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MATHEMATICAL MODELLING AND STATIONARY CHARACTERISTICS OF A TWO-PHASE FLUIDISED-BED BIOREACTOR WITH EXTERNAL AERATION

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A mathematical model for a two-phase fluidised bed bioreactor with liquid recirculation and an external aerator was proposed. A stationary nonlinear analysis of such a bioreactor for an aerobic process with double-substrate kinetics was carried out. The influences of a volumetric fraction of solid carriers in the liquid phase, the rate of active biomass transfer from the biofilm to the liquid, the concentration of carbonaceous substrate, the mean residence time of the liquid and the efficiency of the external aerator on the steady state characteristics of the bioreactor were described. A method for determination of the minimal recirculation ratio related to oxygen demand and fluidised bed conditions was presented. On the basis of the obtained results, it is possible to choose reasonable operating conditions of such plants and to determine constraints, while considering acceptable concentrations of a toxic substrate being degraded.

Keywords: bioreactor, fluidised-bed, external aerator, biofilm, steady states

1. INTRODUCTION

An application of fluidised bed bioreactors in biotechnology and the environment protection engineering has been known for many years. The application of fluidised-bed bioreactors in wastewater treatment can be traced back to the 1940s in England (Rodgers and Zhan, 2003). Both two-phase liquid-solid fluidised-bed bioreactors and three-phase apparatuses are commonly used in industry. These bioreactors are applied to the microbiological degradation of toxic organic compounds (Onysko et al., 2002; Tang and Fan, 1987a; Tang et al. 1987b; Wisecarver and Fan, 1989; Worden and Donaldson, 1987), the microbiological nitrification process (Dunn et al., 1983), and the synthesis of some drugs, e.g. penicillin, oxytetracycline, and other organic compounds (Park et al., 1984). A review of numerous applications of fluidised bed bioreactors is given by Schügerl (1997). Airlift bioreactors with fine particles are also treated by many authors as special cases of fluidised-bed reactors (Tang and Fan, 1987a; Tang et al., 1987b).

Advantages of applications of fluidised bed bioreactors result from the following features:

- High overall biomass concentration, which is caused by immobilisation of microorganisms on the carrier particles.
- The possibility to operate at high liquid flow rates above the velocity of microorganism washout from the liquid phase.
- A large interphase surface between the liquid phase and the biofilm.

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- The usage of carriers in a form of small size particles allows continuous replenishment of the fluidised bed and enables to control the biofilm thickness (Tang and Fan, 1987a).
- Lack of biomass overgrowth and bed clogging.

Depending on the apparatus design and the number of phases, fluidised bed bioreactors may work in three different ways. The first method consists in feeding the apparatus with liquid phase and gas. Hence, we have a three-phase gas-liquid-solid fluidised bed bioreactor. The third phase is constituted by bioparticles. The second method is based on the application of an external aerator working in a recirculating loop. The liquid phase circulates between the bioreactor and the aerator. The bioreactor is fed only with an oxygenated liquid phase. Then we deal with a two-phase liquid-solid fluidised bed. The third method consists in the application of airlift bioreactors. In these apparatuses, three phases coexist and the circulation velocity of liquid and fine particles depends mainly on the intensity of riser aeration. Other types of fluidised-bed bioreactors are also described in the literature. Olivieri et al. (2010) presented a novel bioreactor, which can be treated as a hybrid of a two-phase liquid-solid fluidised bed bioreactor with a two-phase gas-liquid airlift apparatus.

Information concerning the application of a two-phase fluidised bed bioreactor with a cooperating external aerator can also be found in the papers written by Dunn et al. (1983) and Nicolella et al. (2000). Two-phase fluidised-bed bioreactors are also applied in experimental studies on the determination of effective diffusion coefficients in biofilms (Beyenal et al., 1997) and the application of polymers as biofilm carrier (Sevillano et al., 2008). In 2000, in Europe and the USA, there were more than 80 two-phase fluidised-bed bioreactors in industrial use (Rodgers and Zhan, 2003). Dunn et al. (1983), while describing advantages of two-phase fluidised-bed bioreactors with a cooperating external aerator, list two features, which differentiate these apparatuses from three-phase bioreactors. These are described below.

- Separation of the aerator outside the fluidised bed increases the flexibility of the installation by adjusting liquid oxygenation intensity to the bioreactor size and the oxygen demand in a given aerobic process.
- It is assessed that shear stress acting on biofilm is lower in a two-phase fluidised bed than in the three-phase one. It results in lower biomass losses.

A further disadvantage of three-phase fluidised beds, in which very fine carrier particles are used, is a tendency for gas bubbles coalescence. It leads to a decrease of mass transfer between liquid and gas (Dunn et al., 1983).

Research concerning mathematical modelling of fluidised-bed bioreactors is related first of all to three-phase systems (Choi et al., 1999; Hsieng and Lin, 2005). Investigations of dynamics and steady state behaviour of three-phase fluidised-bed bioreactors are also described in the literature (Olivieri et al., 2011; Russo et al., 2008).

Despite the aforementioned features of two-phase fluidised bed bioreactors with an external aerator, till now there has been no information in the literature about the determination of their steady states and their parametric dependence. Therefore, investigations, whose objective was to study stationary properties of these bioreactors, were undertaken. The purpose of these investigations was to obtain information of practical, i.e. process and design, importance. Representative results of these investigations based on our own mathematical model of the analyzed installation are presented below.

2. MATHEMATICAL MODEL OF AEROBIC PROCESS IN TWO-PHASE FLUIDISED BED

Fig. 1 presents a schematic diagram of the analysed installation together with the nomenclature of basic process parameters.

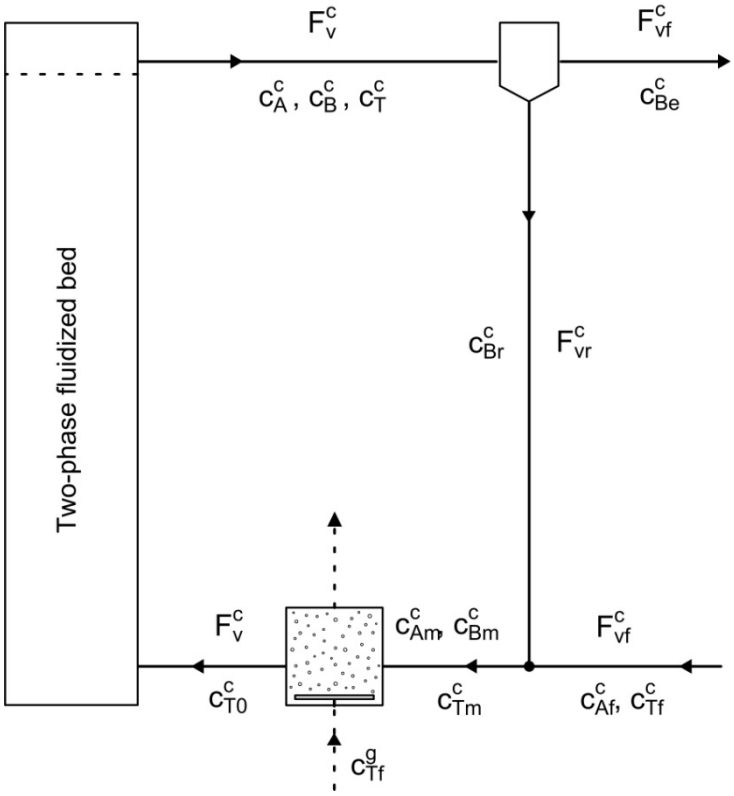

Fig. 1. Scheme of a two-phase fluidised bed bioreactor with an external aerator in the liquid recycle loop

A mathematical model of this object consists of equations which describe the process in the liquid phase and in the biofilm. They are shown below.

For the liquid phase we have:

$$V^c \frac{dc_{\mathrm{A}}^c}{dt} = F_V^c\left(c_{\mathrm{A}m}^c - c_{\mathrm{A}}^c\right) - V^c \cdot r_{\mathrm{A}}^c(c_{\mathrm{A}}^c, c_{\mathrm{B}}^c, c_{\mathrm{T}}^c) - V^c \frac{a_s}{1-\zeta_s} k_{s\mathrm{A}}\left(c_{\mathrm{A}}^c - c_{\mathrm{A}s}\right) \tag{1a}$$

$$V^c \frac{dc_{\mathrm{B}}^c}{dt} = F_V^c\left(c_{\mathrm{B}m}^c - c_{\mathrm{B}}^c\right) + V^c \cdot r_{\mathrm{B}}^c(c_{\mathrm{A}}^c, c_{\mathrm{B}}^c, c_{\mathrm{T}}^c) + V^c \frac{a_s}{1-\zeta_s} r_{det} \tag{1b}$$

$$V^c \frac{dc_{\mathrm{T}}^c}{dt} = F_V^c\left(c_{\mathrm{T}0}^c - c_{\mathrm{T}}^c\right) - V^c \cdot r_{\mathrm{T}}^c(c_{\mathrm{A}}^c, c_{\mathrm{B}}^c, c_{\mathrm{T}}^c) - V^c \frac{a_s}{1-\zeta_s} k_{s\mathrm{T}}\left(c_{\mathrm{T}}^c - c_{\mathrm{T}s}\right) \tag{1c}$$

Particulate terms on the right hand side of Equations (1a), (1b), (1c) describe convection, bioprocess and mass transfer between the liquid phase and the biofilm.
With regard to the presence of a recirculation stream and a settling tank, Equations (1a-c) should be completed with the following dependencies:

- for a mixing node:

$$F_{Vf}^c + F_{Vr}^c = F_V^c \tag{2a}$$

$$\left(F_V^c - F_{Vr}^c\right)c_{\mathrm{A}f}^c + F_{Vr}^c c_{\mathrm{A}}^c = F_V^c c_{\mathrm{A}m}^c \tag{2b}$$

$$\left(F_V^c - F_{Vr}^c\right)c_{\mathrm{T}f}^c + F_{Vr}^c c_{\mathrm{T}}^c = F_V^c c_{\mathrm{T}m}^c \tag{2c}$$

$$F_{Vr}^c c_{\mathrm{B}r}^c = F_V^c c_{\mathrm{B}m}^c \tag{2d}$$

- for a biomass thickening node:

$$F_V^c c_{\mathrm{B}}^c = F_{Vf}^c c_{\mathrm{B}e}^c + F_{Vr}^c c_{\mathrm{B}r}^c \tag{3}$$

Biomass thickening ratio ϑ and recirculation coefficient ξ are respectively defined as:

$$\vartheta = \frac{c_{\mathrm{B}}^c - c_{\mathrm{B}e}^c}{c_{\mathrm{B}}^c}; \quad \xi = \frac{F_{Vr}^c}{F_V^c} \tag{4}$$

Inserting dependences (2), (3) and definitions (4) into the equations for the liquid phase (1) we get for the steady state:

$$0 = \frac{1}{\tau_0^c}\left(c_{\mathrm{A}f}^c - c_{\mathrm{A}}^c\right) - r_{\mathrm{A}}^c(c_{\mathrm{A}}^c, c_{\mathrm{B}}^c, c_{\mathrm{T}}^c) - \frac{a_s}{1-\zeta_s} k_{s\mathrm{A}}\left(c_{\mathrm{A}}^c - c_{\mathrm{A}s}\right) \tag{5a}$$

$$0 = \frac{1}{\tau_0^c(1-\xi)}\left[c_{\mathrm{B}}^c\left(1-(1-\xi)(1-\vartheta)\right) - c_{\mathrm{B}}^c\right] + r_{\mathrm{B}}^c(c_{\mathrm{A}}^c, c_{\mathrm{B}}^c, c_{\mathrm{T}}^c) + \frac{a_s}{1-\zeta_s} r_{det} \tag{5b}$$

$$0 = \frac{1}{\tau_0^c(1-\xi)}\left(c_{\mathrm{T}0}^c - c_{\mathrm{T}}^c\right) - r_{\mathrm{T}}^c(c_{\mathrm{A}}^c, c_{\mathrm{B}}^c, c_{\mathrm{T}}^c) - \frac{a_s}{1-\zeta_s} k_{s\mathrm{T}}\left(c_{\mathrm{T}}^c - c_{\mathrm{T}s}\right) \tag{5c}$$

where τ_0^c is mean residence time of the liquid phase in the analysed installation.

The concentration of oxygen dissolved in the liquid stream fed to the fluidised bed depends mainly on the efficiency of an external aerator. This problem is described later on in the paper.

A heterogeneous model of a fluidised bed demands a separate balance of processes in the biofilm formed on carrier particles. Assuming a spherical shape of both carrier particles and the layer of biofilm formed on them we have:

$$\frac{d^2 c_{\mathrm{A}}^b}{dx^2} + \frac{2}{x}\frac{dc_{\mathrm{A}}^b}{dx} - \frac{1}{D_{e\mathrm{A}}} r_{\mathrm{A}}^b(c_{\mathrm{A}}^b, c_{\mathrm{T}}^b) = 0 \tag{6a}$$

$$\frac{d^2 c_{\mathrm{T}}^b}{dx^2} + \frac{2}{x}\frac{dc_{\mathrm{T}}^b}{dx} - \frac{1}{D_{e\mathrm{T}}} r_{\mathrm{T}}^b(c_{\mathrm{A}}^b, c_{\mathrm{T}}^b) = 0 \tag{6b}$$

where x is a current co-ordinate in the biofilm and D_{ei} is an effective diffusion coefficient of the i-th substrate. Equations (6) are associated with Neumann-Robin boundary conditions, i.e.

$$\frac{dc_{\mathrm{A}}^b(r_0)}{dx} = 0 \tag{7a}$$

$$\frac{dc_{\mathrm{T}}^b(r_0)}{dx} = 0 \tag{7b}$$

$$D_{e\mathrm{A}}\frac{dc_{\mathrm{A}}^b(r_b)}{dx} = k_{s\mathrm{A}}\left[c_{\mathrm{A}}^c - c_{\mathrm{A}}^b(r_b)\right] \tag{7c}$$

$$D_{e\mathrm{T}}\frac{dc_{\mathrm{T}}^b(r_b)}{dx} = k_{s\mathrm{T}}\left[c_{\mathrm{T}}^c - c_{\mathrm{T}}^b(r_b)\right] \tag{7d}$$

Conditions (7a) and (7b) mean that there is no mass transfer between the biofilm and the substratum. Equality between the diffusion and the convection at a bioparticle surface is described by conditions (7c) and (7d). The term r_{det} in Equation (5b) describes a resultant rate of the active biomass transfer from the biofilm to the liquid phase. Quantitative boundaries of this variable may be described as

$$0 \le r_{det} \le r_{det.max} \tag{8}$$

where: $r_{det.max} = L_b \cdot \bar{r}_{\mathrm{B}}^{b}$, while $\bar{r}_{\mathrm{B}}^{b} = \frac{1}{L_b}\int_0^{L_b} r_{\mathrm{B}}^{b}\left[c_{\mathrm{A}}^{b}(x), c_{\mathrm{T}}^{b}(x)\right]\cdot dx$

Thus, the rate of biomass transfer from the biofilm to the liquid phase may be then quantified as

$$r_{det} = X_{\mathrm{B}} L_b \cdot \bar{r}_{\mathrm{B}}^{b}, \qquad 0 \le X_{\mathrm{B}} \le 1 \tag{9}$$

where X_{B} depicts the fraction of active biomass transferred from the biofilm to the liquid phase. At the same time, this is a convenient method of quantitative description of biomass transfer rate between these two phases, which may be applied in simulation and design calculations. For further analysis, dimensionless variables were introduced. They were defined as follows:

$$\alpha = \frac{c_{\mathrm{A}f}^{c} - c_{\mathrm{A}}^{c}}{c_{\mathrm{A}f}^{c}}, \quad \beta = \frac{c_{\mathrm{B}}^{c}}{c_{\mathrm{A}f}^{c}}, \quad \gamma = \frac{c_{\mathrm{T}}^{c}}{c_{\mathrm{A}f}^{c}}, \tag{10a}$$

$$\eta = \frac{c_{\mathrm{A}}^{b}}{c_{\mathrm{A}}^{c}}, \quad \delta = \frac{c_{\mathrm{T}}^{b}}{c_{\mathrm{T}}^{c}}, \quad z = \frac{x - r_0}{L_b} \tag{10b}$$

After introducing dimensionless variables (10) to Equations (5), (6) and to boundary conditions (7), we get a model of the fluidised bed in a dimensionless form. It is a system of nonlinear algebraic and differential equations,

$$-\frac{1}{\tau_0^c}\alpha + r_{\mathrm{A}}^{c}(\alpha,\beta,\gamma) + \frac{a_s}{1-\zeta_s}k_{s\mathrm{A}}(1-\alpha)(1-\eta_s) = 0 \tag{11a}$$

$$\frac{\vartheta - 1}{\tau_0^c}\beta + r_{\mathrm{B}}^{c}(\alpha,\beta,\gamma) + \frac{a_s}{(1-\zeta_s)c_{Af}^{c}} r_{det} = 0 \tag{11b}$$

$$\frac{1}{\tau_0^c(1-\xi)}(\gamma_0 - \gamma) - r_{\mathrm{T}}^{c}(\alpha,\beta,\gamma) - \frac{a_s}{1-\zeta_s}k_{s\mathrm{T}}\gamma(1-\delta_s) = 0 \tag{11c}$$

$$\frac{d^2\eta}{dz^2} + \frac{2L_b}{r_0 + L_b z}\frac{d\eta}{dz} - \Phi_{\mathrm{A}}^{2}\frac{r_{\mathrm{A}}^{b}(\eta,\delta)}{r_{\mathrm{A}}^{c}} = 0 \tag{12a}$$

$$\frac{d^2\delta}{dz^2} + \frac{2L_b}{r_0 + L_b z}\frac{d\delta}{dz} - \Phi_{\mathrm{T}}^{2}\frac{r_{\mathrm{T}}^{b}(\eta,\delta)}{r_{\mathrm{T}}^{c}} = 0 \tag{12b}$$

$$\frac{d\eta(0)}{dz} = 0 \tag{13a}$$

$$\frac{d\delta(0)}{dz} = 0 \tag{13b}$$

$$\frac{d\eta(1)}{dz} - \mathrm{Bi}_{\mathrm{A}}\left[1 - \eta(1)\right] = 0 \tag{13c}$$

$$\frac{d\delta(1)}{dz} - \mathrm{Bi}_{\mathrm{T}}\left[1 - \delta(1)\right] = 0 \tag{13d}$$

Biot numbers and Thiele modulus for the biofilm are defined as follows:

$$\mathrm{Bi}_{\mathrm{A}} = \frac{k_{s\mathrm{A}}L_b}{D_{e\mathrm{A}}}, \quad \mathrm{Bi}_{\mathrm{T}} = \frac{k_{s\mathrm{T}}L_b}{D_{e\mathrm{T}}}, \quad \Phi_{\mathrm{A}}^{2} = \frac{L_b^2}{D_{e\mathrm{A}}c_{\mathrm{A}}^{c}}r_{\mathrm{A}}^{c}, \quad \Phi_{\mathrm{T}}^{2} = \frac{L_b^2}{D_{e\mathrm{T}}c_{\mathrm{T}}^{c}}r_{\mathrm{T}}^{c}$$

In order to solve a system ,of Equations (11), (12) with conditions (13), the shooting method was used. Every solution of the equations determines the steady state of an object presented in Fig.1. Merging this method with a continuation algorithm, one may determine the steady state branches, i.e. dependencies of the state variables, α, β and γ, on a chosen model parameter.

The determination of a steady state consists in calculating five unknowns, i.e. α, β, γ, $\eta(0)$, $\delta(0)$, and concentration profiles in the biofilm, i.e. $\eta(z)$ and $\delta(z)$. For the determination of the mentioned variables, five equations are used, i.e. (11a)-(11c) and (13c)-(13d). The values of $\eta_s=\eta(1)$ and $\delta_s=\delta(1)$, which occur in Equations (11a) and (11c), are obtained with integration of the system of differential equations (12). Equations (11a)-(11c) and (13c)-(13d) were solved with the Newton method. The differential equations system (12) was integrated with the Runge-Kutta method.

3. KINETIC PARAMETERS OF FLUIDISED BED AND EXTERNAL AERATOR

Minimum and maximum fluidisation velocity, i.e. u_{mf} and u_t, were calculated according to commonly used correlations known from the literature (Dziubiński and Prywer, 2009). Mass transfer coefficients between the liquid and the surface of the biofilm in the two-phase fluidised bed were calculated according to Equation (14), which was proposed by Lakhsmi and Setty (2008).

$$\mathrm{Sh}_i = \mathrm{Re}_c^{0.5895} \cdot \mathrm{Sc}_i^{0.2048} \tag{14}$$

where

$$\mathrm{Sh}_i = \frac{k_{si} d_s}{D_i}, \quad \mathrm{Re}_c = \frac{u_t d_s \rho_c}{\eta_c}, \quad \mathrm{Sc}_i = \frac{\eta_c}{\rho_c D_i}, \quad (i = \mathrm{A}, \mathrm{T})$$

The aim of an external aerator is oxygenation of the liquid stream feeding the bioreactor. The concentration of oxygen dissolved in this stream may vary between the concentration in the mixing node and the equilibrium concentration of oxygen in the liquid phase, which may be described as:

$$\frac{c^c_{\mathrm{T}m}}{c^c_{\mathrm{A}f}} = \gamma_m \le \gamma_0 < \gamma^* = \frac{c^g_{\mathrm{T}f}}{K \cdot c^c_{\mathrm{A}f}} \tag{15}$$

and which depends on the efficiency of the aerator. If the efficiency of the aerator is defined as

$$E = \frac{c^c_{\mathrm{T}0} - c^c_{\mathrm{T}m}}{c^*_{\mathrm{T}} - c^c_{\mathrm{T}m}} = \frac{\gamma_0 - \gamma_m}{\gamma^* - \gamma_m} \tag{16}$$

then

$$\gamma_0 = \gamma_m + E\left(\gamma^* - \gamma_m\right) \tag{17}$$

In this paper, the efficiency of oxygenation for various aerators, whose application is possible, was not calculated, because for a nonlinear analysis of steady states, efficiency values E are sufficient. They depend on the construction of an aerator, mass transfer surface, and hydrodynamic conditions inside the aerator.

4. ASSESSMENT OF LIQUID RECIRCULATION RATIO

Based on mass balances of a microbiological process and an assumption of total oxygen consumption while passing through the bioreactor, the minimum value of the recirculation ratio ξ_{min} may be calculated. From the mixing node balance (2b), we have

$$c_{Am}^{c} = (1-\xi)c_{Af}^{c} + \xi \cdot c_{A}^{c} \tag{18}$$

where c_{Am}^{c} is the carbonaceous substrate concentration in the liquid flowing out from the mixing node.

A relation between the quantity of consumed oxygen T and the carbonaceous substrate A can be given by the equation

$$c_{T0}^{c} - c_{T}^{c} = w_{TA}\left(c_{Am}^{c} - c_{A}^{c}\right) \tag{19}$$

where $w_{TA} = \dfrac{w_{BA}}{w_{BT}}$, w_{BA} and w_{BT} are yield coefficients.

The minimum value of the recirculation ratio ξ_{min} may be determined taking into consideration the following boundary assumptions

$$c_{T}^{c} = 0 \quad \text{and} \quad c_{T0}^{c} = c_{T}^{*} = \frac{c_{Tf}^{g}}{K} \tag{20}$$

After inserting these assumptions into Equations (18) and (19), we get

$$\xi_{min} = 1 - \frac{c_{T}^{*}}{w_{TA} \cdot c_{Af}^{c}\alpha} = \xi_{min,1} \tag{21}$$

From formula (21), it follows that the value $\xi_{min,1}$ will be rising with an increase of the yield coefficient w_{TA}, the carbon substrate concentration in the feed stream and its degree of conversion α. In Fig. 2, the function $\xi_{min,1} = f(\alpha)$ for a few values of the yield coefficient w_{TA} is shown. The coefficient w_{TA} is dependent on the aerobic process.

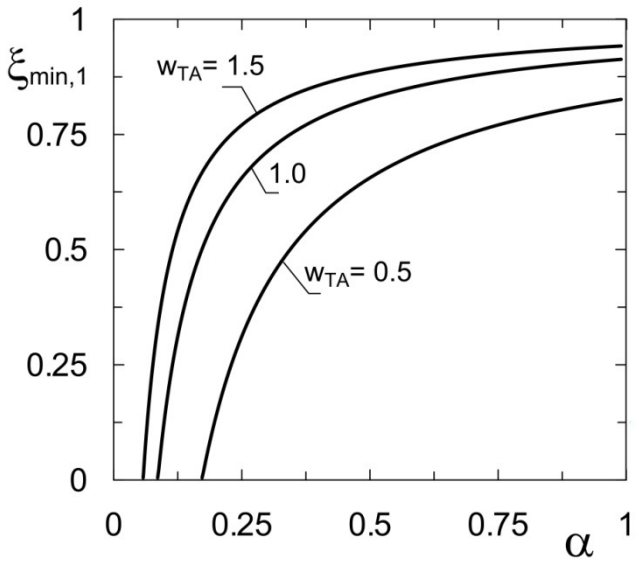

Fig. 2. Dependency of $\xi_{min,1}$ on the degree of conversion of carbonaceous substrate α and the yield coefficient w_{TA} (c_{T}^{*}= 0.0086 kg/m^3; c_{Af}= 0.1 kg/m^3)

Apart from mass balances which were described above, the value of the recirculation ratio ξ is also influenced by demands of maintaining fluidisation. The minimum value of this parameter may be calculated according to

$$\xi_{min} = 1 - \frac{H}{u_{mf}\tau_{0}^{c}} = \xi_{min,2} \tag{22}$$

Hence, it is possible to talk about two minimum values of liquid recirculation ratio. Designing the installation, which is shown in Fig. 1, one should assume at least a bigger value than $\xi_{min,k}$ (k = 1, 2) for the actual recirculation ratio.

5. DISCUSSION OF RESULTS AND STATIONARY CHARACTERISTICS OF THE BIOREACTOR

The model of a bioreactor, which was presented above, is characterised by many kinetic and operating parameters. Assessment of stationary characteristics has been made by determining parametric dependence of the state variables with relation to the chosen operating and design parameters. Hereafter, the influences of liquid mean residence time, carbonaceous substrate concentration in the feed stream, biofilm carrier fraction in the fluidised bed, rate of active biomass transfer from the biofilm to the liquid phase and the efficiency of an external aerator are described.

Such a theoretical analysis may be carried out for an abstract microbiological process or for a chosen microbiological process of industrial importance. Below, the phenol aerobic degradation, which proceeds according to double-substrate kinetics, is chosen as an example. The kinetic model proposed by Seker et al. (1997) was assumed. In accordance with this model, the rate of microbiological process in the liquid phase and in the biofilm may be formulated as :

$$r_{\mathrm{A}}^{c}(\alpha,\beta,\gamma)=\frac{1}{w_{\mathrm{BA}}}\cdot f_1\left(c_{\mathrm{A}}(\alpha)\right)\cdot f_2\left(c_{\mathrm{T}}(\gamma)\right)\cdot\beta \tag{23a}$$

$$r_{\mathrm{B}}^{c}(\alpha,\beta,\gamma)=f_1\left(c_{\mathrm{A}}(\alpha)\right)\cdot f_2\left(c_{\mathrm{T}}(\gamma)\right)\cdot\beta \tag{23b}$$

$$r_{\mathrm{T}}^{c}(\alpha,\beta,\gamma)=\frac{1}{w_{\mathrm{BT}}}\cdot f_1\left(c_{\mathrm{A}}(\alpha)\right)\cdot f_2\left(c_{\mathrm{T}}(\gamma)\right)\cdot\beta \tag{23c}$$

$$r_{\mathrm{A}}^{b}(\eta,\delta)=\frac{1}{w_{\mathrm{BA}}}\cdot f_1\left(c_{\mathrm{A}}(\eta)\right)\cdot f_2\left(c_{\mathrm{T}}(\delta)\right)\cdot\rho_a \tag{24a}$$

$$r_{\mathrm{T}}^{b}(\eta,\delta)=\frac{1}{w_{\mathrm{BT}}}\cdot f_1\left(c_{\mathrm{A}}(\eta)\right)\cdot f_2\left(c_{\mathrm{T}}(\delta)\right)\cdot\rho_a \tag{24b}$$

where

$$f_1(c_{\mathrm{A}})=\frac{kc_{\mathrm{A}}}{K_s+c_{\mathrm{A}}+\dfrac{c_{\mathrm{A}}^2}{K_{in}}}\quad,\quad f_2(c_{\mathrm{T}})=\frac{c_{\mathrm{T}}}{K_{\mathrm{T}}+c_{\mathrm{T}}} \tag{23d}$$

The values of kinetic parameters given by Seker et al. (1997) are listed below.

Table 1. Values of kinetic parametrs for phenol degradation

k [1/h]	K_s [kg/m^3]	K_{in} [kg/m^3]	w_{BA} [kgB/kgA]	K_{T} [kg/m^3]	w_{BT} [kg of oxygen /kgB]
0.365	0.01095	0.113	0.496	10^{-4}	0.354

Fig. 3 presents the stationary characteristics of the analysed bioreactor in a form of steady state branches. The dependencies of:

- a degree of conversion of carbonaceous substrate α,
- dimensionless biomass concentration β,
- dimensionless concentration of oxygen dissolved in the liquid γ

on mean residence time in the whole installation and the fraction of biomass transferred from the biofilm to the liquid X_B are presented. Solid lines depict stable steady states, while dashed lines – unstable states. For comparison, the stationary characteristics of the bioreactor without carrier particles in the liquid, i.e. for $\zeta_s = 0$, were also determined. A steady-state branch corresponding to such a limit case is shown as gray squares in Fig. 3.

It follows from Fig. 3 that the stationary characteristics of a fluidised bed bioreactor are qualitatively different from the characteristics of a bioreactor without carrier particles covered with an immobilised biofilm. With a decreasing mean residence time of the liquid in the bioreactor without bioparticles, biomass washout occurs. It causes a complete loss of production capacity. In the same case, in a fluidised bed, i.e. for $\zeta > 0$, the total loss of production capacity does not occur even for a slight volumetric fraction of particles $\zeta_s = 0.01$. Therefore, the existence of the biofilm allows to carry out a microbiological process without suspended biomass in a planktonic form present.

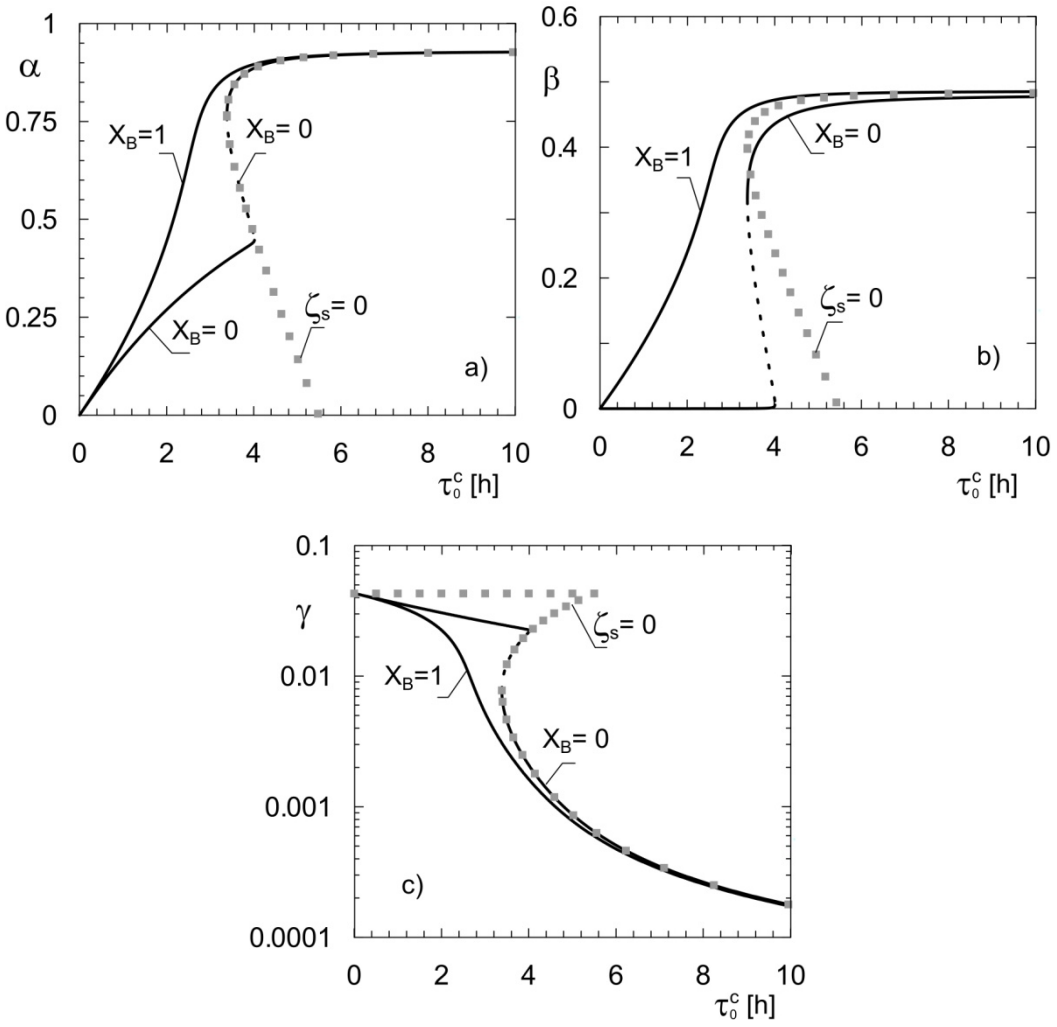

Fig. 3. Stationary properties of a two-phase loop fluidised-bed bioreactor
(c_{Af} = 0.2 kg/m^3; $L_b = 3 \cdot 10^{-5}$ m; $\vartheta = 0$; $\xi = 0.97$; $\zeta_s = 0.01$)
—— stable steady states; - - - - - - unstable steady states

Steady state branches presented in Fig. 3 make it also possible to determine the value of mean residence time of the liquid phase, which is necessary to obtain the required degree of conversion of carbonaceous substrate.

Fig. 4 presents how the degree of conversion α, of dimensionless biomass concentration β, and concentration of oxygen dissolved in the liquid γ changes depending on the concentration of the

carbonaceous substrate in the feed stream c_{Af}. Each diagram in this figure shows two curves corresponding to the limit values of active biomass transfer rate from the biofilm to the liquid, i.e. for $X_B = 0$ and $X_B = 1$. On the basis of the stationary characteristics shown in Fig. 4, it is possible to deduce the limits of applicability of such bioreactors when it comes to an allowable range of concentrations of the toxic substrate subjected to biodegradation. It follows from Fig. 4a that for substrate concentrations exceeding the value of $c_{Af} \approx 0.2$ kg/m^3 its degree of conversion decreases significantly. It may be concluded from Fig. 4c that the limitation of carbon substrate concentration in the feed stream is related to the lack of oxygen in the liquid phase. Hence, these diagrams are also of practical importance both at a step of installation design, and during its operation.

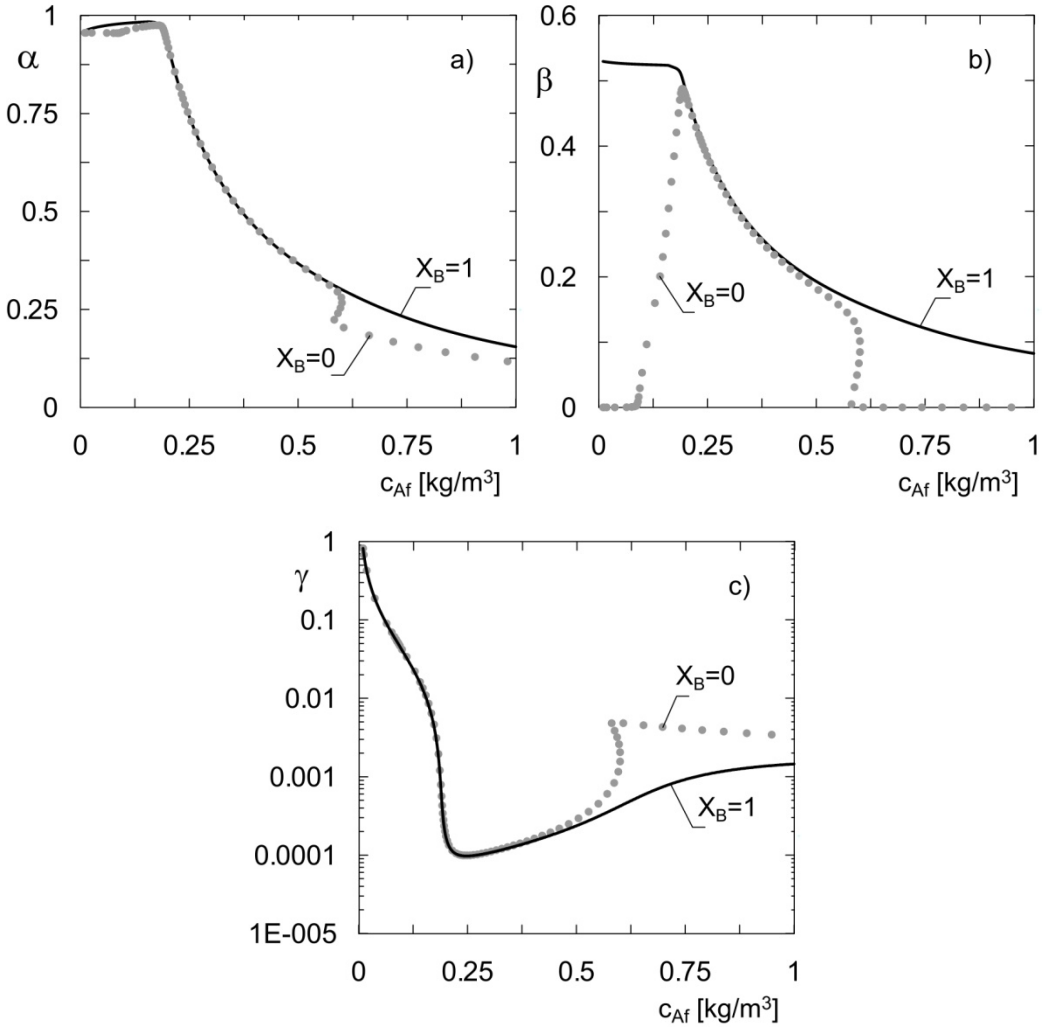

Fig. 4. Steady state branches $\alpha(c_{Af})$, $\beta(c_{Af})$ and $\gamma(c_{Af})$ of a two-phase loop fluidised-bed bioreactor ($\tau_0^c = 10$ h; $L_b = 3 \cdot 10^{-5}$ m; $\vartheta = 0$; $\xi = 0.97$; $\zeta_s = 0.01$)

One of the key operating parameters is the fraction of biofilm carriers in the liquid depicted by the parameter ζ_s. The surface available for microbial growth and, thus, the overall biomass concentration in the apparatus is highly dependent on this parameter. It is proved that even a small fraction of biofilm carriers, of an order of 1 per cent, has a significant influence on the bioreactor stationary characteristics. This property is illustrated in Fig. 5. It shows the dependence of only one state variable, i.e. a degree of conversion of carbonaceous substrate α, on the mean residence time of the liquid in the installation for three values of the parameter ζ_s. The value $\zeta_s = 0$ corresponds to the lack of particles in the apparatus. The presence of particles, which means an application of fluidised bed, causes a qualitative change of bioreactor stationary characteristics due to three reasons:

- prevention of biomass washout,
- apparent reduction of substrate inhibition phenomenon,

- decrease of the number of possible steady states.

The last two features result from the disappearance of a turning point on the steady state branch for the considered fluidised bed bioreactor.

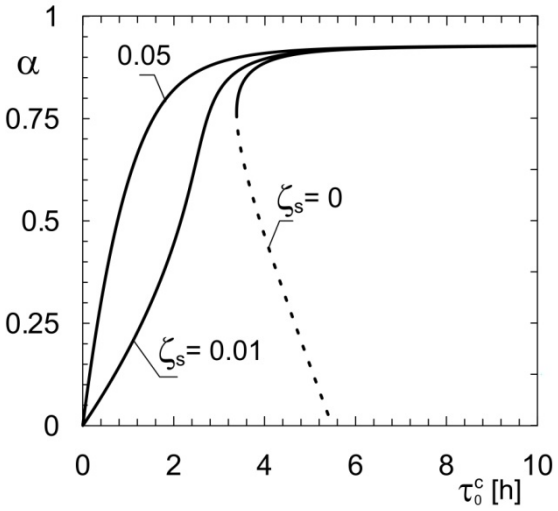

Fig. 5. Effect of the fraction of biofilm carriers in the liquid phase ζ_s on the degree of conversion of carbonaceous substrate (c_{Af}= 0.2 kg/m^3; L_b= 3·10^{-5} m; ϑ= 0; ξ= 0.97)

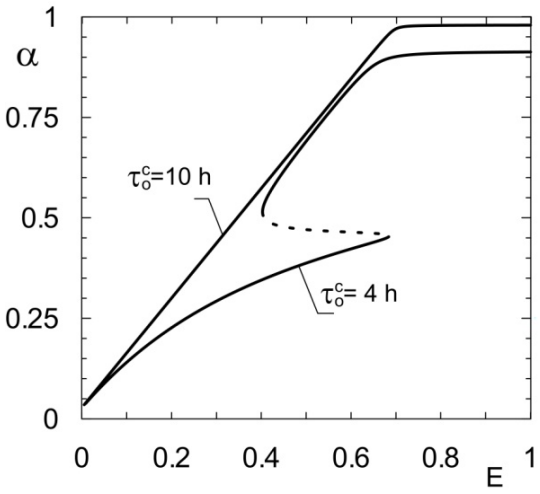

Fig. 6. Effect of external aerator efficiency E on a degree of conversion of carbonaceous substrate (c_{Af}= 0.2 kg/m^3; L_b= 3·10^{-5} m; ϑ= 0; ξ= 0.98; ζ_s= 0.01)

An effective bioreactor operation presented in Fig. 1, depends on the efficiency of the external aerator. This parameter is described by formula (16). Hence, it seems to be justified to determine the influence of this parameter on the efficiency of the whole installation. Fig. 6 shows how the degree of conversion of carbonaceous substrate changes depending on the efficiency of the aerator. Calculations were performed for two different values of liquid mean residence time in the installation. It is proved that a change of the aerator efficiency may have an influence even on the occurrence of the multiple steady states phenomenon. However, regardless of the dependence of the degree of conversion α on the aerator efficiency E, there exists a range of values E, for which a high degree of conversion of the carbonaceous substrate may be obtained. Furthermore, the influence of the aerator efficiency E on the degree of biodegradation α is insignificant within this range. It is caused by intensive oxygenation of the liquid phase. For sufficiently high values of E, the concentration of oxygen dissolved in the liquid and in the biofilm is so high that this substrate does not limit the growth of microorganisms. Hence,

only the carbonaceous substrate is growth-limiting. Obtaining such diagrams as in Fig. 6 gives the possibility to determine this limit value of the parameter E.

6. CONCLUSIONS

The paper presents a mathematical model of a two-phase fluidised bed bioreactor, in which an aerobic microbiological process according to Haldane-Monod kinetics occurs. The analysed installation is equipped with an external mass exchanger, which works in a recirculation loop and is used for liquid oxygenation. Complete mixing of the liquid phase was assumed, which is justified taking into consideration operating conditions of the apparatus and the hydrodynamics of a fluidised bed.

Nonlinear analysis of the steady states of the bioreactor was performed. Phenol degradation taking place in the presence of the *Pseudomonas putida* microorganisms was chosen as a process example. The location of steady state branches depending on liquid mean residence time, carbonaceous substrate concentration and efficiency of the external aerator was determined. On this basis, a rational selection of process parameters and the analysis of their influence on the bioreactor's state variables are possible. Furthermore, the determination of conditions, in which the application of two-phase fluidised bed loop bioreactors with an external aerator is justified from a process point of view, is possible.

A method for the determination of minimum liquid recirculation ratio values, which are based on both oxygen demand and maintenance of fluidised bed conditions, was proposed.

SYMBOLS

a_s	external specific area of biofilm, m^{-1}
Bi	Biot number
c_A, c_T	mass concentration of carbonaceous substrate and oxygen, $kg \cdot m^{-3}$
c_B	biomass concentration in liquid phase, $kg \cdot m^{-3}$
D_{Ae} , D_{Te}	effective diffusion coefficients in biofilm, $m^2 \cdot h^{-1}$
d	diameter, m
E	efficiency of oxygenator
F_V	volumetric flow rate, $m^{-3} \cdot h^{-1}$
H	height of fluidised bed, m
k_{sA}, k_{sT}	mass transfer coefficients at external surface of biofilm, $m \cdot h^{-1}$
k, K_s, K_{in}	constants in kinetic equations
K	oxygen gas-liquid equilibrium constant
L_b	biofilm thickness, m
r_A, r_T	uptake rate of carbonaceous substrate and oxygen, respectively, $kg \cdot m^{-3} \cdot h^{-1}$
r_b	radius of bioparticle, m
r_{det}	detachment rate of biofilm, $kg \cdot m^{-2}$
r_0	radius of inert pellet, m
Re	Reynolds number
Sc	Schmidt number
Sh	Sherwood number
t	time, h
u	velocity, $m \cdot h^{-1}$
w_{BA}, w_{BT}	yield coefficients
x	current co-ordinate in biofilm, m
X_B	fraction of active biomass transferred from biofilm to liquid

z dimensionless coordinate in biofilm

Greek symbols

α degree of conversion of carbonaceous substrate
β dimensionless concentration of biomass in liquid phase
γ dimensionless concentration of oxygen in liquid phase
δ dimensionless concentration of oxygen in biofilm
η dimensionless concentration of carbonaceous substrate in biofilm
ζ_s volumetric fraction of solid carriers in liquid phase
ϑ biomass thickening coefficient
ξ recirculation ratio
ρ density, kg·m^{-3}
τ mean residence time, h
Φ Thiele modulus

Superscripts

b refers to biofilm
c refers to liquid phase
g refers to gas phase

Subscripts

A, T refers to limiting carbonaceous substrate and oxygen, respectively
B refers to biomass
f refers to feed stream
m refers to mixing node
s refers to external surface of biofilm

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