

LMS Algorithm Step Size Adjustment for Fast Convergence

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In the areas of acoustic research or applications that deal with not-precisely-known or variable conditions, a method of adaptation to the uncertainty or changes is usually necessary. When searching for an adaptation algorithm, it is hard to overlook the least mean squares (LMS) algorithm. Its simplicity, speed of computation, and robustness has won it a wide area of applications: from telecommunication, through acoustics and vibration, to seismology. The algorithm, however, still lacks a full theoretical analysis. This is probably the cause of its main drawback: the need of a careful choice of the step size – which is the reason why so many variable step size flavors of the LMS algorithm has been developed.

This paper contributes to both the above mentioned characteristics of the LMS algorithm. First, it shows a derivation of a new necessary condition for the LMS algorithm convergence. The condition, although weak, proved useful in developing a new variable step size LMS algorithm which appeared to be quite different from the algorithms known from the literature. Moreover, the algorithm proved to be effective in both simulations and laboratory experiments, covering two possible applications: adaptive line enhancement and active noise control.

Keywords: signal processing, adaptive algorithms, least mean squares, active noise control, system identification.

1. Introduction

It is a long time since adaptive techniques and algorithms were a domain of control science researchers only. In acoustics, for example, application of adaptive techniques includes (but is not limited to) adaptive beamforming, adaptive channel equalization, acoustic feedback reduction in digital hearing aids, many flavors of acoustic signal processing including underwater acoustics and active noise control, speech processing, and acoustic echo cancellation. The last example is very common nowadays, as adaptive echo cancelers are used in mobile phones. All these applications require a method of adaptation. The most popular adaptation algorithms are Recursive Least Squares (RLS) and Least Mean Squares (LMS). The former, although more complicated, is better known from a theoretical point of view. Nevertheless, due to its qualities, it is the latter that is a common choice for many applications.

LMS algorithm has been used for many years. Its features include simplicity, speed of computation, and robustness (proved by HASSIBI *et al.*, 1993; 1996). Therefore, the LMS algorithm covers a wide area

of use, including telecommunication, acoustics, vibration, automatic control, and even seismology (HAYKIN, 2002). The main drawback of the LMS algorithm is that it requires a careful choice of the only parameter used for adjusting its behavior, called step size (see Eq. (7)). A too large step size gives a fast response to plant changes but results in a large excess mean square error (MSE), and may even cause loss of convergence. A too small step size degrades tracking capabilities of the algorithm.

An optimal step size, giving a trade-off between the speed of convergence and residual error, depends on the power of the input data. Therefore, the main, commonly used modification of the LMS algorithm is a normalization of the step size. This leads to the normalized LMS (NLMS) algorithm, in which the step size is scaled (divided) by an estimated power of the input data. However, even the NLMS algorithm requires a “base” step size choice.

The problem of an automatic step size adjustment has been addressed in the literature for many years. The earliest work reported is probably the paper by HARRIS *et al.*, dated by 1986 (HARRIS *et al.*, 1986).

The topic was popular in the 90-ies (see KWONG, JOHNSTON, 1992; EVANS *et al.*, 1993; MATHEWS, XIE, 1993; ABOULNASR, MAYYAS, 1997), and returned in the recent years (see (HWANG, LI, 2009), but also conference papers (BISMOR, 2008; 2009; LIU *et al.*, 2009; SUN *et al.*, 2009; ZOU, ZHAO, 2009; WANG *et al.*, 2009; BI *et al.*, 2010)). The problem has not been solved, which probably can be explained by an inherent complexity of the LMS algorithm. In fact, in the well-known adaptive filtering textbook (HAYKIN, 2002), we read: “(...) although the LMS filter is very simple in physical terms, its mathematical analysis is profoundly complicated because of its highly nonlinear nature. Indeed, despite the extensive effort that has been expended in the literature to analyze the LMS filter, we still do not have a direct mathematical analysis of its stability and steady-state performance, and quite probably we never will.”

The algorithm proposed by HARRIS *et al.* in (HARRIS *et al.*, 1986) uses a diagonal matrix instead of a single step size. Each element of the diagonal is allowed to vary between chosen minimum and maximum values. The step sizes are decreased if the value of the gradient of the mean square error alternates in sign, and increased if the sign is constant over some number of successive samples. However, the idea to use individual step sizes for each filter tap did not have many followers – see (ANG, FARHANG-BOROJENY, 2001).

Another large group of variable step size algorithms is based on the idea of one but changable step size for all filter taps. The origin of the idea is connected with the observation that the step size should be small when the adaptive filter is near its optimum value and large otherwise. However, simple algorithms selecting the step size based on the magnitude of the estimation error did not prove to be effective (MATHEWS, XIE, 1993). Therefore, the main effort was directed towards the selection of the criterion of when to increase and when to decrease the step size.

A few variable step size algorithms known from the literature update the step size based on the gradient of the squared error with respect to the step size itself (MATHEWS, XIE, 1993; ANG, FARHANG-BOROJENY, 2001). An update of the step size in a step manner, where the step size is a multiple of the squared error (KWONG, JOHNSTON, 1992), seems to be a simplification of this technique. Numerous algorithms use a function of the squared error as a basis to the step size adjustment (LIU *et al.*, 2009; SUN *et al.*, 2009).

There are also algorithms that use a correlation between two consecutive error samples (or a function of this correlation) as a main factor for the step size update (ABOULNASR, MAYYAS, 1997; ZOU, ZHAO, 2009). A relatively new conference paper presents an approach where the step size is not updated in a step fashion but is rather computed in each iteration “from anew” (WANG *et al.*, 2009). Finally, there are more

sophisticated algorithms combining a couple of approaches like (HWANG, LI, 2009) which uses the step size controlled by the squared norm of the weighted-averaged gradient vector.

The common denominator of the algorithms discussed above is that they update the step size based on a function of the error. In this paper, the author presents an approach where the step size is updated based solely on the input data. The algorithm was developed after discovering a new, weak condition for the LMS filter convergence. The algorithm proved to be effective in simulations, as well as in practice of active noise control, providing fast convergence and a small excess mean square error.

The rest of the paper is organized as follows. The LMS algorithm together with the convergence criterions known from the literature is summarized in Sec. 2. The new convergence condition for the LMS adaptive filter is derived in Sec. 3. The algorithm for the step size adjustment is described in Subsec. 3.2. Simulation experiments for an adaptive line enhancer are described in Sec. 4. Finally, results of active noise control (ANC) experiments with the new algorithm are described in Sec. 5. The paper is concluded in Sec. 6.

2. The LMS algorithm

2.1. Statistical filtering

Consider the statistical filtering problem presented in Fig. 1 (HAYKIN, 2002). The input signal $u(n)$ is composed of discrete values. This signal is filtered with the filter $W(z^{-1})$ which is a linear, discrete time, transversal filter. The result of this filtration, the filter output $y(n)$, is compared with the desired signal values $d(n)$ to produce the error $e(n)$. The statistical filtering problem is about how to choose the filter coefficients to minimize – in some sense – the error.

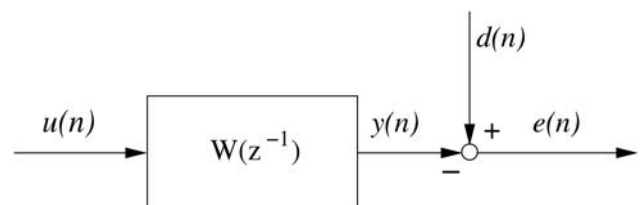


Fig. 1. Statistical filtering problem.

Without a loss of generality we will assume that $u(n)$ and $y(n)$ are discrete time random variables with a zero mean value, and are jointly stationary in a wide sense. Furthermore, we will assume that $W(z^{-1})$ is a finite impulse response (FIR) filter. Therefore, the output value at a discrete time n can be expressed as:

$$y(n) = \sum_{i=0}^{N-1} w_i^* u(n-i), \quad (1)$$

where w_i is the i -th element of the filter impulse response (called also filter coefficient or filter tap), N is the filter length and $*$ denotes a complex conjugate.

After defining:

$$\mathbf{w} = [w_0, w_1, \dots, w_{N-1}]^T, \quad (2)$$

$$\mathbf{u}(n) = [u(n), u(n-1), \dots, u(n-N)]^T, \quad (3)$$

Eq. (1) can be expressed as:

$$y(n) = \mathbf{w}^H \mathbf{u}(n) = \mathbf{u}^H(n) \mathbf{w}, \quad (4)$$

where superscript H denotes Hermitian transposition. The error can, therefore, be expressed as:

$$e(n) = d(n) - y(n) = d(n) - \mathbf{w}^H \mathbf{u}(n). \quad (5)$$

If the minimization criterion for the error signal is the most commonly used *mean square value of the error* $e(n)$ (MSE), the well-known solution of the optimization is the Wiener-Hopf equation (HAYKIN, 2002):

$$\mathbf{w}_o = \mathbf{R}^{-1} \mathbf{p}, \quad (6)$$

where $\mathbf{R} = E(\mathbf{u}(n)\mathbf{u}^T(n))$ is the autocorrelation matrix of the input signal, $\mathbf{p} = E(\mathbf{u}(n)d(n))$ is the cross-correlation vector between the input and the desired signal, and E denotes an expectation operator.

As the Wiener-Hopf equation (6) involves inversion of a possibly large autocorrelation matrix, the equation is rarely used in practical applications. Instead, many applications use the idea of a *local iterative descent* resulting in a recursive algorithm, updating filter coefficients in a step-by-step manner (HAYKIN, 2002). The following three recursive algorithms are the most prominent: the method of steepest descent, the least mean squares (LMS) algorithm, and the recursive least squares (RLS) algorithm.

The method of steepest descent requires an exact knowledge of statistical properties of the input and the desired signal in the form of an autocorrelation matrix \mathbf{R} and cross-correlation vector \mathbf{p} . The RLS algorithm, on the other hand, is much more computationally expensive. Therefore, it is the LMS algorithm that gained the biggest attention and the highest number of practical implementations.

2.2. Convergence of the LMS algorithm

The LMS algorithm updates filter coefficients according to the following formula:

$$\widehat{\mathbf{w}}(n+1) = \widehat{\mathbf{w}}(n) + \mu \mathbf{u}(n) e^*(n), \quad (7)$$

where $\widehat{\mathbf{w}}(n)$ is the vector of estimated filter coefficients in step n , μ is the step size and $e^*(n)$ is the complex conjugate of the estimation error, which is now expressed by:

$$e(n) = d(n) - \widehat{\mathbf{w}}^H(n) \mathbf{u}(n). \quad (8)$$

Assuming $\widehat{\mathbf{w}}(0) = \mathbf{0}$, the filter coefficients in the n -th step can be expressed as:

$$\widehat{\mathbf{w}}(n) = \mu \sum_{i=0}^{n-1} \mathbf{u}(i) e^*(i). \quad (9)$$

Substituting Eq. (8) into the above equation gives:

$$\widehat{\mathbf{w}}(n) = \mu \sum_{i=0}^{n-1} \mathbf{u}(i) \left(d^*(i) - \widehat{\mathbf{w}}^H(i) \mathbf{u}^*(i) \right) \quad (10)$$

and reveals the previously mentioned highly nonlinear nature of the LMS algorithm (HAYKIN, 2002). Thus, a detailed mathematical analysis of the LMS algorithm is not currently known. However, it is apparent from the above equation, and is even more clear from Eq. (7), that in convergence of the LMS algorithm the step size μ plays the main role.

There are two main results concerning conditions of convergence of the LMS algorithm. Both of them require stronger assumptions than those described at the beginning of Subsec. 2.1. In practice, both sets of assumptions contain, among others, the *small step size assumption*, which means that the step size should tend to zero as the discrete time increases: $\mu \xrightarrow{n \rightarrow \infty} 0$. The first condition, attributed to BUTTERWECK (1995), states that the convergence necessary condition can be expressed as:

$$0 < \mu < \frac{2}{\lambda_{\max}}, \quad (11)$$

where λ_{\max} is the largest eigenvalue of the autocorrelation matrix \mathbf{R} (for a detailed derivation see, e.g. (HAYKIN, 2002)). The second main result, presented by SAYED (2003), states that the LMS algorithm is convergent in the mean square provided that:

$$\sum_{i=1}^N \frac{\mu \lambda_i}{1 - \mu \lambda_i} < 2, \quad (12)$$

where λ_i are the eigenvalues of the autocorrelation matrix \mathbf{R} .

The small step size assumption describes very well the situation where the adaptive filter is near its optimal value. Particularly in the case of variable step size modifications of the LMS algorithm, the step size is required to be small near the end of convergence to minimize the excess mean square error. However, during the beginning of adaptation it is desired to keep the step size large to allow for a fast adaptation. This is also the case of many practical applications when the filter should adapt very fast after rapid plant changes. As an example, industrial hall, room, or office ANC application can be considered, when the ANC filter should adapt very fast after changes in the environment caused by persons walking in the room. Also, speech enhancement in a noisy environment is an application that requires a fast adaptation – see e.g. (LATOS, PAWELCZYK, 2010).

3. New convergence condition and variable step size algorithm

3.1. Convergence condition

Let us assume now the input and desired signals $u(n)$ and $d(n)$ are discrete and finite valued. For the simplicity and without a loss of generalization, let us assume that their values are real. Substituting Eq. (8) into Eq. (7) yields:

$$\widehat{\mathbf{w}}(n+1) = \widehat{\mathbf{w}}(n) + \mu \mathbf{u}(n) \left[d(n) - \widehat{\mathbf{w}}^T(n) \mathbf{u}(n) \right]. \quad (13)$$

After regrouping, the above equation becomes:

$$\widehat{\mathbf{w}}(n+1) = [\mathbf{1} - \mu \mathbf{u}(n) \mathbf{u}^T(n)] \widehat{\mathbf{w}}(n) + \mu \mathbf{u}(n) d(n). \quad (14)$$

Notice that Eq. (14) can be viewed as a discrete, non-stationary system state equation:

$$\mathbf{x}(n+1) = \mathbf{A}(n) \mathbf{x}(n) + \mathbf{B}(n) d(n), \quad (15)$$

with the state matrix:

$$\mathbf{A}(n) = \mathbf{1} - \mu \mathbf{u}(n) \mathbf{u}^T(n). \quad (16)$$

From the control theory (KACZOREK, 1993) it follows that for a discrete, *stationary* system instability it suffices that:

$$\text{tr}(|\mathbf{A}|) = \sum_{i=1}^N |a_{ii}| > N, \quad (17)$$

where a_{ii} is the i -th element on the diagonal of the state matrix \mathbf{A} , and N is the number of states (or the size of matrix \mathbf{A}). The reason is that for the stability of a discrete system it is necessary that all the eigenvalues of the system state matrix \mathbf{A} are within the unit disk. On the other hand, from the linear algebra it follows that the trace of a matrix is equal to the sum of its eigenvalues. Thus, a trace greater than the size of a matrix means that at least one eigenvalue of a matrix is greater than one.

Combining Eqs. (16) and (17), the divergence sufficient condition for the adaptive LMS filter can be formulated as:

$$\begin{aligned} & \text{tr}(|\mathbf{1} - \mu \mathbf{u}(n) \mathbf{u}^T(n)|) \\ &= \sum_{i=0}^{N-1} |1 - \mu u^2(n-i)| > N \quad \text{sufficiently long.} \end{aligned} \quad (18)$$

The term “sufficiently long” has been added because of the nonstationarity of the system under consideration. In such a system, if the condition in Eq. (18) is true for a limited number of samples but turns false afterwards, the filter may still remain convergent. Therefore, the convergence necessary condition can be expressed as:

$$\sum_{i=0}^{N-1} |1 - \mu u^2(n-i)| \leq N \quad \text{almost always.} \quad (19)$$

It must be emphasized that the above condition has been obtained without any meaningful assumptions, especially without the small step assumption. In fact, it was not even assumed that the signals are random or stationary. Note also that the condition regards only the filter input $u(n)$; the error values are not considered. However, this condition defines the largest step size – the step size which may be used for a fast adaptation at the beginning of (a phase of) a convergence. In practical implementations, to obtain acceptable excess mean square error the step size must be even smaller. Intuitively, this means that the condition in Eq. (19) must always be true.

3.2. Variable step size algorithm

Performed simulations revealed that the step sizes chosen close to a violation of the condition from Eq. (19) result in large excess mean square errors and even in a sudden filter coefficients growth, similar to parameter explosion in the RLS algorithm. Therefore, it may be reasonable to strengthen this condition in the following way:

$$\begin{aligned} & \forall_{0 \leq i \leq N-1} |1 - \mu u^2(n-i)| \leq 1 \\ & \Rightarrow \sum_{i=0}^{N-1} |1 - \mu u^2(n-i)| \leq N. \end{aligned} \quad (20)$$

Please note that when the left hand side inequality is fulfilled, the right hand side inequality holds, but the opposite relation is not necessarily true.

The left hand side of Eq. (20) can be expressed as:

$$\forall_{0 \leq i \leq N-1} -1 \leq 1 - \mu u^2(n-i) \leq 1. \quad (21)$$

Thus, for real input data, considering that the above inequality should hold throughout the whole adaptation, the final condition is:

$$\forall_n 0 \leq \mu \leq \frac{2}{u^2(n)}. \quad (22)$$

The above condition specifies the upper limit on the step size when a fast adaptation is needed. The condition is very easy to check in practical implementations. Particularly, when the data comes from A/D converters, the condition can be hard-coded and does not even have to be checked against in each adaptation step. It may also serve as a basis for a preliminary choice of the step size. Unfortunately, it proved to be only a very weak condition.

3.3. Further refinements

The derivation of the new LMS convergence condition directed the attention towards Eq. (19). For further development, the convergence function has been defined as:

$$J(n) = \frac{1}{N} \sum_{i=0}^{N-1} |1 - \mu(n)u^2(n-i)|. \quad (23)$$

Based on this function and condition in Eq. (19) a simple algorithm has been constructed:

$$\text{if } J(n) \geq \epsilon \text{ then } \mu(n+1) = 0.99\mu(n). \quad (24)$$

The above algorithm was used, with $\epsilon = 0.99$, for a simulation of estimation of the parameter of the autoregressive process given by HAYKIN (2002):

$$u(n) = -au(n-1) + v(n). \quad (25)$$

The algorithm performed better than LMS and NLMS algorithms, giving a fast adaptation and small excess MSE.

Next, the same algorithm has been applied to simulations of adaptive line enhancer (ALE) in Fig. 2. The input for the ALE was a recorded speech signal contaminated with four sinusoids – see spectrogram in Fig. 3. The adaptive filter length N and the delay Δ were both equal to 100. The experiments with different *constant* step sizes revealed a great robustness of ALE to the step size changes: good results were obtained for $0.0005 \leq \mu \leq 0.002$. However, the algorithm Eq. (24) failed to produce good results, ending with $\mu = 0.0022$. Furthermore, changing ϵ to different values resulted only in a slightly better, but still unsatisfactory performance.

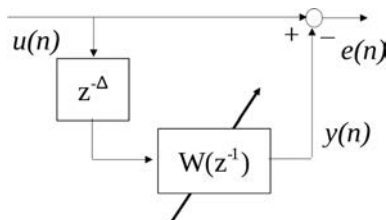


Fig. 2. Adaptive Line Enhancer.

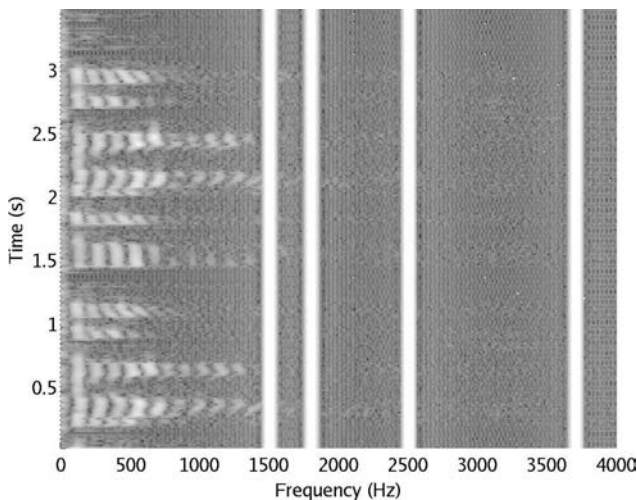


Fig. 3. Spectrogram of ALE input signal.

The experiments with the ALE drew the attention to the fact that it is not the value of the convergence function $J(n)$ in Eq. (23), that is most important for the convergence of the algorithm, but rather the absolute value of $1 - J(n)$. Many experiments with different filter lengths showed also that the longer the filter is, the smaller the value of $|1 - J(n)|$ should be. Therefore, the function describing a desired relationship between the filter length and the $|1 - J(n)|$, for filter lengths greater than 10, was chosen experimentally as:

$$f(N) = \frac{1}{N} - \frac{10}{N^2}, \quad (26)$$

and the algorithm was modified to the following form:

$$\text{if } |1 - J(n)| \geq f(N) \text{ then } \mu(n+1) = 0.99\mu(n). \quad (27)$$

The choice of the function $f(N)$ is not critical: any monotonously decreasing function with similar values in the range of interesting filter lengths can be used.

The algorithm in Eq. (27) was performing well in Matlab/Simulink simulations. However, it appeared to be too time-consuming for real-time applications. Particularly, the summation in Eq. (23) was not well-suited for real-time computations. Therefore, the rectangular summation window from $i = 0$ to $N - 1$ was substituted by an exponential window with the exponent factor equal to 0.9:

$$J_e(n) = 0.9J_e(n-1) + 0.1|1 - \mu(n)u^2(n)|. \quad (28)$$

Moreover, as a decrease in the step size with the multiplication factor 0.99 (i.e. $\mu(n+1) = 0.99\mu(n)$ in Eq. (27)) sometimes appeared to be too slow to prevent the filter from going unstable, the vector of multiplication factors was introduced:

$$\mathbf{mv} = [0.99 \ 0.97 \ 0.95 \ 0.93 \ 0.9]^T. \quad (29)$$

In the final algorithm, the multiplication factor was selected to be a next smaller vector element if the condition for changing the step size occurred in a current, as well as in a previous adaptation step; otherwise the first element of the vector \mathbf{mv} was used.

4. Simulation experiments

To test the performance of the algorithm described above in active noise control applications the simulation experiments described in this section were performed. First, to simulate the nonstationarity, models of electro-acoustic paths for different error microphone locations were obtained. The models were in the form of FIR filters with 300 parameters. Next, the ANC system model with no acoustic feedback, presented in Fig. 4, was assumed. A perfect modeling ($\hat{S}(z^{-1}) = S(z^{-1})$) of the secondary path was assumed for these simulations.

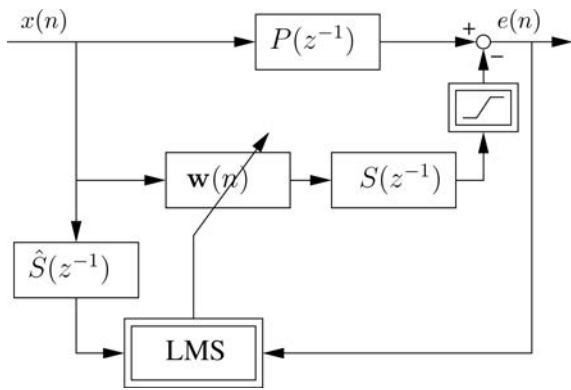


Fig. 4. Block diagram of ANC system used during the simulations.

Nonstationarity was modeled as a switched system, with switches among four different models occurring at regular intervals. The magnitudes of three exemplary transfer functions of the models of the secondary path used during the simulation are presented in Fig. 5. The basic input signal was composed of two sinusoids, at $\omega = 0.2\pi$ and $\omega = 0.888\pi$, embedded in white noise (BISMOR, 2009). The levels of sinusoids and of the noise was altered for different experiments. The length of the ANC filter was 128.

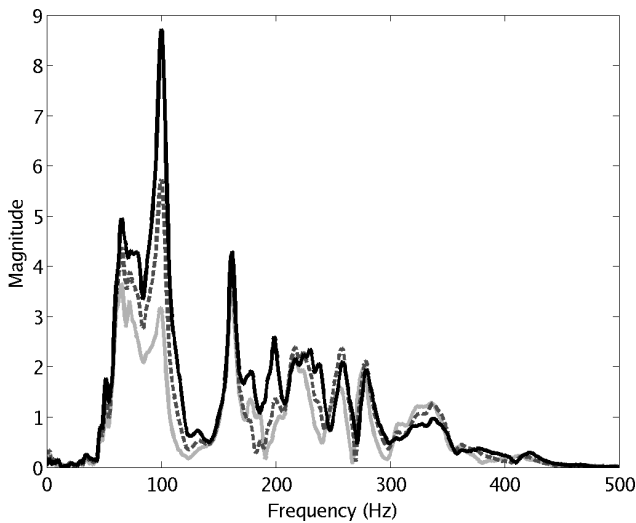


Fig. 5. Magnitude of the transfer function of the three models used during the simulation (for clarity, the fourth model has not been included).

The goal of the simulation experiments was to compare the behavior of the NLMS algorithm with the proposed algorithm. The comparison not only considered the attained attenuation, but also took into the consideration the variance of the attenuated signal during the whole experiment. In fact, the latter criterion was considered superior because the development was concentrated on a fast adaptation. Thus, the main criterion calculated for the comparison of both algorithms was the mean square error (MSE) defined as:

$$D(n) = \sum_{i=0}^{L-1} e^2(n-i), \quad (30)$$

where L is the length of the window used for smoothing.

The proposed variable step size algorithm proved to be superior to the NLMS algorithm in spite of many trials to optimally set the NLMS step size. The algorithm responded very fast after each change of the secondary path transfer function. The attenuation was always slightly better than in the case of the NLMS. The example of attenuation for the NLMS and the proposed algorithm is presented in Fig. 6. The values of the MSE calculated over the whole simulation time for four different experiments are presented in Table 1.

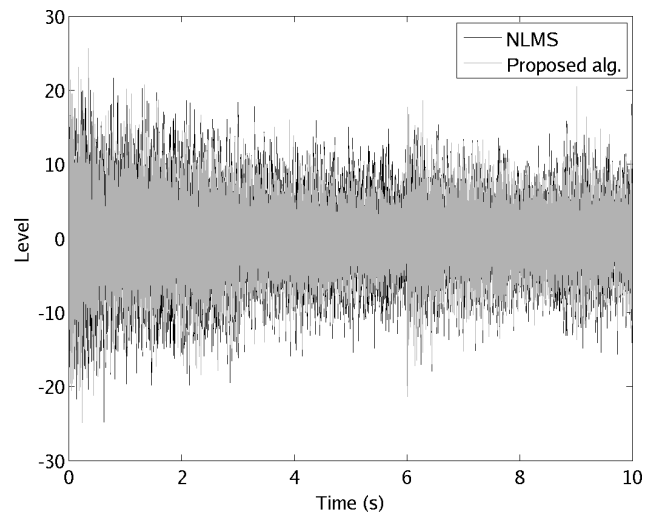


Fig. 6. Time plot of the error signal with ANC algorithms.

Table 1. The MSE in simulations.

| Experiment No. | NLMS | VS LMS |
|----------------|------|--------|
| 1 | 28.6 | 25.1 |
| 2 | 55.5 | 48.3 |
| 3 | 28.7 | 24.9 |
| 4 | 32.9 | 28.7 |

5. Laboratory experiments

Good simulation results encouraged an application of the new algorithm in a real environment. The application of ANC in an industrial hall has been chosen as one with requirements for a fast adaptation. Moreover, as a good model of the secondary path is usually crucial for a room ANC (ELLIOTT, 2001), online adaptation of the secondary path, with a random noise source and additional adaptive filter for improvement of the convergence rate (KUO, MORGAN, 1996), was performed.

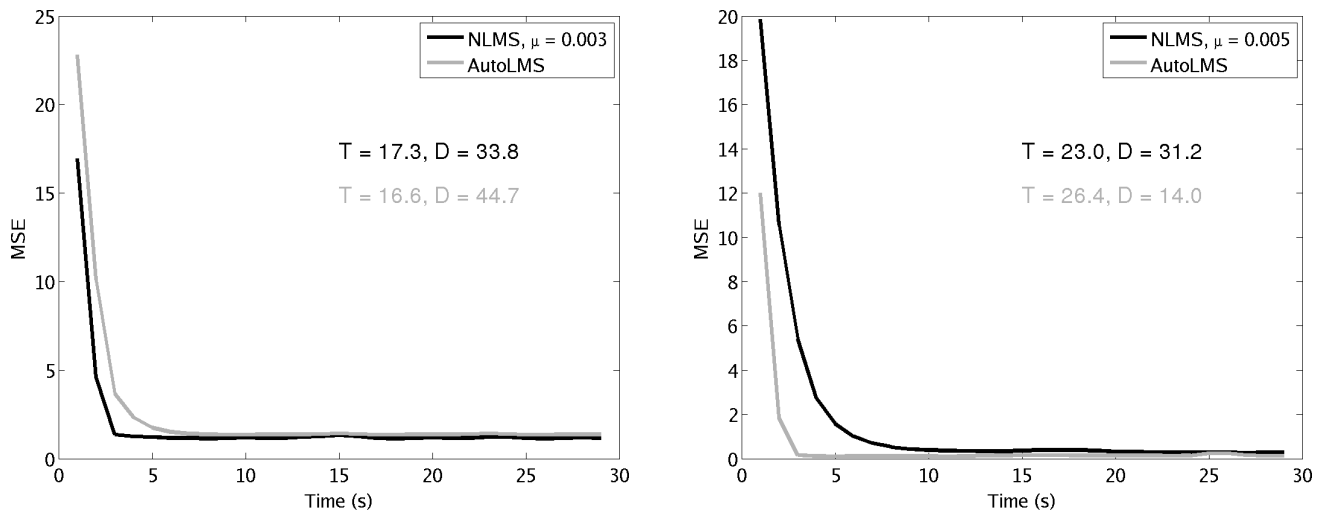


Fig. 7. MSE for experiments with 500 Hz tone.

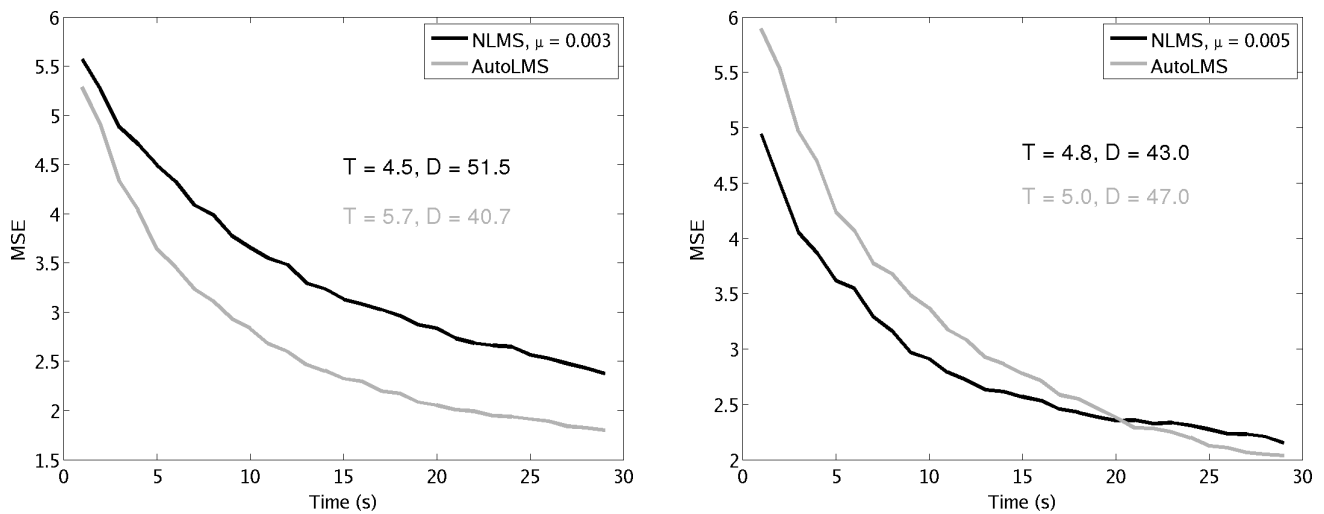


Fig. 8. MSE for experiments with band-limited, 500–600 Hz noise.

The laboratory setup was built around a DSpace DS1104 board, and contained all the necessary equipment like microphones, preamplifiers, filters, amplifiers, and loudspeakers. The algorithms were implemented in C and downloaded into the board. To control the system, a C++ application was built using a DSpace Clib.

Many different noise signals were tested, starting from single tones, through wide-band signals, ending with signals recorded from a real, noisy environment. For each signal, three experiments were performed: two with the NLMS algorithm with different step sizes, and one with the proposed algorithm. The step sizes chosen for the NLMS algorithm were 0.003 and 0.005. For each experiment maximum attenuation after 25 seconds was calculated, as well as the MSE (Eq. (30)).

Generally, the results can be categorized into three groups. The first group contains the signals for which the proposed algorithm gave better results when com-

pared with the NLMS parametrized with one step size, while it was inferior to the NLMS parametrized with the other step size. This group is referred to as “neutral”, although it still proves one advantage of the proposed algorithm: no necessity to choose the step size, or to change it if the noise changes. The MSE for two examples of this group are presented in Figs. 7 and 8. The first one is the attenuation of 500 Hz tone. In this case, the NLMS algorithm is better for the 0.003 step size, but worse for the 0.005 one. The differences are not substantial but noticeable. The second is the attenuation of white noise limited to 500–600 Hz band. In this case, the NLMS is better with the 0.005 step size, but worse with the 0.003 one. There were about 50% of “neutral” cases during the laboratory tests.

The second group contains those signals for which the proposed algorithm was superior to the NLMS with both step sizes. Two examples of this group are presented in Figs. 9 and 10. The first one is the atten-

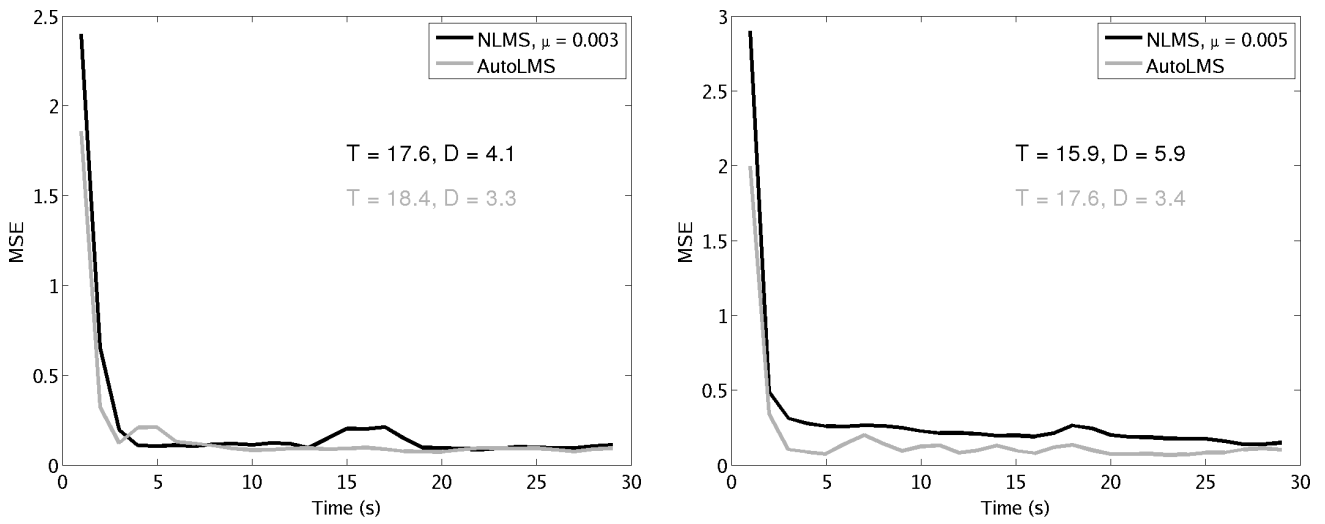


Fig. 9. MSE for experiments with 140 Hz tone.

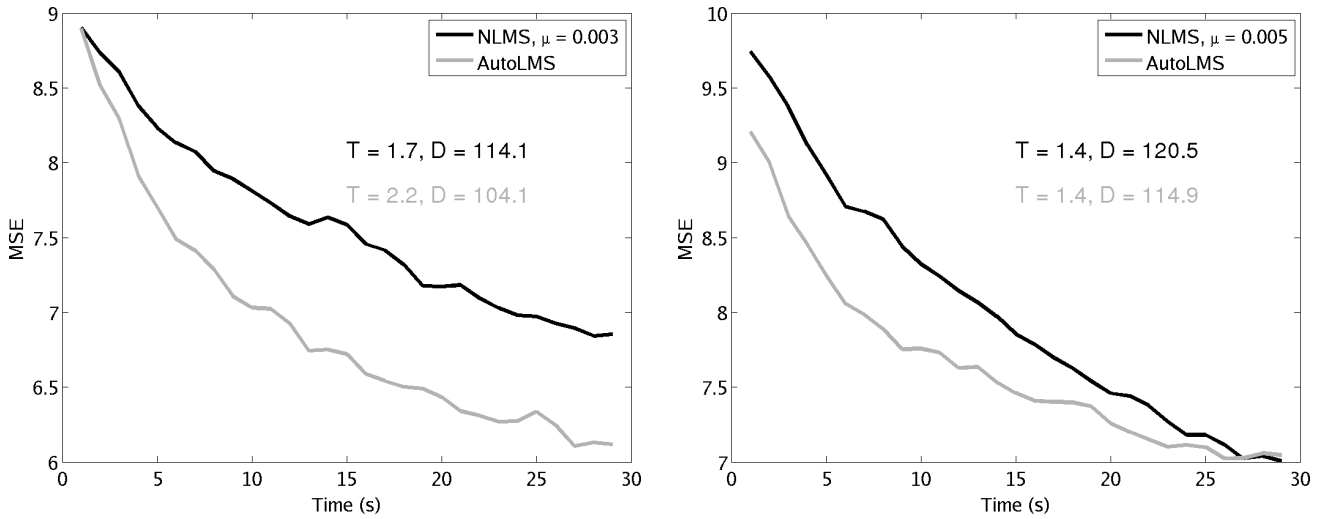


Fig. 10. MSE for experiments with power generator noise.

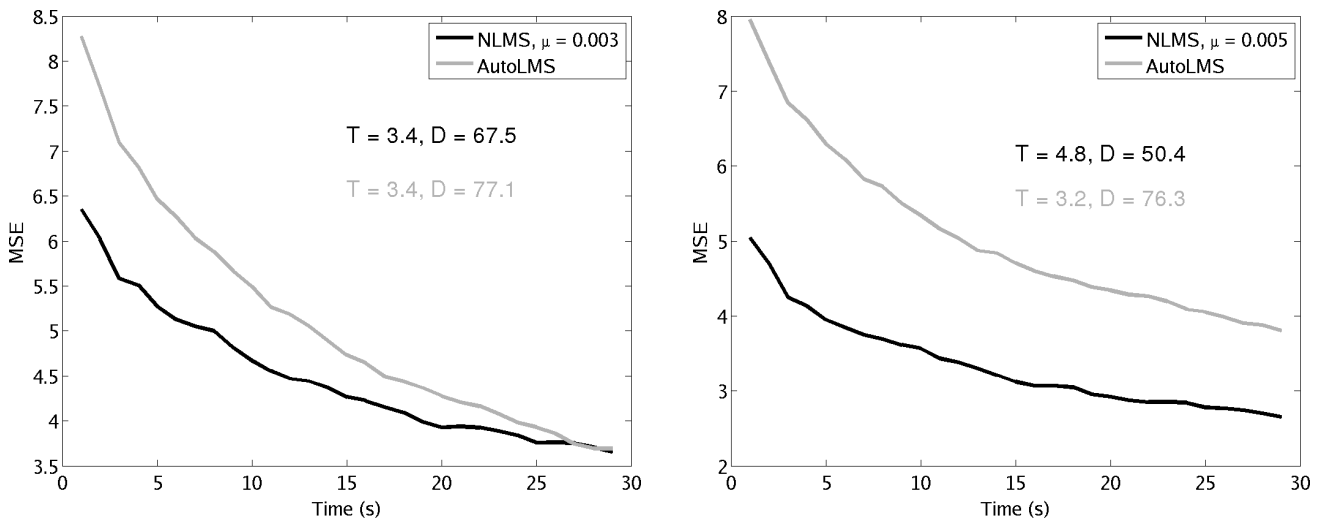


Fig. 11. MSE for experiments with band-limited, 200–300 Hz noise, with sine at 250 Hz.

uation of a 140 Hz tone, the second is the attenuation of noise recorded near a power generator. In those cases, the proposed algorithm usually provides not only a faster convergence, but also a higher attenuation. This group contains about 37% results of attenuation of single tones and about 43% results of attenuation of complex signals.

There were also cases where the NLMS algorithm won. They were more numerous while attenuating single tones (about 13%) than while dealing with complex signals (only two cases among 38 tested complex signals). If they happened, either the attenuation at the end of the experiment was comparable for both of the algorithms, or it was a case of a signal difficult to attenuate. One example of this group is presented in Fig. 11.

6. Conclusions

The LMS algorithm is nowadays a standard adaptation algorithm for many practical applications. Its drawback is the lack of a complete mathematical analysis and rules for choosing the step size, particularly, when a fast adaptation is essential. The paper presents an innovative analysis of the LMS convergence. Based on this analysis, a new algorithm for an automatic step size adjustment has been proposed. The algorithm proved to be effective both in simulations and in real experiments of active noise control.

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