10.2478/acsc-2014-0014

Archives of Control Sciences
Volume 24(LX), 2014
No. 2*, pages 235–251*

Design of multivariable fractional order PID controller using covariance matrix adaptation evolution strategy

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This paper presents an automatic tuning of multivariable Fractional-Order Proportional, Integral and Derivative controller (FO-PID) parameters using Covariance Matrix Adaptation Evolution Strategy (CMAES) algorithm. Decoupled multivariable FO-PI and FO-PID controller structures are considered. Oustaloup integer order approximation is used for the fractional integrals and derivatives. For validation, two Multi-Input Multi- Output (MIMO) distillation columns described by Wood and Berry and Ogunnaike and Ray are considered for the design of multivariable FO-PID controller. Optimal FO-PID controller is designed by minimizing Integral Absolute Error (IAE) as objective function. The results of previously reported PI/PID controller are considered for comparison purposes. Simulation results reveal that the performance of FO-PI and FO-PID controller is better than integer order PI/PID controller in terms of IAE. Also, CMAES algorithm is suitable for the design of FO-PI / FO-PID controller.

Key words: fractional order controller, PID control, CMAES algorithm, distillation column

1. Introduction

Fractional order dynamic systems and controllers, based on fractional calculus have been gaining attention in several applications [1-3]. The idea of using fractional-order controllers for the dynamic system control is well-addressed by Podlubny [4]. The design of fractional order controller is also reported by many researchers. Petras presented a method based on pole distribution of characteristics equation in complex plane [5]. Frequency domain based fractional order controller design for the expected crossover frequency and phase margin is proposed by Vinegar *et al.* [6]. Padula *et al.* have suggested set of rules based on the minimization of the integrated absolute error for tuning of fractional order PI and PID controllers for a first-order-plus-dead-time model [7]. The hardware realization of the FO-PID controller is given by Chen *et al.* [8].

In FO-PID controller, the order of integration and differentiation is of fractional order. Hence, in the design of FO-PID controllers, besides the setting of proportional (K_P),

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Received 08.05.2014.

integral (K_I) and differential constant (K_D), two more parameters, order of integral fraction (λ) and order of differentiation fraction (μ) are to be set. The optimal design of K_P, K_I, K_D, (λ) and (μ) to meet the user specification leads to a real parameter optimization problem.

Recently, Evolutionary Algorithms (EA) like Genetic Algorithms (GA) [9], Particle Swarm Optimization (PSO) [10], and Differential Evolution (DE) [11], are employed for the design of the FO-PID controllers for single input single output (SISO) system. Cao *et al*. [9] have applied GA to FO-PID optimization problem by minimizing combination of ITAE and control input for integer order SISO systems. Cao *et al*. [10] have also applied PSO to this problem by using IAE along with square of control input as objective for SISO systems. Biswas et.al [11] have applied DE algorithm by minimizing error between user specified time response specifications such as peak overshoot, rise time and steady state error and corresponding obtained values with FO-PID controller.

In real world, most of the industrial processes belong to the category of MIMO system which requires MIMO control techniques to improve the performance of the process. The presence of cross-coupling between plant inputs and outputs complicates the design of controller for MIMO system as compared to SISO system. The need to simultaneously adjust multiple control inputs to produce a desired system response is a typical attribute of MIMO systems. Obviously, this makes the control system development procedure more difficult.

Recently, Covariance Matrix Adaptation Evolutionary Strategy (CMAES) algorithm is applied successfully for the optimization test problems [12] and the design of multivariable PID controller for MIMO systems [13] and [14].

In this paper, CMAES algorithm is proposed for the design of Multivariable FO-PI and FO-PID controller for MIMO systems namely binary distillation column described by Wood and Berry and Ogunnaike and Ray. For comparison purpose, the integer order PI/PID controller designed by using CMAES and conventional Biggest Log-modulus Tuning (BLT) method are taken.

This paper is organized as follows: section 2 gives a brief overview of fractional order systems. Section 3 explains the CMAES algorithm. Section 4 describes the Test Systems considered. Section 5 presents the CMAES algorithm based design of multivariable FO-PI / FO-PID controller. Simulation results are given in section 6 and the conclusion in section7 7.

2. Fractional-order systems

Fractional calculus is a branch of mathematical analysis that deals with real number power of differential and integral operator. The generalized continuous differ-integrator operator is represented as follows

$$ {}_aD_t^q = \begin{cases} d^q/dt^q & \Re(q) > 0 \\ 1 & \Re(q) = 0 \\ \int_a^t (d\tau)^{-q} & \Re(q) < 0 \end{cases} \tag{1}$$

where q represents the real order of differ-integrator operator, t is the parameter for which the differ-integral is taken and a is the lower limit. Various definitions are used for the general fractional differ-integral, like Grunwald-Letnikov (GL) definition and Riemann-Liouville (RL) definition. These definitions are explained briefly in [15]. The GL definition is given as

$$ {}_aD_t^q f(t) = \lim_{h\to 0} h^{-q} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \begin{pmatrix} q \\ j \end{pmatrix} f(t-jh) \tag{2}$$

where [.] means integer part. The time domain of the RL definition is given as

$$ {}_aD_t^q f(t) = \frac{1}{\Gamma(m-q)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{q-n+1}} d\tau \tag{3}$$

for $(m-1 < q < m)$ and $\Gamma(\cdot)$ is Euler's Gamma function.

The Laplace transform of the above RL definition has the following form

$$ \int_0^\infty e^{-st} {}_0D_t^q f(t) dt = s^q F(s) - \sum_{p=0}^{m-1} s^k {}_0D_t^{q-p-1} f(t) \,|_{t=0} \tag{4}$$

for $(m-1 < q \leqslant m)$, where s denotes the Laplace transform operator. For $q < 0$ (i.e. the case of fractional integral) the sum in the right-hand side must be omitted.

2.1. Integer order approximation

For transfer function with fractional order q, the most common way is to approximate them with usual (integer order) transfer function. To perfectly approximate a fractional transfer function, an integer transfer function with an infinite number of poles and zeros is required. For the simulation purpose, it is possible to obtain an approximation with a finite number of zeros and poles. An oustaloup filter approximation is widely used in fractional calculus. A generalized filter approximation is given by the transfer function as

$$ s^q \approx k_s \bigsqcup_{b=1}^{N_o} \frac{1+(s/w_{z,b})}{1+(s/w_{p,b})}. \tag{5}$$

The approximation is made reasonable in the frequency domain range $[w_l,\ w_h]$, where w_l, w_h are lower and upper bound of the operating frequency domain. Gain, k_s is regulated to have unit gain at 1 rad/s. The number of poles and zeros (N_o) is chosen in advance and the desired performance of the approximation depends on the order of N_o. Lower order No causes simpler approximation, but may cause ripples in both gain and phase behaviors. Such ripples can be avoided by increasing the order N_o, but the approximation will become computationally burden. The further details about the approximation can be found in [15].

2.2. Fractional order PID controller

Integer order PID control, commonly known as PID control, offers the simplest and efficient solution in many real-world control problems. The three term functionality of PID controller covers treatment of both transient and steady state performances. PID controllers have been widely used in the industries in process control for several decades. The reason for the wide popularity is for the simplicity, clear functionality, applicability and robustness in nature [14]. The transfer function representation of the PID controller is as follows

$$G_{IC}(s) = K_P + K_I s^{-1} + K_D s \tag{6}$$

FO-PID controllers are described by fractional order differential equations. The mathematical representation of FO-PID in s-domain is given as

$$G_{FC}(s) = K_P + K_I s^{-\lambda} + K_D s^{\mu}, \quad 0 \leqslant \lambda, \mu \leqslant 1 \tag{7}$$

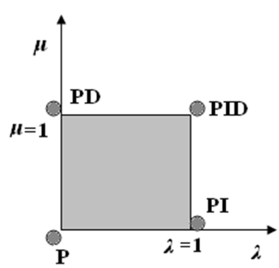

Figure 1. PID controllers with fractional order.

Fig.1 shows the pictorial representation of various controllers. Any point inside the shaded square represents the fractional order PID controller. The extreme points represent the integer order controllers.

The design of FO-PID controllers involve the optimal parametrization of K_P, K_I, K_D, λ and μ to meet the user specifications for a given process. The design of these parameters involves the parameter optimization in five-dimensional space for a SISO system.

3. Covariance matrix adaptation evolutionary strategy (CMAES)

Evolutionary algorithms like GA, DE and CMAES algorithms differ in selection, offspring generation and replacement mechanisms. CMAES is a class of continuous EA that generates new population members from a probability distribution that is constructed during the optimization process [12]. One of the key concepts of this algorithm involves the self-adaptation of learning of correlations between parameters and the use of the correlations to accelerate the convergence of the algorithm. The adaptation mechanism of CMAES consists of two parts, 1) the adaptation of the covariance matrix C and 2) the adaptation of the global step size σ. The covariance matrix C is adapted by the evolution path P_c and vector difference between the μ best individuals in the current and previous generation. The algorithm is explained in following steps [12].

Step 1:
Set the parameters c_σ, c_c, c_{cov} and d to their default values as per Tab. 1.

Step 2:
Set the evolution path $P_\sigma^{(0)} = 0$, $P_c^{(0)} = 0$ and covariance matrix $C(0) = I$. Choose step size $\sigma(0)$ and frame the stopping criteria.

Step 3:
Generate an initial random solution.

Step 4:
The offspring at $g+1$ generation, x_k^{g+1} are sampled from a Gaussian distribution using covariance matrix and global step size at generation g as given in (8).

$$x_k^{(g+1)} = z_k, \quad z_k = N\left(\langle x\rangle_\mu^{(g)}, \sigma^{(g)^2}\mathbf{C}^{(g)}\right) \quad k = 1, \ldots, \lambda \tag{8}$$

where $\langle x\rangle_\mu^{(g)} = \sum_{i=1}^{\mu} x_i^{(g)}$ with μ being the selected best individuals from the population, x_k^{g+1} is k-th offspring (search point) from generation $g+1$, $N(x,C)$ denotes a normally distributed random vector with mean x and covariance matrix C, λp is the population size, μp parent number, σ the step size and C covariance matrix at generation g.

Step 5:
Adaptation of global step size σ^{g+1} is based on an evolution path as given in (9-10).

$$\mathbf{P}_\sigma^{(g+1)} = (1-c_\sigma)\cdot\mathbf{P}_\sigma^{(g)} + \sqrt{c_\sigma(2-c_\sigma)\mu_{eff}}.(C^{(g)})^{\frac{-1}{2}}\frac{\left(\langle x\rangle_{\mu p}^{(g+2)} - \langle x\rangle_{\mu p}^{g}\right)}{\sigma^{(g)}}. \tag{9}$$

The global step size $\sigma^{(g+1)}$ is determined by

$$\sigma^{(g+1)} = \sigma^{(g)} \exp\left(\frac{c_\sigma}{d}\left(\frac{\left\|\mathbf{P}_\sigma^{(g+1)}\right\|}{E\left(\|N(0,\mathbf{I})\|\right)}\right) - 1\right). \tag{10}$$

The damping factor d can be adapted if maximum iteration number is small.

Step 6:

The covariance matrix adaptation is based on the evolution path $\mathbf{P}_c^{(g+1)}$ as follows

$$\mathbf{P}_c^{(g+1)} = (1-c_c)\cdot\mathbf{P}_c^{(g)} + \sqrt{c_c(2-c_c)}\cdot\frac{\sqrt{\mu p}}{\sigma^{(g)}}\left(\langle x\rangle_{\mu p}^{(g+1)} - \langle x\rangle_{\mu p}^{g}\right), \tag{11}$$

$$\begin{aligned} \mathbf{C}^{(g+1)} &= (1-c_{\text{cov}})\cdot\mathbf{C}^{(g)} \\ &+c_{\text{cov}}\cdot\left(\begin{array}{l} \frac{1}{\mu p}\mathbf{P}_c^{(g+1)}(\mathbf{P}_c^{(g+1)})^{\mathrm{T}} + \\ (1-\frac{1}{\mu p})\frac{1}{\mu p}\sum\limits_{i=1}^{\mu p}\frac{1}{\sigma^{(g)^2}}(x_i^{(g+1)} - \langle x\rangle_{\mu p}^{(g)})(x_i^{(g+1)} - \langle x\rangle_{\mu p}^{(g)})^{\mathrm{T}} \end{array}\right). \end{aligned} \tag{12}$$

Strategy parameter $c_{\text{cov}} \in [0,\ 1]$ determines the rate of change of the covariance matrix C.

Step 7:

Repeat Steps 4-6 until the stopping criteria are satisfied.

Table 21. CMAES parameter.

Step size control
$c_\sigma = \frac{\mu_{eff}+2}{D+\mu_{eff}+3}$, $\quad d = 1+2\max\left(0, \frac{\sqrt{\mu_{eff}-1}}{D+1} - 1\right) + c_\sigma$
Covariance matrix adaptation
$c_c = \frac{4}{D+4}$
$c_{\text{cov}} = \frac{1}{\mu p}\frac{2}{\left(D+\sqrt{2}\right)^2} + \left(1-\frac{1}{\mu p}\right)\min\left(1, \frac{2\mu p-1}{(D+2)^2+\mu p}\right)$

4. Test of the system

Most of the industrial process belongs to the category of MIMO system, which requires MIMO control techniques to improve the system performance. In order to validate the performance of the CMAES algorithm on the design of the fractional-order PI/PID controller two MIMO systems, namely binary distillation column described by Wood and Berry (WB) [13] (Test system-I) and Ogunnaike and Ray distillation column (OR) [14] (Test system-II) are considered and their details are given below.

4.1. Test system–I

A binary distillation column having two inputs and two outputs described by Wood and Berry is considered for validation. The transfer function of WB process is given in (13). The transfer function of WB process has first order dynamics and significant time delays. It has a strong interaction between the inputs and outputs. The inputs are the reflux flow rate and steam flow rate in the rebolier. The outputs are percentage of methanol in distillate (X_D) and bottom products (X_B).

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{1+16.7s} & \dfrac{-18.9e^{-3s}}{1+21s} \\ \dfrac{6.6e^{-7s}}{1+10.9s} & \dfrac{-19.4e^{-3s}}{1+14s} \end{bmatrix}. \tag{13}$$

4.2. Test system–I

Ogunnaike and Ray distillation column (OR) is considered for a 3×3 system. The transfer function of OR process is represented by (14).

$$G(s) = \begin{bmatrix} \dfrac{0.66e^{-2.6s}}{6.7s+1} & \dfrac{-0.61e^{-3.6s}}{8.64s+1} & \dfrac{-0.0049e^{-s}}{9.06s+1} \\ \dfrac{1.11e^{-6.5s}}{3.25s+1} & \dfrac{-2.36e^{-3s}}{5s+1} & \dfrac{-0.01e^{-1.2s}}{7.09s+1} \\ \dfrac{-34.68e^{-9.2s}}{8.15s+1} & \dfrac{46.2e^{-9.4s}}{10.9s+1} & \dfrac{0.87(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix}. \tag{14}$$

5. CMAES implementation of FO-PI / FO-PID controller

CMAES implementation of multivariable decoupled FO-PI / FO-PID controller for MIMO test systems is described below. The general representation of decoupled fractional order controller for the test systems considered is given in

$$G_{FC}(s) = \begin{bmatrix} G_{FC1}(s) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & G_{FCi}(s) \end{bmatrix}, \quad i = 1, 2 \ldots, n \tag{15}$$

where n is number of system inputs. The transfer function of $G_{FC1}(s)$ is given by (7). The fractional order integral and derivative are approximated as per (5). The controller approximations made are system order 1 and the bandwidth for the fractional order controller [1e-4, 1e2]. In order to obtain the optimum performance of FO-PI controller, the parameters of $G_{FCi}(s) = \{K_{Pi}, K_{Ii}, \lambda_i\}$ are optimized using the CMAES algorithm. Similarly for the FO-PID controller the optimum parameter set of $G_{FCi}(s) = \{K_{Pi}, K_{Ii}, K_{Di}, \lambda_i, \mu_i\}$ are determined. The lower and upper bounds of the controller parameters considered for the systems are given in Tab. 2. Minimization of integral of absolute value of error (IAE) given by (16) is considered as the objective function. Error e_i is calculated as per (17)

$$IAE = \int_0^\infty (|e_1(t)| + |e_2(t)| + \ldots |e_n(t)|)dt \tag{16}$$

$$e_i = y_{di} - y_i, \quad i = 1, 2 \ldots, n \tag{17}$$

where y_{di} is system desired response and y_i is system actual response.

In general, the number of design parameters for a FO-PI design is $3n$ and for a FO-PID design is $5n$. For example, the ith parameter vector in CMAES algorithm for FO-PID controller of WB process is $X_i = \{K_{P1}, K_{I1}, K_{D1}, \lambda_1, \mu_1, K_{P2}, K_{I2}, K_{D2}, \lambda_2, \mu_2\}$.

Table 22. Range of fractional-order controller parameter set [14].

Test system	Controller	Parameter range
WB	FO-PI	$-1 \leqslant K_{Pi}, K_{Ii} \leqslant 1$, $-1 \leqslant \lambda_i \leqslant 0,\ i = 1, 2$
	FO-PID	$-1 \leqslant K_{P1}, K_{I1}, K_{D1} \leqslant 1$, $-1 \leqslant \lambda_i \leqslant 0,\ 0 \leqslant \mu_i \leqslant 1,\ i = 1, 2$
OR	FO-PI	$-5 \leqslant K_{Pi}, K_{Ii} \leqslant 5$, $-1 \leqslant \lambda_i \leqslant 0\ i = 1, 2, 3$
	FO-PID	$-5 \leqslant K_{P1}, K_{I1}, K_{D1} \leqslant 5$, $-1 \leqslant \lambda_i \leqslant 0,\ 0 \leqslant \mu_i \leqslant 1,\ i = 1, 2$

6. Simulation results

Simulations are carried out using a PC Intel Core 2 Duo operating @2.2 GHz with 2 GB RAM. IAE is determined for step response over 150 min and 100 min time duration for Test system-I and Test system-II respectively. For simulating MIMO plants, MATLAB-SIMULINK software is used. Due to the randomness of the EAs, statistical performance of the results in 20 trials is reported. For comparison purpose, reported results of CMAES based PI/PID controller for Test system-I [13] and for Biggest Log-modulus Tuning (BLT) algorithm based PI controller for Test system-II [14]. The parameters employed for the simulations are given in Tab. 3. For fair comparison of results, maximum number of function evaluations (Fevalmax) is selected as that of [13] for Test system-I. Original MATLAB codes for CMAES algorithm is taken from the website [16].

Table 23. CMAES parameter.

Parameter	Test system-I		Test system-II	
	FO-PI	FO-PID	FO-PI	FO-PID
Design variables (n)	6	10	9	15
Population size	50	50	100	100
Fevalmax	6000	6000	15000	15000

6.1. Design of multivariable FO-PI controller for Test system-I

CMAES algorithm is applied to the design of decoupled multivariable FO-PI controller for Test system-I using IAE as objective. Best FO-PI parameter set and the corresponding IAE value in 20 independent trials and reported results of PI controller are given in Tab. 4. For comparison, reported values of PI controller [13] are also given in Tabs 4-5.

Table 24. Optimum controller parameters FO-PI controller: Test system-I.

Controller	Optimum parameter set of multivariable PI controller						IAE
	K_{P1}	K_{I1}	λ_1	K_{P2}	K_{I2}	λ_2	
PI [13]	0.8485	0.0026	-1	-0.0132	-0.0069	-1	10.4378
FO-PI	0.8760	0.0187	-0.4035	-0.0123	-0.0070	-0.9984	10.2069

Table 25. Statistical performance of FO-PI controller: Test system-I.

Controller	Best value	Mean value	Worst value	Standard deviation
PI [13]	10.4378	10.4378	10.4378	0
FO-PI	10.2069	10.2081	10.2088	5e-4

The performance of the proposed FO-PI controller is comparatively better than the PI controller in terms of IAE. The statistical performance of the proposed controller is tabulated in Tab. 5. From the Tab. 5, it is clear that CAMES algorithm gives better performance and consistency achieving IAE. The convergence characteristics of the CAMES algorithm is given in Fig. 2. Due to the self learning behavior of CMAES algorithm, convergence characteristics show large variations during the initial search. Fig. 3 shows the convergence characteristics of the parameters of the multivariable FOPI controller. In Fig.3 ‘Lambda’ represents the parameter λ. Figs. 4 and 5 shows the step response characteristics of the best multivariable FO-PI controller and PI controller designed by CMAES.

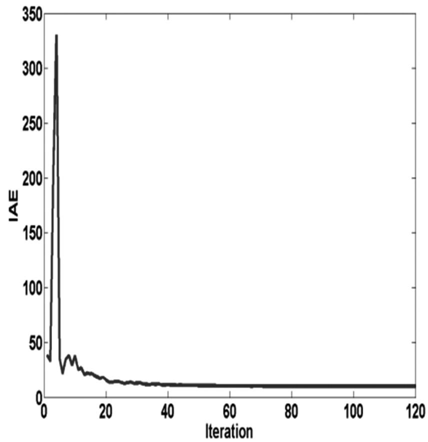

Figure 2. Convergence characteristics of CMAES algorithm for multivariable FO-PI controller: Test system-I.

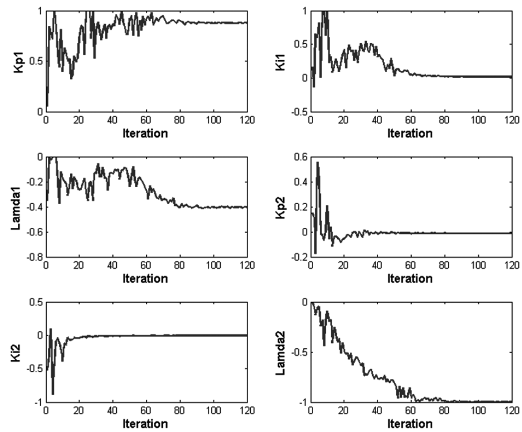

Figure 3. Convergence characteristics of multivariable FO-PI controller parameters-Test system-I.

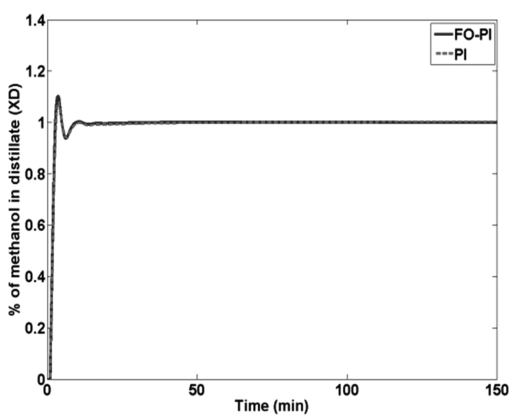

Figure 4. Step response X_B for WB system with FO-PI controller.

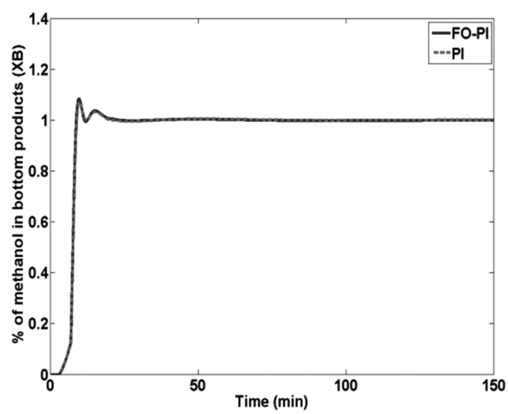

Figure 5. Step response X_D for WB system with FO-PI controller.

6.2. Design of multivariable FO-PID controller for Test system-I

Best FO-PID parameter set and the corresponding IAE value of decoupled multivariable FO-PID controller using CMAES algorithm are reported in Tab. 6. For comparison, reported values of PID controller [13] are also given in Tabs 6-7.

From the Tab. 6, it is clear that the performance of the proposed FO-PID controller is better than the PID controller in terms of IAE. The statistical performance of the FO-PID controller in 20 trials is tabulated in Tab. 7. From the Tab. 7, it is clear that the statistical performance of the CMAES algorithm based design of FO-PID gives better performance in consistency achieving good results of IAE. Figs. 6 and 7 show the step response characteristics of the best multivariable FO-PID controller and PID controller. From the Figs. 6 and 7, it is clear that the rise time of the response X_D and X_B are improved by 13% and 76% with the compromise in the overshoot and settling time. The improvement in rise time is due to inclusion of parameter μ.

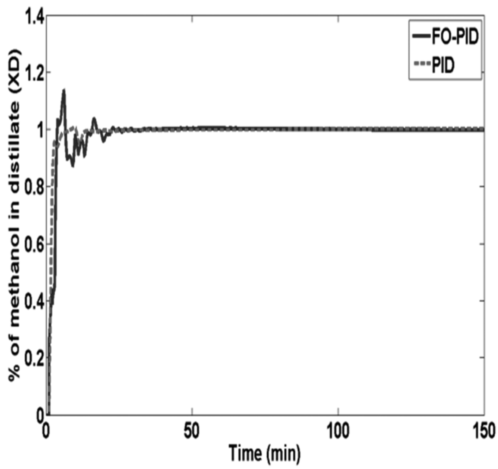

Figure 6. Step response X_D for WB system with FO-PID controller.

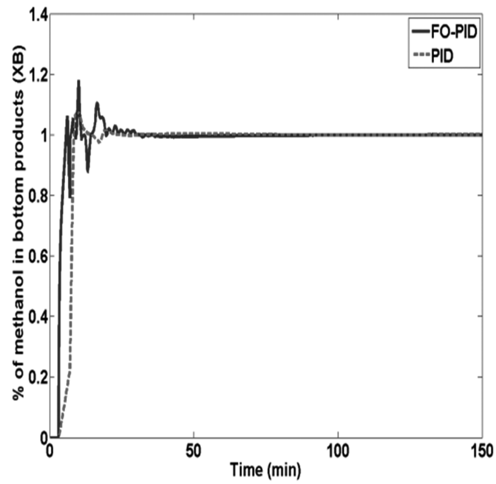

Figure 7. Step response X_B for WB system with FO-PID controller.

Table 26. Optimum FO-PID controller parameters: Test system-I.

Controller	Parameters					IAE
PID [13]	K_{P1}	K_{I1}	K_{D1}	λ_1	μ_1	9.6824
	1.0	0.0025	0.3872	-1	1	
	K_{P2}	K_{I2}	K_{D2}	λ_2	μ_2	
	-0.0332	-0.0073	-0.0909	-1	1	
FO-PID	K_{P1}	K_{I1}	K_{D1}	λ_1	μ_1	8.2549
	0.2714	0.0241	0.3217	-0.3092	1.0	
	K_{P2}	K_{I2}	K_{D2}	λ_2	μ_2	
	-0.1373	-0.0138	-0.1413	-1.0	0.5207	

Table 27. Statistical performance of FO-PID controller: Test system-I.

Controller	Best value	Mean value	Worst value	Standard deviation
PID [13]	9.6824	10.4451	–	0.7116
FO-PID	8.2549	8.3898	8.4393	0.0711

6.3. Design of multivariable FO-PI/FO-PID controller for Test system-II

Best FO-PI/FO-PID parameters and their corresponding IAE value are reported in Tab. 8. For comparison, reported results of PI controller using BLT method [14] and CMAES based PI/PID controllers are also reported in Tabs 8-9. From the Tab. 8, it is clear that the performance of the proposed decoupled multivariable FO-PI / FO-PID controller is comparatively better than the PI/PID controller designed by conventional BLT method and CMAES algorithm. From the statistical performance of the proposed controller as given in Tab. 9, it is clear that the CMAES algorithm gives better performance and consistency in achieving better results for multivariable systems.

Fig. 8 shows the convergence characteristics of the CMAES algorithm based FO-PID controller. The initial higher values of IAE during generations are omitted for clarity purposes. Fig. 9 shows the convergence characteristics of the multivariable FO-PID controller parameters. In Fig. 9 ‘Mu’ represents the parameter μ.

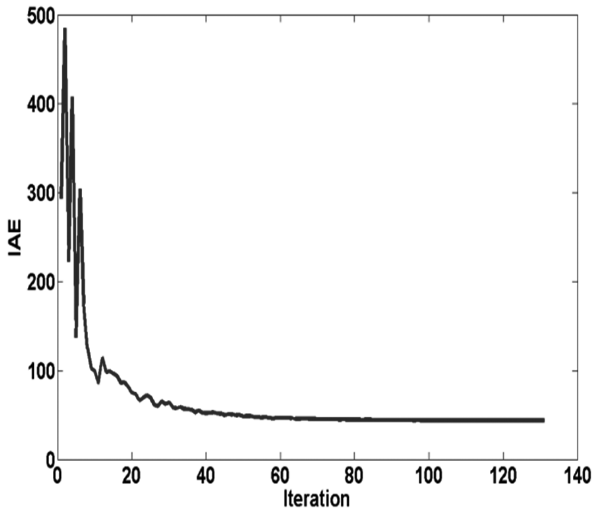

Figure 8. Convergence characteristics of CMAES algorithm for multivariable FO-PID controller: Test system-II.

Figs. 10, 11 and 12 shows the step response of the best multivariable FO-PI controller. FO-PI controller simulation results show the improvement in both in rise time

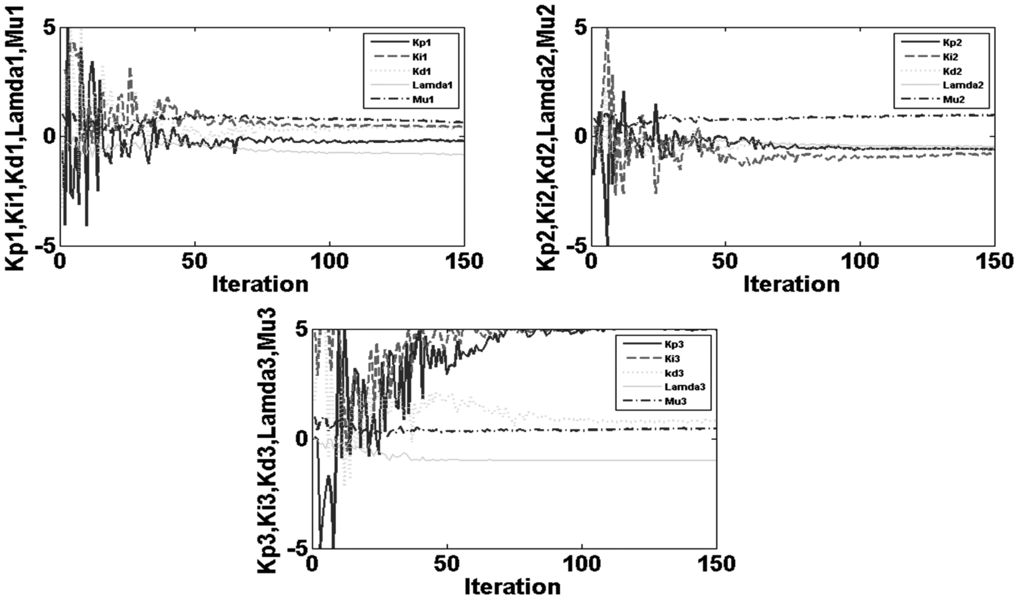

Figure 9. Convergence characteristics of parameters of the multivariable FO-PID controller: Test system-II.

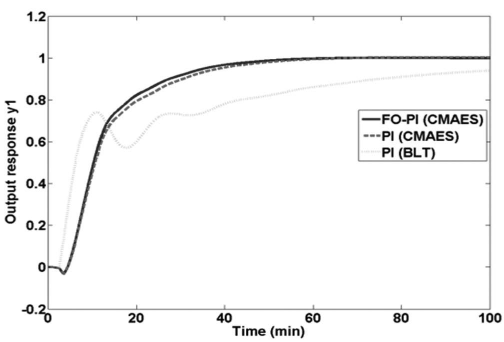

Figure 10. Step response y_1 for OR system with FO-PI controller.

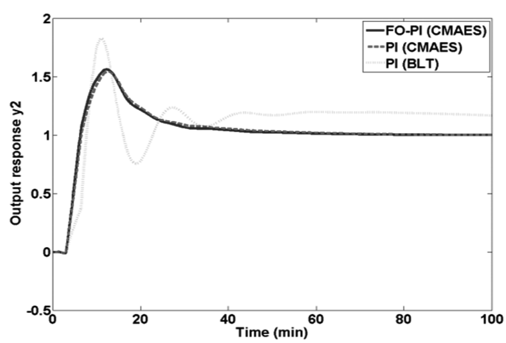

Figure 11. Step response y_2 for OR system with FO-PI controller.

and settling time. Figs. 13, 14 and 15 show the step response of the best multivariable FO-PID controller. Simulation results of FO-PID controller improves the rise time by 33%, overshoot by 9% and settling time by 26% than CMAES based PID controller for output response y_2.

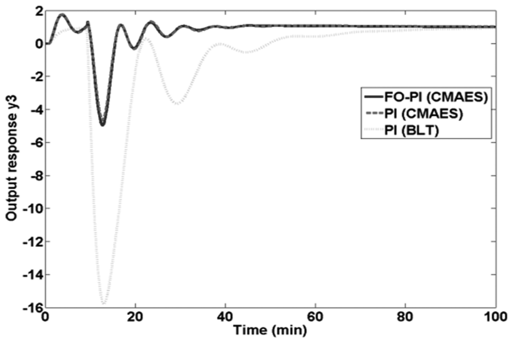

Figure 12. Step response y_3 for OR system with FO-PI controller.

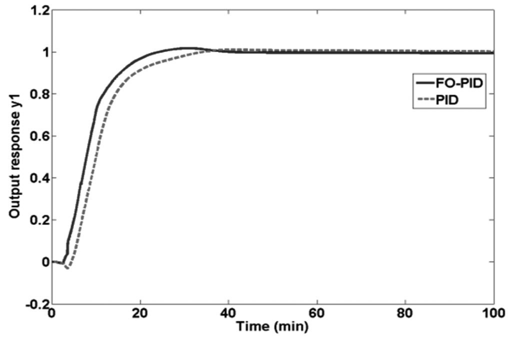

Figure 13. Step response y_1 for OR system with FO-PID controller.

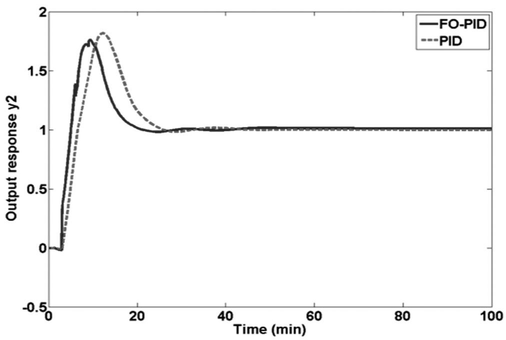

Figure 14. Step response y_2 for OR system with FO-PID controller.

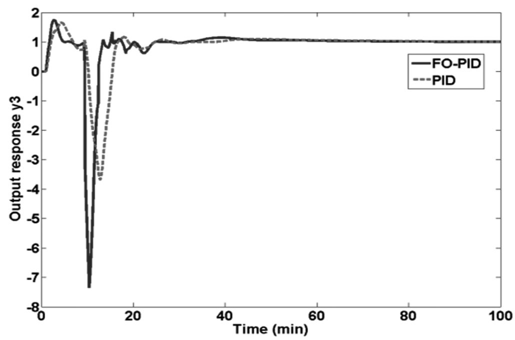

Figure 15. Step response y_3 for OR system with FO-PID controller.

7. Conclusion

Decoupled multivariable fractional order PI/PID controller based on CMAES algorithm is designed in this paper. Oustaloup integer order approximation is considered for the approximations of fractional order integrals and derivatives. Decoupled multivariable FO-PI and FO-PID controller are designed for MIMO distillation columns namely WB and OR systems. Reported results of CMAES algorithm based PI and PID controller for WB system and PI controller by BLT method for OR system are considered for comparison. The FO-PI / FO-PID controller designed by the CMAES algorithm exhibits minimum IAE as compared to PI/PID controller. Also, the proposed FO-PI/FO-PID controller achieves favorable closed loop performance.

Table 28. Optimum Controller Parameters: Test system-II.

Parameters	PI	PI	FO-PI	PID	FO-PID
Method	BLT [14]	CMAES	CMAES	CMAES	CMAES
K_{P1}	1.5070	-0.3323	-0.3663	-0.3755	-0.3717
K_{I1}	0.0920	0.1686	0.2178	0.2225	0.6211
K_{D1}	–	–	–	2.5362	0.2264
λ_1	-1	-1	-0.9206	-1	-0.6811
μ_1	–	–	–	1	0.3719
K_{P2}	-0.2930	-0.5964	-0.6279	-0.5132	-0.4813
K_{I2}	-0.0163	-0.1320	-0.1456	-0.1791	-1.1370
K_{D2}	–	–	–	-0.3064	-0.7118
λ_2	-1	-1	-0.9886	-1	-0.3678
μ_2	–	–	–	1	0.9731
K_{P3}	2.6326	5.0000	4.9996	5.0000	4.9232
K_{I3}	0.3975	3.1980	4.9998	5.0	5.0
K_{D3}	–	–	–	5.0000	1.0540
λ_3	-1	-1	-0.7971	-1	-1.0
μ_3	–	–	–	1	0.4218
IAE	245.4433	62.1182	61.0379	48.9158	43.1575

Table 29. Statistical performance: Test system-II.

Controller	Best value	Mean value	Worst value	Standard deviation
PI	62.1182	62.1182	62.1182	7.2e-7
FO-PI	61.0379	61.0422	61.0450	2.3e-3
PID	48.9158	48.9158	48.9158	5.3e-6
FO-PID	43.1575	43.8588	44.4412	0.4127

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