proven, even for such a simple fuzzy system, differences between the on-line and fuzzy MPC structures are not significant.

4. Simulation test

The proposed control structures were tested under a variety of simulation tests. The sampling period of the optimized torque control loop is set to 100 μs, and speed control loop to 1 ms (MPC controller and the Kalman filter). Systems with three values of T_2 are tested, namely: $T_2 = 0.5T_{2N}$; $T_2 = T_{2N}$ and $T_2 = 2T_{2N}$.

Firstly, the system with an adaptive on-line controller is tested. The transients of the system are shown in Fig. 7, 8.

The drive system works under cyclic reversal. The reference signal is set to ±1 (Fig. 7) and ±0.1 (Fig. 8). The transients of the electromagnetic and shaft torques for the nominal value of the reference speed are shown in Fig. 7a, c, respectively. The constraints set on the system are not validated. The electromagnetic torque (red line) for the case of $T_2 = 2T_{2N}$ is under the constrained value in order to keep the shaft torque limit.

Next, the control structure with a fuzzy MPC controller was investigated. The driving system works under identical condi-

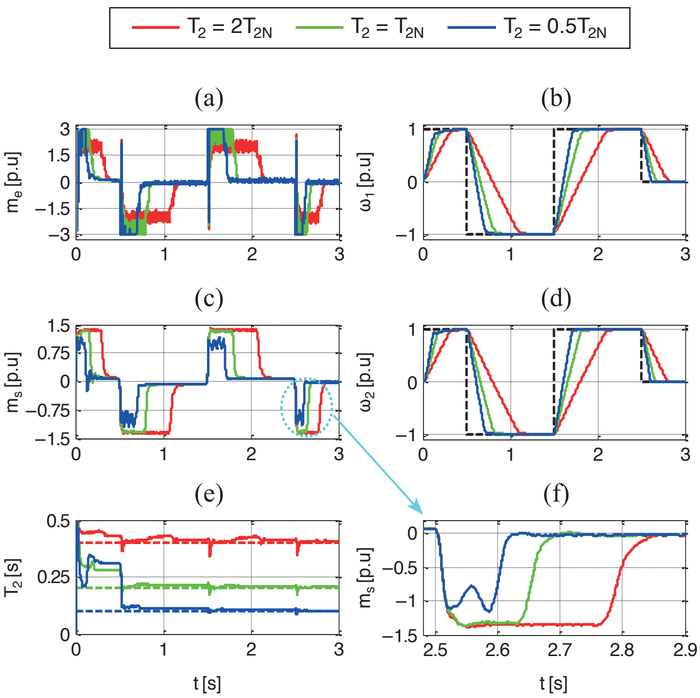

Fig. 7. Transients of the system states: electromagentic torque (a), motor speed (b), shaft torque (c, f), load speed (d) and estimated value of time constant of the load machine (e) for nominal reference speed for adaptive on-line MPC controller

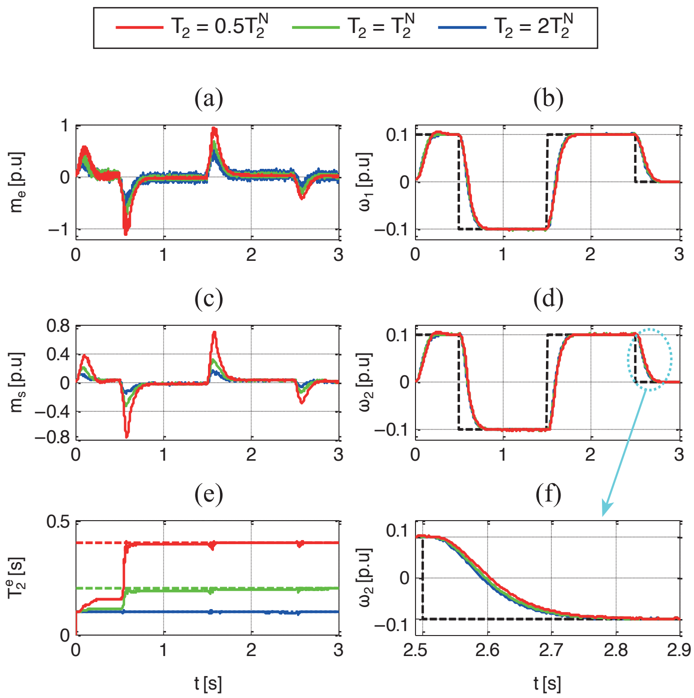

Fig. 8. Transients of the system states: electromagentic torque (a), motor speed (b), shaft torque (c, f), load speed (d) and estimated value of time constant of the load machine (e) for low reference speed for adaptive on-line MPC controller

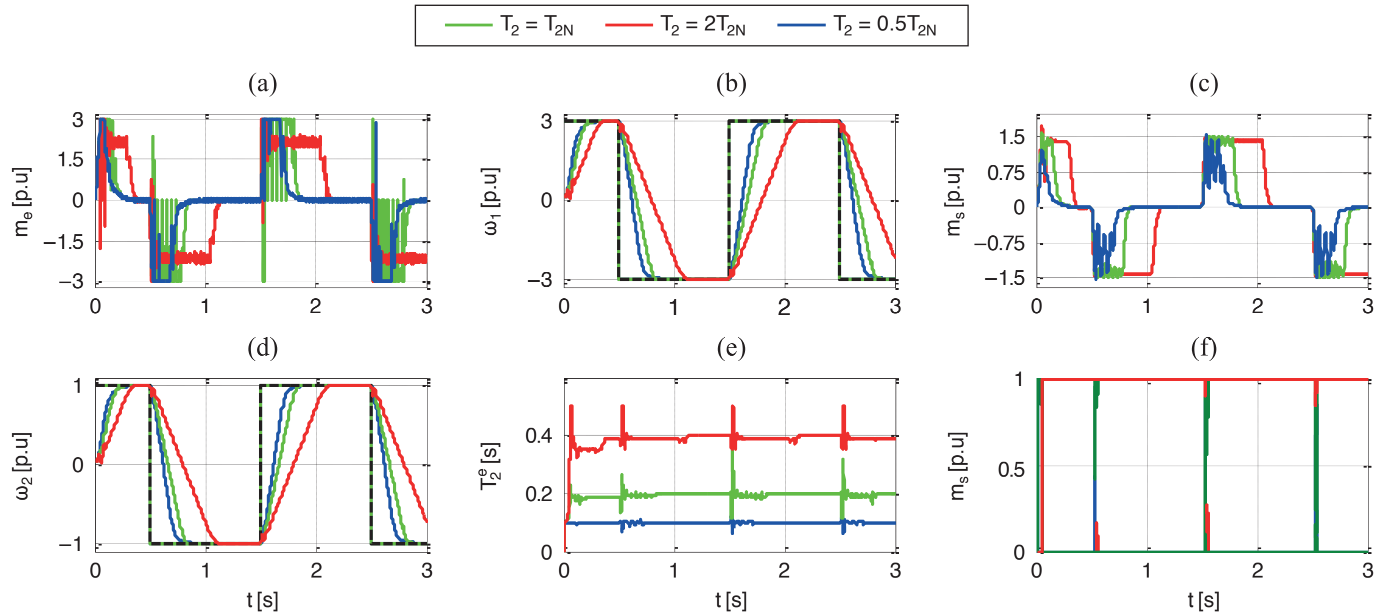

Fig. 9. Transients of the system states: electromagnetic torque (a), motor speed (b), shaft torque (c), load speed (d), estimated value of time constant of the load machine (e) and output of fuzzy system (f) for nominal reference speed for fuzzy MPC controller

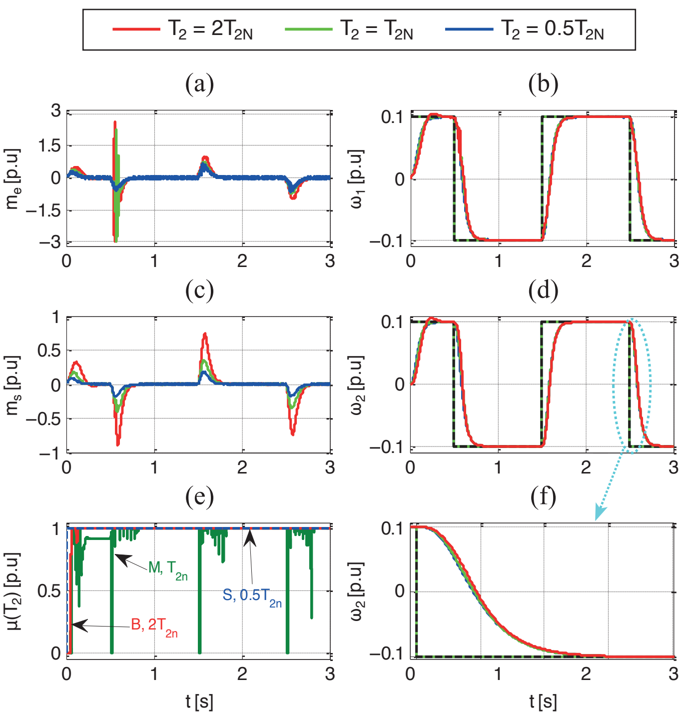

Fig. 10. Transients of the system states: electromagentic torque (a), motor speed (b), shaft torque (c), load speed (d, f) and output of fuzzy system (e) for low reference speed for fuzzy MPC controller

tions as the previously described system. The transients of the drive are shown in Fig. 9 (nominal reference speed) and Fig. 10. (low reference speed).

The obtained transients are very similar to the results obtained previously. For the bigger values of the reference speed the constraints are not validated. The load speed transients for the smaller values of reference speed are almost identical. The drive system with a fuzzy MPC controller works properly.

The system transients taken for a small reference value are presented in Fig. 8. In this case the system is below the constraints. The presented transients allow to observe that there is a small difference between the load speed transients, which confirms the proper work of the adaptive algorithm. The estimates of changeable parameter T2 are shown in Fig. 7.e and 8.e.

Additionally, as a comparative criterion, ITAE integral of time-weighted absolute error is selected. The computed values are located in Table 1. All systems have similar values, which means that the load speed transients have a similar shape.

Table 1.
Values of ITAE

T_2	$0.5T_{2N}$	T_{2N}	$2T_{2N}$
ITAE on-line	0.0213	0.0221	0.0231
ITAE fuzzy off-line	0.0218	0.0219	0.0220

Next the two analysed systems are compared with respect to the linear changeable mechanical time constant of the load machine. The transients obtained for both systems are shown in Fig. 11. The shaft torque and load speed transients are shown in Fig. 11a, b. Despite the changes of T_2 the load speed has no overshoot. The enlarged transients of those two variables are presented in Fig. 11c, d. They have almost identical shapes. The values of the control index ITAE for both systems calculated for every second are shown in Fig. 11e. They have similar values. The sums of this index for the whole period have the following values: 0.1799 and 0.1791 for on-line and Fuzzy MPC controllers. The real and estimated values of T_2 are shown in Fig. 11f.

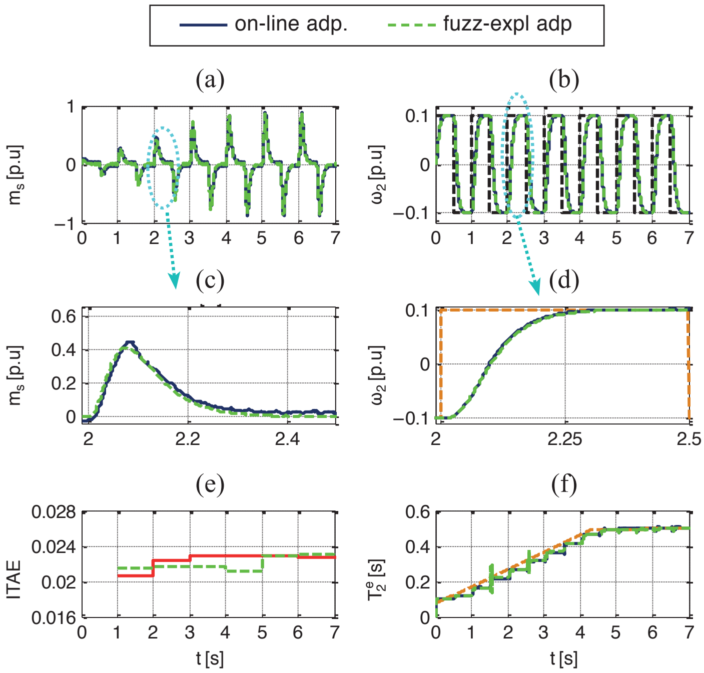

Fig. 11. Transients of the systems with adaptive on-line MPC and fuzzy MPC: shaft torque (a, c) load speed (b, d) ITAE (e), real and estimated value of T_2 (f)

5. Experimental tests

A pilot-scale laboratory set-up composed of two 500 W DC-motors connected by a long steel shaft (length 600 mm, diameter 7 mm) was used in the tests. This system is characterized by anti-resonance frequency from 7.3 to 10.2 Hz. The driving motor and the load machine are controlled by a dSpace 1103 control platform via two separate power converters. The inner torque control loop consists of a PI torque controller, power converter (transistor *H*-bridge), armature winding and an armature current sensor. The parameters of the torque controller are tuned to ensure fast aperiodic step response of the torque control loop. In the experimental set-up the motor speed measurement is made with an incremental encoder (36000 pulses per rotation). The sampling period of the optimized torque control loop is set to 0.1 ms, and the sampling time of *MPC* controller is equal to 1 ms also for the Kalman filter. The experimental system with two values of T_2 is tested $T_2 = T_{2N}$ and $T_2 = 2T_{2N}$. The system works under a reversal cycle.

Firstly, an attempt is made to implement an adaptive on-line MPC controller. Unfortunately, with the sampling period of 1 ms the computations cannot be done on-line. The system was implemented with a 6 ms sampling period but in this case the performances are very poor and contain oscillations. Therefore, these transients are not demonstrated in the paper.

Then, the fuzzy MPC controller is implemented with the sampling period of 1ms. The system is tested for two values of T_2 for the small and nominal values of the reference speed. The obtained transients are presented in Fig. 12. First the system for the value of the reference speed equal to 0.25 is investi-

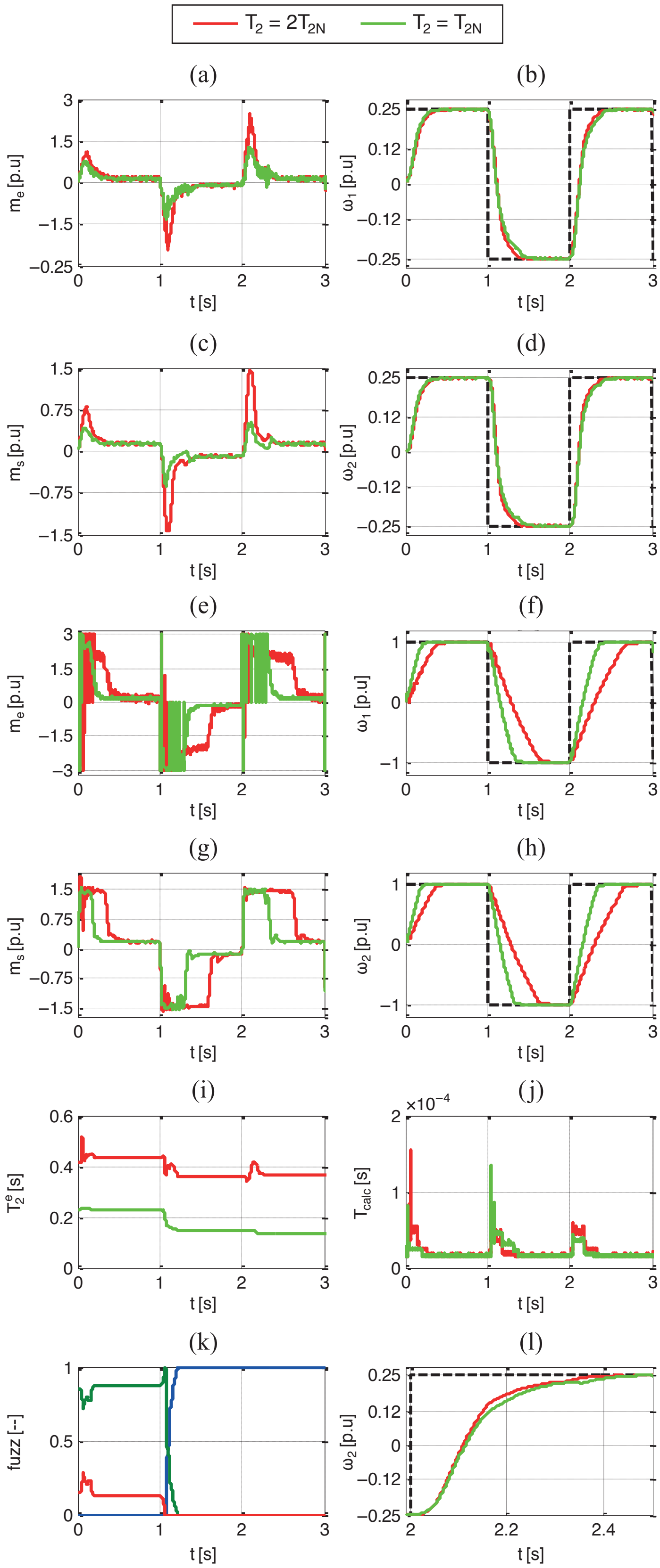

Fig. 12. Transients of the system states: electromagentic torque (a, e), motor speed (b, f), shaft torque (c, g), load speed (d, h, l) and estimated value of time constant of the load machine (i), computation time (j), output of fuzzy system (k) for nominal (e, f, g, h) and small value (a, b, c, d, l, i, j, k), of the reference signal for fuzzy MPC controller

gated. The transients of electromagnetic and shaft torques for the two values of T_2 are shown in Fig. 12a, c respectively. In this case the system does not reach limitation values. The load speed transients are presented in Fig 12d, e. It is clearly visible that these transients are very similar. The estimated value of T_2 and the output of the fuzzy system are shown in Fig. 12i, k. The computation time is demonstrated in Fig. 12j. For the most complicated case the computation takes 0.15 ms – a far smaller value as compared to the on-line adaptive MPC controller.

Then the system performances for the nominal value of the reference speed is tested. The system transients are shown in Fig. 12e, f, g, h. The system works properly and the constraints are not validated.

6. Conclusions

In the paper the novel fuzzy MPC controller for the drive system with an elastic joint is proposed.

Based on the theoretical considerations, simulations and experimental tests performed, following remarks can be formulated:

- The proposed fuzzy MPC controller ensures similar transient shape to the adaptive MPC controller.
- The values of the performance index calculated for different values of the inertia are almost identical for the fuzzy MPC controller. Contrary to this, the values computed for the adaptive controller vary in a wider range.
- The total implementation cost of the fuzzy controller is approximately six times smaller than the adaptive MPC solution. So in the used rapid prototyping system, the adaptive controller cannot be implemented with a sufficient sampling period.
- The torsional vibrations are effectively suppressed in the elastic drive system in both control structures. Despite parameter changes the shaft torque limits are not validated. So, from the theoretical point of view, both structures work correctly.

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Appendix: extended nonlinear Kalman filter

In the work, the Kalman filter described in detail in [26] is applied. The original state vector of the system is extended by the load torque and inverse of changeable parameters of the drive–time constant of the load machine:

$$\mathbf{X_K}(t) = \left[\omega_{1e}(t) \;\; \omega_{2e}(t) \;\; \omega_{se}(t) \;\; \omega_{Le}(t) \;\; \frac{1}{T_{2e}}(t)\right]. \quad \text{(A1).}$$

The estimated value T_{2e} is used to modify the MPC controller as well as to change the covariance matrices [26]. The state equation can be formulated as follows:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{dt}}\mathbf{X_K}(t) &= \mathbf{A_K}\left(\frac{1}{T_2}(t)\right) + \mathbf{B_K}u(t) + \mathbf{w}(t) = \\ &= f_K\big(\mathbf{x_K}(t), u(t)\big) + \mathbf{w}(t) \qquad \text{(A2).}\\ \mathbf{y_K}(t) &= \mathbf{C_K}\mathbf{X_K}(t) + \mathbf{v}(t).\end{aligned}$$

Matrices of the system are presented below:

$$\mathbf{A_K}\left(\frac{1}{T_2}(t)\right) = \begin{bmatrix} 0 & 0 & \frac{-1}{T_1} & 0 & 0 \\ 0 & 0 & \frac{1}{T_2}(t) & \frac{-1}{T_1}(t) & 0 \\ \frac{1}{T_c} & \frac{-1}{T_c} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{B_K} = \begin{bmatrix} \frac{1}{T_1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(A3).}$$

$$\mathbf{C_K} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The values of the covariance matrices are set with the help of a global optimization technique based on genetic algorithm and direct search method.